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Onset of Slugging Criterion Based on Singular Points and Stability Analyses of Transient One-Dimensional Two-Phase Flow Equations of Two-Fluid Model

Chang-Kyung Sung and Moon-Hyun Chun
Korea Advanced Institute of Science and Technology
Department of Nuclear Engineering
373-1 Kusong-dong, Yusong-gu, Taejon 305-701, Korea
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Abstract

A two-step approach has been used to obtain a new criterion for the onset of slug formation : (1) In the first step, a more general expression than the existing models for the onset of slug flow criterion has been derived from the analysis of singular points and neutral stability conditions of the transient one-dimensional two-phase flow equations of two-fluid model. (2) In the second step, introducing simplifications and incorporating a parameter into the general expression obtained in the first step to satisfy a number of physical conditions a priori specified, a new simple criterion for the onset of slug flow has been derived. Comparisons of the present model with existing models and experimental data show that the present model agrees very closely with Taitel & Dukler's model and experimental data in horizontal pipes. In an inclined pipe ($\theta = 50°$), however, the difference between the predictions of the present model and those of existing models is appreciably large and the present model gives the best agreement with Ohnuki et al.'s data.

1. Introduction

Because of its importance in thermal-hydraulic designs of two-phase flow systems, the phenomenon of the transition from a stratified wavy to a slug flow has been studied by many investigators[1-9] in the past 25 years since Kordyban and Ranov[1] first analyzed it for water and air between horizontal parallel plates.

If the gas phase velocity is sufficiently high, the pressure component in phase with the wave becomes so large that the resulting suction overcomes the downward forces on the liquid, and the wave motion changes from periodic to essentially exponential. This is known, in general, as a Kelvin-Helmholtz instability. In the case of the two-phase flow in a closed conduit, in particular, this effect is significantly enhanced by the proximity of the upper wall. Kordyban and Ranov[1] proposed that the formation of slugs in the two-phase flow is due to a modified Kelvin-Helmholtz instability of the waves. A review of the literature of two-phase flow applications shows that the most widely used correlation to predict the initiation of the slug flow and flooding is the correction to the Kelvin-Helmholtz inviscid theory proposed by Taitel and Dukler[3].

The main purpose of this paper is to present the two-step approach used to extend the method of characteristics and stability analyses of one-dimensional two-phase flow equations presented by Lyczkowski et al. [10] and Bilicki et al. [11] to the derivation of a new criterion for the onset of slug formation. In the first step, a more general form for the onset of slug flow criterion is derived based on the bifurcation theory (nonlinear analysis) : more specifically, analyses of singular points and neutral stability conditions of the transient one-dimensional two-phase flow equations of the two-fluid model in a manner similar to the procedure used in the derivation of a flooding correlation by Lee and No [12]. In the second step, the final form of a new criterion for the onset of slug flow has been obtained by simplification of the general expression derived in the first step and incorporation of a parameter to satisfy a number of physical conditions a priori specified.

2. Two-Phase Flow Equations

Consider a one-dimensional transient stratified two-phase flow shown in Fig. 1. The two phases are assumed to be weakly coupled co-current gas-liquid in a pipe of diameter D, (or a channel of height H and infinite width) with an inclination θ to the horizontal.

The one-dimensional, two-fluid transient model [7, 13] is formulated by considering each phase separately in terms of two sets of conservation equations governing the balance of mass and momentum of each phase as follows :

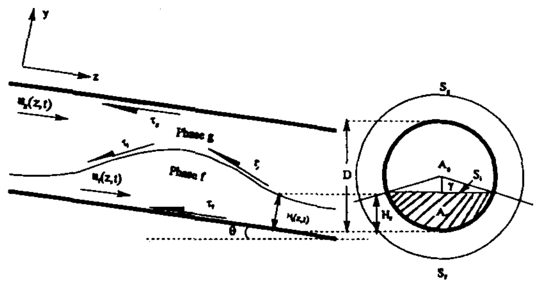

Fig. 1. Physical Model for the Onset of Slug Flow Analysis

$$\frac{\partial}{\partial t}(\alpha_f \rho_f) + \frac{\partial}{\partial z}(\alpha_f \rho_f u_f) = \Gamma_f \quad (1a)$$

