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《Technical Note》

A New Design Procedure for the Evaluation of Rod Bow DNBR Penalty

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Abstract

In the thermal-hydraulic design, the effect of fuel rod bow is quantified by the rod bow DNBR penalty which is a key design parameter to assure the coolability of fuel assembly in the pressurized water reactor. In this work, a computer program for the evaluation of the rod bow DNBR penalty based on Westinghouse methodology is developed and its application procedure is proposed. The computer simulation is based on the Monte-Carlo method. The qualification of developed computer program is performed by a comparison of calculational results with that given by Westinghouse's document. A new application procedure is built using batch mean and batch standard deviation. The normality of sample population generated by the batch calculation is confirmed by means of a chi-square test for goodness of fit. On the view point of statistics it is expected that the more reliable design value may be produced by the new application procedure.

1. Introduction

During reloading of LOPAR(low parasitic) 14×14 fuel assembly in 1972, severe fuel rod bowing was found in the PWR's (Pressurized Water Reactors) built by Westinghouse. The thimble tubes of LOPAR fuel assembly were made of Zircaloy and the fuel rods are supported by grid springs, while those of HIPAR(high parasitic) fuel assembly made of stainless steel and the fuel rods stand freely on the bottom nozzle of fuel assembly. In 1975, a rod bow correlation as a function of burnup was developed using rod bow data base. In 1976, a report describing the evalulation of fuel rod bow effects on DNBR (Departure from Nucleate Boiling Ratio) and power peaking factor was submitted to U.S. NRC (Nuclear Regulation Committee). An approved report of Westinghouse was finally published in 1979 after a long and elaborated discussion with NRC, which was titled by "Fuel Rod Bow Evaluation." [1]

This paper develops a computer program which calculates the rod bow DNBR penalty, qualifies the computer program, and establishes a new design procedure to be used in core design. The design procedure of Westinghouse needs to be modified on the point of view of statistics in order to obtain the more reliable design value of the rod bow DNBR penalty.

Section 2 reviews briefly the Westinghouse model of fuel rod bow evaluation. Section 3 describes the Monte-Carlo method to develop a computer program. Section 4 shows some results and discussion, and then establishes a new design procedure. The conclusion is drawn in Section 5.

2. Fuel Rod Bow Evaluation Model of Westinghouse

2.1. Correlation of Channel Closure

In order to quantify the decrease of flow channel area due to rod bowing, a variable "channel closure" is introduced. It is defined as a ratio of measured gap to normal gap as a function of burnup. And then, the standard deviation of channel closure is calculated and is fitted using a first order linear function as follows :

$$s_{be} = A_1 + B_1\mu, \tag{1}$$

where, s_{be} is the best-estimated standard deviation of channel closure, A_1 and B_1 are fitting constants, and μ is burnup. The probabilistic density function of the channel closure is assumed to be a normal distribution function after a proper normality test. A 95% tolerance limit of the standard deviation of channel closure, s_w, is evaluated depending on number of channel closure measurements :

$$s_w = A_2 + B_2\mu, \tag{2}$$

where, A_2 and B_2 are constants. Since the measurement of channel closure is usually done in spent fuel storage pool, the standard deviation of channel closure is modified using a multiplication factor, f_m, to account for the high pressure and high temperature reactor core condition, which results in :

$$s_{w,core} = f_m s_w, \tag{3}$$

where, f_m is a constant greater than 1. In case when the channel closure measurement data are not available, e.g., an advanced fuel which is not commercially used yet, a scaling factor may be applicable to predict the standard deviation of channel closure : [1]

$$Scaling\ Factor = L^2/I, \tag{4}$$

where, L is distance between grids of fuel assembly and I is the moment of inertia of fuel cladding.

2.2. DNB Variation by Rod Bow

The reduction of flow area due to fuel rod bow deteriorates coolability to fuel rod, and can decrease the CHF (Critical Heat Flux). The reduction of CHF as a function of channel closure is estimated by means of CHF test on 4×4 rod bundle. [1] The experimental results are quantified using following parameter :

$$\delta_{bow} = \frac{(M/P)_{no\ bow} - (M/P)_{bow}}{(M/P)_{no\ bow}}, \tag{5}$$

where,
P = predicted critical heat flux on unbowed rod bundle,
M = measured critical heat flux,
no bow = condition where rods are not bowed,
bow = condition where rods are bowed.