$$\frac{\partial}{\partial t}(\alpha_g \rho_g) + \frac{\partial}{\partial z}(\alpha_g \rho_g u_g) = \Gamma_g \quad (1b)$$

$$\frac{\partial}{\partial t}(\alpha_f \rho_f u_f) + \frac{\partial}{\partial z}(\alpha_f \rho_f u_f^2) = \alpha_f \rho_f g \sin\theta - \frac{\partial(\alpha_f P_f)}{\partial z} + P_{if}\frac{\partial \alpha_f}{\partial z} - \tau_i \frac{\partial \alpha_f}{\partial z} + \frac{\partial}{\partial z}(\alpha_f \tau_f) + \Gamma_f (u_{if} - u_f) + M_{if} \quad (2a)$$

$$\frac{\partial}{\partial t}(\alpha_g \rho_g u_g) + \frac{\partial}{\partial z}(\alpha_g \rho_g u_g^2) = \alpha_g \rho_g g \sin\theta - \frac{\partial(\alpha_g P_g)}{\partial z} + P_{ig}\frac{\partial \alpha_g}{\partial z} - \tau_i \frac{\partial \alpha_g}{\partial z} + \frac{\partial}{\partial z}(\alpha_g \tau_g) + \Gamma_g (u_{ig} - u_g) + M_{ig} \quad (2b)$$

Here α, ρ, u, P, θ and τ_i are the volume fraction, the density, the average velocity of the two fluids, the pressure, the inclination to the horizontal, and the interfacial shear stress, respectively. Γ_k and M_{ik} are the rate of mass generation of phase k at the interface and the rate of momentum generation of phase k at the interface (which results from surface tension, and depends on the geometric state of the interface). The subscripts f and g denote liquid-phase and gas-phase, respectively, and i stands for the value at the interface. In Eqs.(1) and (2), the spatial coordinate is denoted by z and the time is denoted by t. From the interfacial momentum and mass conservation, $M_{if} + M_{ig} = 0$ and $\Gamma_f + \Gamma_g = 0$.

In the present derivation of the onset of slugging criterion based on characteristics and stability analyses of the transient one-dimensional two-fluid formulations of two-phase flow, the energy conservation equations are not included since the effect of thermal energy transfer on the rapid flow regime transition phenomenon such as the onset of slugging can be considered to be a secondary factor in comparison with the effect of mass and momentum transfer on the flow regime transition.

The average pressures P_f and P_g differ from the corresponding values at the common interface P_{if} and P_{ig} since the pressure of each phase varies due to gravity. Also, P_{if} and P_{ig} may be different due to surface tension effects. The average pressure of each phase, in terms of its pressure level at the liquid-gas interface, $y=H_f$, is given by [7]

$$\frac{\partial}{\partial z}(P_f\alpha_f)=\frac{\partial}{\partial z}\int_0^{\alpha_f}[P_{if}+\rho_f g\cos\theta(H_f-y)]d\alpha_f$$
$$=\frac{\partial}{\partial z}(P_{if}\alpha_f)+\rho_f g\cos\theta\alpha_f\frac{\partial H_f}{\partial z} \quad (3a)$$
$$=\alpha_f\frac{\partial}{\partial z}(P_{if})+P_{if}\frac{\partial}{\partial z}(\alpha_f)+\rho_f g\cos\theta\alpha_f\frac{\partial H_f}{\partial z}$$

and

$$\frac{\partial}{\partial z}(P_g\alpha_g)=\frac{\partial}{\partial z}\int_0^{\alpha_g}[P_{ig}-\rho_g g\cos\theta(y-H_f)]d\alpha_g$$
$$=\frac{\partial}{\partial z}(P_{ig}\alpha_g)+\rho_g g\cos\theta\alpha_g\frac{\partial H_f}{\partial z} \quad (3b)$$
$$=\alpha_g\frac{\partial}{\partial z}(P_{ig})+P_{ig}\frac{\partial}{\partial z}(\alpha_g)+\rho_g g\cos\theta\alpha_g\frac{\partial H_f}{\partial z}$$

The spatial derivative of the interfacial pressure difference of the two phases (i.e., $\Delta P_i=P_{ig}-P_{if}$) is a function of surface tension and curvature of the interface (surface) and ΔP_i can be represented in terms of the principal curvatures of the interface R_1 and R_2 as follows :

$$\Delta P_i=\sigma(\frac{1}{R_1}+\frac{1}{R_2}) \quad (3c)$$

The curvature of the liquid-gas interface, on the other hand, is related to void fraction. Therefore, spatial derivative of the interfacial pressure force ($\partial\Delta P_i/\partial z$) can be expressed as

$$\frac{\partial}{\partial z}\Delta P_i=\frac{\partial\Delta P_i}{\partial\alpha_g}\frac{\partial\alpha_g}{\partial z} \quad (4)$$

and

$$\frac{\partial}{\partial z}P_{if}=\frac{\partial}{\partial z}P_{ig}-\frac{\partial}{\partial z}\Delta P_i \quad (5)$$

The spatial derivative of the interfacial pressure difference in Eq.(5) is given by

$$\frac{\partial}{\partial z}\Delta P_i=\frac{\partial}{\partial z}[\sigma(\frac{1}{R_1}+\frac{1}{R_2})] \quad (6)$$

For a separated flow in a tube it can be shown that [13]

$$\tau_i\frac{\partial\alpha_g}{\partial z}=\tau_i\frac{S_i}{A},\qquad \tau_i\frac{\partial\alpha_f}{\partial z}=-\tau_i\frac{\partial\alpha_g}{\partial z}=-\tau_i\frac{S_i}{A} \quad (7)$$

where S_i is the wetted perimeter of the interface. In the present study, the phasic Reynold stresses (i.e., viscous stress) for the separated flow can be modeled as follows :

$$\frac{\partial(\tau_k\alpha_k)}{\partial z}=-\tau_k\frac{S_k}{A} \quad (8)$$

Referring to an inclined stratified gas-liquid flow in a pipe of diameter D shown in Fig. 1, $\partial H_f/\partial z$ terms in Eq.(3) can be expressed as (Appendix I) :

$$\frac{\partial H_f}{\partial z}=-\frac{\pi D}{4\sin\gamma}\frac{\partial\alpha_g}{\partial z} \quad (9)$$

Noting that $\Gamma_g=-\Gamma_f=\Gamma$, the mass conservation equations for liquid and gas, Eqs.(1a) and (1b), can be rewritten as

$$\frac{\partial}{\partial t}(\alpha_f)+\frac{\partial}{\partial z}(\alpha_f u_f)=-\Gamma/\rho_f \quad (10)$$

$$\frac{\partial}{\partial t}(\alpha_g)+\frac{\partial}{\partial z}(\alpha_g u_g)=\Gamma/\rho_g \quad (11)$$

Substituting the above relations, Eqs.(3a)~(9), into Eqs.(2a) and (2b), and multiplying Eqs.(2a) and (2b) by and α_g and α_f, respectively, and subtracting the latter from the former, one can obtain the combined momentum equation for two-phase flow as follows :

$$\alpha_f\alpha_g\left[\rho_f\frac{\partial u_f}{\partial t}-\rho_g\frac{\partial u_g}{\partial t}\right]$$
$$+\alpha_f\alpha_g\left[\rho_f u_f\frac{\partial u_f}{\partial z}-\rho_g u_g\frac{\partial u_g}{\partial z}\right]-F\frac{\partial\alpha_g}{\partial z}=G \quad (12)$$

where F and G are defined as

$$F\equiv\alpha_f\alpha_g[(\frac{\partial\Delta P_i}{\partial\alpha_g})+\frac{\pi}{4}\frac{\Delta\rho g D\cos\theta}{\sin\gamma}], \quad (13)$$