A model for δ_{bow} obtained by an experiment plotted in Figure 1. As shown in Figure 1, there is no change in CHF characteristics if the channel closure is less than 0.5. If the channel closure is 1.0, i.e., two adjacent rods are contacted each other, the CHF reduction is maximized. Figure 1 gives also δ_{bow} value

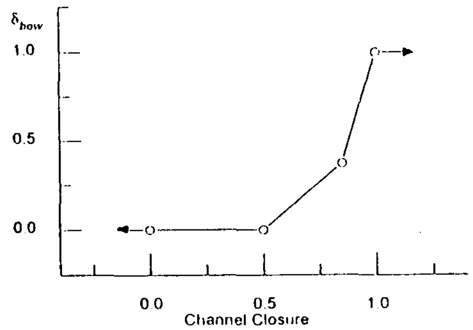

Fig. 1. Variation of δ_{bow} as a Function of Channel Closure

for the channel closure is greater than 1. 0 and less than 0.0, since the probabilistic distribution of channel closure can be distributed from $-\infty$ to $+\infty$ in statistics.

2.3. Determination of DNBR Limit

The DNBR limit equation to assure that DNB will not occur at 95% probability and 95% confidence level is determined using following equation :

$$DNBR\ Limit = \frac{1}{\overline{(M/P)} - K_{95x95}\,\sigma(M/P)}, \qquad (6)$$

where, $\overline{(M/P)}$ = mean value of the ratio of measured to predicted CHF value,

$\sigma\overline{(M/P)}$ = standard deviation of (M/P)

$$= \left[\frac{\sum_{i=1}^{n}\{\overline{(M/P)} - (M/P)_i\}^2}{n-1}\right]^{1/2},$$

n = number of data points used in CHF correlation development,

K_{95x95} = Owen's factor. [2]

The DNBR limit for the bowed fuel bundle is determined by the same manner as Eq.(6) :

$$DNBR\ Limit\ (B) = \frac{1}{\overline{(M/P)}_B - K^B_{95x95}\,\sigma_B(M/P)}, \qquad (7)$$

where, the meaning of $\overline{(M/P)}_B$, K^B_{95x95} and $\sigma_B(M/P)$ will be explained in the following section. Finally, the rod bow DNBR penalty(RBP) is determined using the DNBR limit values obtained by Eq.(6) and Eq.(7) :

$$RBP = \frac{DNBR\ Limit\ (B) - DNBR\ Limit}{DNBR\ Limit\ (B)}. \qquad (8)$$

2.4. Determination of $\overline{(M/P)}_B$ K^B_{95x95} and $\sigma_B(M/P)$

As described in the section 2.1, the channel closure is assumed to be normally destributed with standard deviation $s_{w,core}$ and mean 0. Therefore, the effect of rod bow to DNBR is also described as a probabilistic density function which can be evaluated using the Monte-Carlo method. From Eq.(3), a channel closure is selected randomly and is applied in Figure 1 to determine corresponding δ_{bow} value. This two step procedure is repeated sufficiently to generate statistically reliable mean $(\bar{\delta})$ and standard deviation (σ_δ) of δ_{bow}. From the definition of δ_{bow} as given in Eq.(5), $\overline{(M/P)}_B$, $\sigma_B(M/P)$ and the degree of freedom to evaluate K^B_{95x95} can be calculated using following relationships : [1]

$$\overline{(M/P)}_B = \overline{(M/P)}\,(1-\bar{\delta}), \qquad (9)$$

$$\sigma_B^2 = (1-\bar{\delta})^2\,[\sigma(M/P)]^2 + \overline{(M/P)}^2\,\sigma_\delta^2, \qquad (10)$$

$$n_B - 1 = \frac{\sigma_B^4}{\frac{(1-\bar{\delta})^4\,[\sigma(M/P)]^4}{n-1} + \frac{\overline{(M/P)}^4\,\sigma_\delta^4}{n_\delta - 1}} \qquad (11)$$

where, n = number of data points used in CHF correlation development,

n_δ = number of data points used in the channel closure correlation development,

$n_B - 1$ = degree of freedom.

3. Computation with the Monte-Carlo Method

3.1. Monte-Carlo Simulation

The probabilistic density function of the channel closure is assumed to be a normal distribution function with standard deviation $s_{w,core}$ at a given burnup :

$$f(x) = \frac{1}{\sqrt{2\pi}\,s_{w,core}} \exp\left[-\frac{1}{2}\left(\frac{x}{s_{w,core}}\right)^2\right], \qquad (12)$$

where, X is the channel closure.

In order to select randomly the channel closure in the Monte-Carlo simulation, the probabilistic density function (Eq.(12)) should be integrated to produce a cumulative density function :

$$F(x) = \int_{-\infty}^{x} f(x')\,dx'. \qquad (13)$$

Since there is not an analytical solution for Eq. (13), an approximate integration function may be utilized. Once an integration function is selected, the channel closure is obtained by the inverse function of Eq.(13):

$$channel\ closure = F^{-1}(RN), \quad (14)$$

where, RN is a bigger number chosen from two random numbers which are fairly and uniformly distributed from 0 to 1. The negative value of random number is out of our interest, because it corresponds to increase of rod gap. It is the reason why a bigger number is selected from two random number that when a rod is bowed there are two gaps decreased with respect to the normal gap at horizontal plain of fuel assembly. The rod bow DNBR penalty is a unique value for a bowed rod, so it should be calculated on the bigger channel closure (small rod gap).

Once δ_{bow} value is evaluated for an arbitrary rod using the randomly generated channel closure which comes from the cumulative density function, it is stored. And then another δ_{bow} value is evaluated for another rod. This procedure is repeated to accumulate sufficient number of δ_{bow} values, and then the mean $(\overline{\delta})$ and standard deviation (σ_δ) of δ_{bow} is estimated to calculated the rod bow DNBR penalty.

Because the Monte-Carlo simulation is a time consuming procedure, some efficiency enhancement sampling techniques, including importance sampling, may be utilized. [3]

3.2. Confidence of Random Sampling

In the Monte-Carlo method, a question whether the random numbers are fairly generated without bias is difficult to be answered. The statistical characteristics of random number set is, in general, dependent on random number generator. For example, the numerical generation of random number using digital computer is dependent on the seed value which is selected arbitrarily by a user. The rod bow DNBR penalty is, in turn, also dependent on the characteristics of random number set. On this regard, the batch calculation is tried on this study which is not the stanard design procedure of Westinghouse method. [1] A rod bow DNBR penalty is calculated in a batch, and another rod bow DNBR penalty in another batch, and so on. In each batch, the seed value of random number generator is changed arbitrarily to produce different set of random numbers.

4. Results and Discussion

4.1. Sensitivity Study

For a sensitivity study on the rod bow DNBR penalty calculation, a reference case is selected as 35 GWD/MTU burnup, 0.374 inch rod with 20 inches span between spacer grids, WRB-2 CHF correlation, [4] 100,000 generations of random samples and 10 batches.

The variation of rod bow DNBR penalty to the number of random samples is investigated. As shown

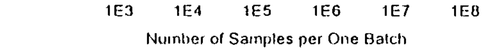

Fig. 2. Standard Deviation of Rod Bow DNBR Penalty as a Function of Sample Size (lEl to be read as 1.0×10^1)

Table 1. Mean and Standard Deviation of Rod Bow DNBR Penalty as a Function of the Number of Batches

Number of Batches	Mean Value	Standard Deviation
10	5.430	0.050
100	5.434	0.053
1,000	5.423	0.057
10,000	5.425	0.059
100,000	5.425	0.059

in Figure 2, the standard deviation of rod bow DNBR penalty is decreased as the number of random samples is increased.

For the given number of samples as defined in the reference case, the number of batches is varied and the result is shown in Table 1. Table 1 shows that the mean and standard deviation of the rod bow DNBR penalty is a few affected by the number of bat ches. The table shows that from 1000 batches the values become stabilized.

The optimum combination of the number of random samples and that of batches is investigated in Figure 3. The figure shows that for the given number of total samples (i.e., number of random samples multiplied by number of batches), increase of the number of random samples gives better results than increase of the number of batches.

Figure 4 shows the calculations with various CHF correlations. There exists some difference between CHF correlation families, i.e., W-3 family[5, 6] and

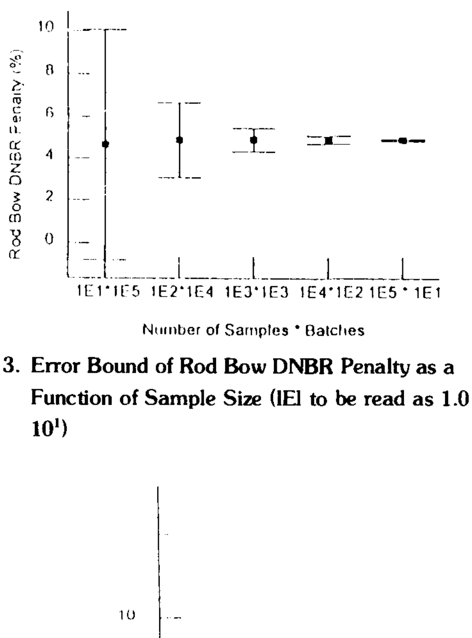

Fig. 3. Error Bound of Rod Bow DNBR Penalty as a Function of Sample Size (1E1 to be read as 1.0×10^1)