$$G\equiv\alpha_f\alpha_g\Delta\rho g\sin\theta-\tau_f\frac{S_f}{A}\alpha_g+\tau_g\frac{S_g}{A}\alpha_f+\tau_i\frac{S_i}{A}(\alpha_g+\alpha_f)$$
$$-\alpha_g\Gamma(u_{if}-u_f)-\alpha_f\Gamma(u_{ig}-u_g) \quad (14)$$

and the constitutive relations for shear stresses, $\Delta\rho$ and Γ and are given by the following :

$$\tau_f=\frac{f_f}{2}\rho_f|u_f|u_f \quad (15a)$$

$$\tau_g = \frac{f_g}{2}\rho_g|u_g|u_g \quad (15b)$$

$$\tau_i = \frac{f_i}{2}\rho_g|u_r|u_r \quad (15c)$$

$$\Delta\rho = \rho_f - \rho_g \quad (16)$$

$$\Gamma = \frac{Q_w}{i_{fg}} \quad (17)$$

In Eq.(15c), denotes the relative velocity between the two-phase (i.e., $u_r = u_g \pm u_f$), where the positive sign is applicable for countercurrent flow whereas the negative sign is applicable for cocurrent.

3. Irregular Singular Points and Hyperbolicity Breaking for Transient One-Dimensional Two-Phase Flow Equations

The mathematical models of the above transient one-dimensional two-phase flow equations, i.e., Eqs. (10), (11), and (12) can be brought to the form of a system of first-order, quasi-linear, partial differential vector equation as follows :

$$A_{ij}(X_j)\frac{\partial X_j}{\partial t} + B_{ij}(X_j)\frac{\partial X_j}{\partial z} = C_j \quad (18)$$

Here the matrices and A_{ij} and B_{ij} the column vectors X_{ij} and C_{ij} are defined as follows :

$$A_{ij} = \begin{bmatrix} -\alpha_g\alpha_f\rho_g & \alpha_g\alpha_f\rho_f & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} \quad (19a)$$

$$B_{ij} = \begin{bmatrix} -\alpha_g\alpha_f\rho_g u_g & \alpha_g\alpha_f\rho_f u_f & -F \\ \alpha_g & 0 & u_g \\ 0 & \alpha_f & -u_f \end{bmatrix} \quad (19b)$$

$$X_j = \begin{bmatrix} u_g \\ u_f \\ \alpha_g \end{bmatrix} \quad (19c)$$

$$C_j = \begin{bmatrix} G \\ \Gamma/\rho_g \\ -\Gamma/\rho_f \end{bmatrix} \quad (19d)$$

It may be noted here that Eq.(18) is similar in form to Lyczkowski et al.'s Eq.(A1) [10]. Here the 3×3 square matrices A_{ij} and B_{ij} and the column vector C_j depend only on (X_j, t, z), whereas X_j is a column vector of 3 dependent variables of two-phase flow dynamic quantities (i.e., u_g, u_f, and α_g) which are functions of two independent variables z and t.

Introducing the new coordinate $\xi = \lambda t + z$, Equation (18) can be transformed into the following :

$$(A_{ij}\lambda + B_{ij})\frac{\partial X_j}{\partial \xi} = C_j \quad (20)$$

For this hyperbolic equation, the characteristics must be real and non-zero. To find the condition under which Eq. (18) becomes singular points, Eq. (20) is rewritten by application of Cramer's rule as follows :

$$\frac{\partial X_j}{\partial \xi} = (A_{ij}\lambda + B_{ij})^{-1}C_j = \frac{N_j}{\Delta(X_j)} \quad (21)$$

where $\Delta(X_j)$ is determinant given by

$$\Delta(X_j) = |A_{ij}\lambda + B_{ij}| \quad (22)$$

and $N_j(X_j, \xi)$ are also determinants, each obtained from $(A_{ij}\lambda + B_{ij})$ by replacing the j-th column by C_j. The phase space of Eq.(21) is constructed of the $n+1$ dimensions which consists of the n components X_j (i.e., u_g, u_f, and α_g) and of coordinate ξ. In this phase space, there are three classes of points : (1) regular points (if $\Delta(X_j) \neq 0$), (2) turning points (if $\Delta(X_j) = 0$ and $N_j \neq 0$), and (3) singular points (if $\Delta(X_j) = 0$ and $N_j = 0$) [11] as illustrated in Appendix Ⅱ.