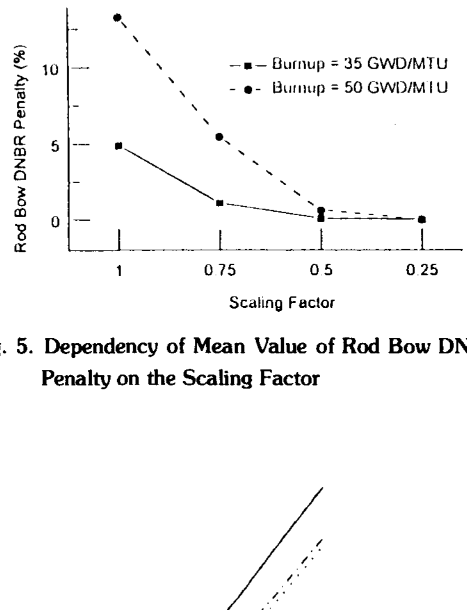

Fig. 5. Dependency of Mean Value of Rod Bow DNBR Penalty on the Scaling Factor

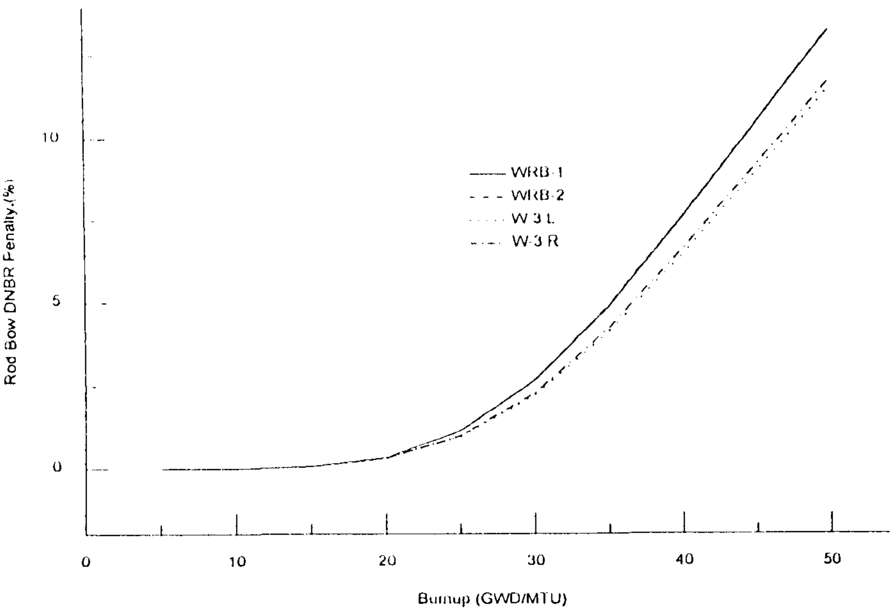

Fig. 4. Mean Values of Rod Bow DNBR Penalty Evaluated by Various CHF Correlations

WRB family[4, 7]. However, there is no notable difference within same family.

Figure 5 shows the results with the scaling factor given in Eq.(4). The rod bow DNBR penalty is very sensitive to the selected scaling factor. In case when the grid span is reduced by half of the normal span which corresponds to the scaling factor to be 0.25, the rod bow DNBR penalty is practically zero up to 50 GWD/MTU.

4.2. Qualification of Computer Program

Figure 6 shows the results of qualification for the program developed. Consistency between the rod bow DNBR penalty given by Westinghouse[1] and that calculated by the program developed in this study is excellent.

4.3. Modified Design Procedure

As discussed in the section 4.1, the bigger number of random samples gives the smaller standard deviation of the rod bow DNBR penalty. However, for a core to be analyzed, there is finite number of fuel rods. A typical 900 MWe PWR contains about 35, 000 fuel rods whose probability density of channel closure may be given by Eq.(3). For the population of 35,000 fuel rods. the rod bow DNBR penalty with acceptable confidence level should be drawn. This problem may be simplified assuming that the burnup of a hot rod is supposed to be the pre-defined design burnup. The higher design burnup results in the bigger rod bow DNBR penalty as shown in Figure 6. The highly burned rod, however, becomes less probable to be the hot rod because of burn out of fissile materials. So there may exist a optimum value for the design burnup.

Besides the number of rods and design burnup, the number of batches should be determined. According to Table 1, the optimum number of batches may be selected as 1,000 without losing statistical confidence and with reasonable computing cost.