If $\Delta(X_j) \neq 0$, a point in the phase space is a 'regular point', whereas all points in the space which satisfy the condition are either 'turning points' or 'singular points' [11]. However, these conditions depend on the values of characteristics in Eq.(22). Lyczkowski [10] showed that most two-phase flow models proposed in the literature yield complex-valued characteristics in the practical regions of interest for the two-phase steam-water system.

There are three different types of solutions to the characteristic equation corresponding to Eq.(22) (see Eq.(24)) : (1) two different real roots (hyperbolic equation domain), (2) only one real root (parabolic equation domain), and (3) no real roots and two complex roots (elliptic equation domain). Physically, the hyperbolic equation domain represents a regular flow, the elliptic equation domain represents a different type regular flow (instability), and the parabolic equation domain represents the neutral stable state between regular flow and unstable flow [12].

If the characteristics at singular points are inflected nodes ($\lambda_1 = \lambda_2$ real), the hyperbolicity of the two-phase flow equation is broken and a regular flow pattern changes to another flow pattern. (The definition of inflected nodes is given in Appendix Ⅲ.)

For the purpose of the present work, the above mathematical background can be briefly summarized as follows : The 'irregular singular points' occur when both conditions of (1) $\Delta(X_i)$ and the hyperbolicity breaking are satisfied and (2) $N_i = 0$. The conditions of hyperbolicity breaking, on the other hand, are (1) inflected nodes ($\lambda_1 = \lambda_2$ real) and (2) parallel lines ($\lambda_1 \neq 0$, $\lambda_2 = 0$ real).

4. 'Onset of Slugging Criterion' from Analyses of Singular Point and Neutral Stability Conditions

4.1. Singular Point and Neutral Stability Conditions

Since the parabolic domain that corresponds to the single real root of Eq.(22) represents the bifurcation of the instability, this 'neutral stability condition' in addition to the condition of $\Delta(X_i) = 0$ (i.e., 'turning points' or 'singular points') is used to derive the 'onset of slugging criterion' as follows :

From Eq.(21), the gradient of the void fraction along the gradient of is given as

$$\frac{\partial \alpha_g}{\partial \xi} = \frac{N_{\alpha_g}}{\Delta(X_i)} \tag{23}$$

where

$$\Delta(X_i) = |A_{ii}\lambda + B_{ii}|$$

$$= \begin{bmatrix} -\alpha_g\alpha_l\rho_g(u_g+\lambda) & \alpha_g\alpha_l\rho_l(u_l+\lambda) & -F \\ \alpha_g & 0 & (u_g+\lambda) \\ 0 & \alpha_l & -(u_l+\lambda) \end{bmatrix}$$

$$= \alpha_l\rho_g(\lambda+u_g)^2 + \alpha_g\rho_l(\lambda+u_l)^2$$

$$-\alpha_l\alpha_g[(\frac{\partial \Delta P}{\partial \alpha_g}) + \frac{\pi}{4}\frac{\Delta\rho g D\cos\theta}{\sin\gamma}] = 0 \tag{24}$$

and

$$N_{\alpha_g} = \begin{bmatrix} -\alpha_g\alpha_l\rho_g(u_g+\lambda) & \alpha_g\alpha_l\rho_l(u_l+\lambda) & G \\ \alpha_g & 0 & \Gamma/\rho_g \\ 0 & \alpha_l & -\Gamma/\rho_l \end{bmatrix}$$

$$= \alpha_l\alpha_g(\rho_l - \rho_g)g\sin\theta - \tau_l\frac{S_l}{A}\alpha_g$$

$$+\tau_g\frac{S_g}{A}\alpha_l + \tau_i\frac{S_i}{A}(\alpha_g + \alpha_l)$$

$$+\Gamma[\alpha_l(\lambda+u_g) + \alpha_g(\lambda+u_l)] = 0 \tag{25}$$

Now, the condition of hyperbolicity breaking at singular (and/or turning) points can be found by first finding the characteristics from the solutions of the characteristic equation $\Delta(X_i) = 0$ and then by checking whether the characteristics obtained from this equation become inflected nodes (discriminant in Eq.(29) is zero) or parallel lines (In Eq.(29), $q = 0$) :

Equation (24) can be rewritten as follows :

$$\lambda^2 + 2\lambda(\frac{\alpha_l\rho_g u_g + \alpha_g\rho_l u_l}{\alpha_l\rho_g + \alpha_g\rho_l}) + \frac{\alpha_l\rho_g u_g^2 + \alpha_g\rho_l u_l^2 - F}{\alpha_l\rho_g + \alpha_g\rho_l} = 0 \tag{24a}$$

Rewriting Eq.(24a) as

$$\lambda^2 + 2r\lambda + q = 0 \tag{26}$$

where

$$r = \frac{\alpha_l\rho_g u_g + \alpha_g\rho_l u_l}{\alpha_l\rho_g + \alpha_g\rho_l} \tag{27}$$

$$q = \frac{\alpha_l\rho_g u_g^2 + \alpha_g\rho_l u_l^2 - F}{\alpha_l\rho_g + \alpha_g\rho_l} \tag{28}$$

Solving Eq.(26) for λ, one can obtain

$$\lambda = -r \pm \sqrt{r^2 - q} \tag{29}$$

Now, the neutral stability condition can be obtained by putting the discriminant of Eq.(29) to be zero.

$$r^2 - q = 0 \tag{30}$$

Using Eqs.(27) and (28) in Eq.(30), following relations can be obtained:

$$(u_g - u_f)^2 = u_r^2$$

$$= [(\frac{\partial \Delta P_i}{\partial \alpha_g}) + \frac{\pi}{4}\frac{\Delta \rho g D \cos\theta}{\sin\gamma})](\frac{\alpha_f \rho_g + \alpha_g \rho_f}{\rho_f \rho_g}) \tag{31}$$

Equation (31) is a primitive form of the onset of slugging criterion that results from the analysis of a singular point and neutral stability conditions for the transient one-dimensional two-fluid governing equations of two-phase flow. This equation may be considered to be a more general expression than the existing models for the onset of slugging shown in Table 1 in the sense that Eq.(31) includes the effects of both surface tension and pipe inclination. Equation (31) in its present form, however, is not useful for comparisons with existing models and experimental data to examine its applicability and accuracy. To obtain a simpler and more useful criterion for the onset of slugging, therefore, simplifications and modifications of Eq. (31) are made as shown in the following.