To test the hypothesis that random samples come from a population having a normal distribution, the data may be fitted to a normal curve and then be tested to see whether the hypothesis is justified. The tes

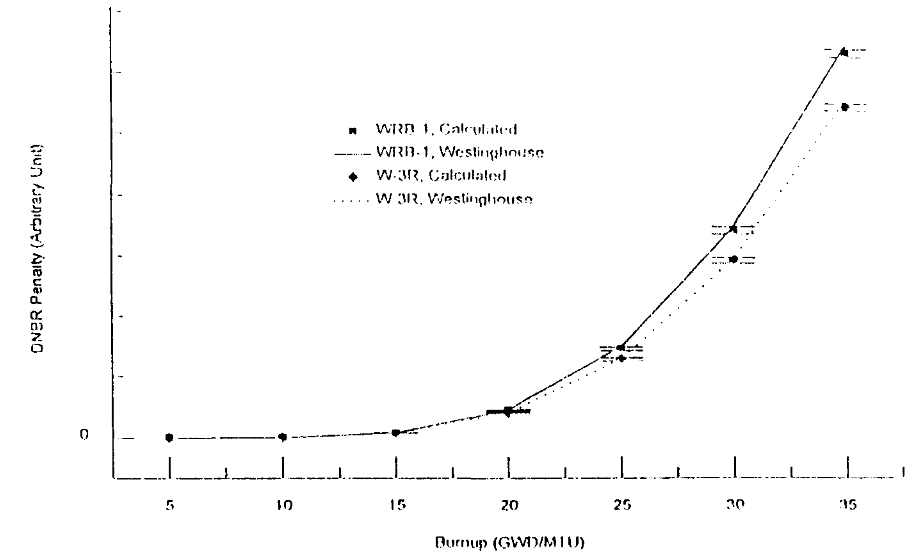

Fig. 6. Comparision of Calculated Results with Westinghouse's Results

Table 2. Chi-square Test of Normality on Grouped 1000 Batches RBP Values

RBP Range	Sampled density, f_i	Theoretical density, F_i	$\frac{(f_i - F_i)^2}{F_i}$
$-\infty \sim 5.239$	29	30.98	0.126
5.239~5.269	32	29.35	0.139
5.269~5.300	41	47.09	0.788
5.300~5.330	65	68.74	0.204
5.330~5.361	101	91.29	1.033
5.361~5.392	114	110.30	0.124
5.392~5.422	112	121.23	0.704
5.422~5.453	129	121.23	0.499
5.453~5.483	105	110.30	0.254
5.483~5.514	99	91.29	0.651
5.514~5.545	66	68.74	0.110
5.545~5.575	42	47.09	0.551
5.575~5.606	34	29.35	0.726
$5.606 \sim +\infty$	30	30.98	0.031
Sum Total	1000	1000.00	6.049

mean = 5.422, standard deviation = 0.099, d.f. = 11, $\chi^2_{11,05} = 19.68$

ting may be done by means of a chi-square test for goodness of fit [8], in which case the mean and standard deviation for the fitted normal curve are estimated from the grouped sample data. To confirm the normality of 1,000 populations of rod bow penalty at 35 GWD/MTU burnup, the chi-square test is done as shown in Table 2.

As given in Table 2, the summation of the value in column 4 is far smaller than $\chi^2_{11,05}$. Therefore, it is concluded that at the 5% level of significance the sample distribution is consistent with the hypothesis that the parent distribution is normal.

Assembling the above studies, a new design procedure is established as follow:

(1) Identify the target fuel assembly and reactor core in order to determine the CHF correlation and number of fuel rods.

(2) Determine the design burnup and number of batches.

(3) Calculate the rod bow DNBR penalty for each batch and evaluate the batch mean (μ_{batch}) and standard deviation (σ_{batch}).

(4) Evaluate the rod bow DNBR penalty from following equation ;

$$RBP = \mu_{batch} + K_{95x95}\,\sigma_{batch}, \qquad (15)$$

where, K_{95x95} is Owen's factor. [2] For the case shown in Table 2, the rod bow DNBR penalty obtained by the new design procedure is to be 5.593%.

5. Conclusions

A computer program to evaluate the rod bow DNBR penalty based on a Westinghouse methodology is developed and qualified. Upon the program developed, a new design procedure is proposed introducing a batch calculation on an actual number of fuel rods in the reactor core. The normality of sample population generated by the batch calculation is confirmed by means of a chi-square test for goodness of fit. The new design procedure is expected to generate statistically the more confident design value.

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