Table 1. Summary of Various Models for the Onset of Slugging Criterion

Authors	Geometry (Wave)	Basic Equation	Slugging Criterion	Dimensionless Form in Horizontal Condition
Wallis & Dobson (1973)	Rectangular Duct (Small- amplitude)	Laplace Bernoulli	$u_g = 0.5(\frac{gH_g(\rho_f - \rho_g)}{\rho_g})^{1/2}$	$j_g^* = 0.5\alpha_g^{1.5}$
Taitel & Dukler (1976)	Duct & Pipe (Solitary-Roll)	Bernoulli Momentum	$u_g \geq (1 - \frac{H_f}{D})(\frac{gH_g(\rho_f - \rho_g)}{\rho_g})^{1/2}$	$j_g^* = \alpha_g^{2.5}$
Gardner (1979)	Duct (Large-amplitude)	Energy Continuity	$u_r = F_{1\Delta}(\frac{gH(\rho_f - \rho_g)}{\rho_g})^{1/2}$ $F_{1\Delta} = f(\alpha_g, \rho_g^*)$	$\frac{j_g^*}{\alpha_g} - \sqrt{\frac{\rho_g}{\rho_f}}\frac{j_f^*}{\alpha_f} = F_{1\Delta}$
Mishima & Ishii (1980)	Rectangular Duct (Finite-amplitude)	Laplace Wave instability	$u_r \geq 0.487(\frac{gH_g(\rho_f - \rho_g)}{\rho_g})^{1/2}$	$\frac{j_g^*}{\alpha_g} - \sqrt{\frac{\rho_g}{\rho_f}}\frac{j_f^*}{\alpha_f} = 0.487\alpha_g^{0.5}$
Lin & Hanartty (1986)	Pipe (Film wave)	Momentum Linear instability	$u_g = K_1(\frac{gD(\rho_f - \rho_g)}{\rho_g})^{1/2}$ $K_1 = f(\alpha_g, \nu, u_g)$	$j_g^* = K_1\alpha_g^{1.5}$
Crowely et al. (1992)	Pipe (Large-amplitude Long wave)	Momentum 1-D wave theory	$(\frac{\rho_g}{gD(\rho_f - \rho_g)})^{1/2} u_r =$ $[\alpha_g^3(1 + \frac{\rho_g\alpha_f}{\rho_f\alpha_g})\frac{D}{S_i}]^{0.5}(\frac{\pi}{4})^{1.5}$	$\frac{j_g^*}{\alpha_g} - \sqrt{\frac{\rho_g}{\rho_f}}\frac{j_f^*}{\alpha_f} = [(1 + \frac{\rho_g\alpha_f}{\rho_f\alpha_g})\frac{D}{S_i}]^{1.5}(\frac{\pi}{4}\alpha_g)^{1.5}$
Brauner & Moalem Maron (1992)	Pipe (Large-amplitude Long wave)	Momentum Linear instability	$u_r^2 = \frac{D}{\rho_{ab}}(\Delta\rho g\cos\theta + \sigma k^2)$ $\rho_{ab} = \frac{\rho_g\tilde{A}_f}{\tilde{\rho}_g A_g}, \tilde{A}_{g,f} = \frac{A_{g,f}}{D^2}, \tilde{A}_f = \frac{d\tilde{A}_f}{dH}$	$\frac{j_g^*}{\alpha_g} - \sqrt{\frac{\rho_g}{\rho_f}}\frac{j_f^*}{\alpha_f} =$ $[(1 + \frac{\rho_g\alpha_f}{\rho_f\alpha_g})]^{0.5}(\frac{\pi}{4}\alpha_g)^{0.5}(\frac{d\tilde{A}_f}{dH_f})^{-0.5}$
Chun et al. (1995)	Duct & Pipe (Small-amplitude)	Energy Wave instability	$u_g \geq 0.470(\frac{gH_g(\rho_f - \rho_g)}{\rho_g})^{1/2}$	$j_g^* = 0.470\alpha_g^{1.5}$
Present Model	Pipe (Large-amplitude Long wave)	Momentum Nonlinear instability	$u_r = (1 - \frac{H_f}{D})[(\frac{\pi\Delta\rho\alpha_g gD\cos\theta}{4\sin\gamma})K]^{0.5}$ $K = 1 + \frac{\alpha_f\rho_g}{\alpha_g\rho_f}$	$\frac{j_g^*}{\alpha_g} - \sqrt{\frac{\rho_g}{\rho_f}}\frac{j_f^*}{\alpha_f} =$ $(\frac{1+\cos\gamma}{2})[(1 + \frac{\rho_g\alpha_f}{\rho_f\alpha_g})(\frac{\pi}{4}\frac{\alpha_g}{\sin\gamma})]^{0.5}$

4.2. Final Form of the Onset of Slug Flow Criterion

The first term on the right hand side (R.H.S.) of Eq.(31) represents the effect of surface tension force whereas the second term includes the effect of gravitational force. In those waves which form slugs, it can be shown that the first term on the R.H.S. of Eq.(31) is negligibly smaller than the second term. That is,

$$\frac{\partial \Delta P_i}{\partial \alpha_g} << \frac{\pi}{4}\frac{\Delta\rho g D\cos\theta}{\sin\gamma} \tag{32}$$

When the first term on the R.H.S. of Eq.(31) is neglected, Eq.(31) reduces to the following :

$$u_{r,crit} = [(\frac{\pi}{4}\frac{\Delta\rho\alpha_g gD\cos\theta}{\rho_g \sin\gamma})(1+\frac{\alpha_f\rho_g}{\alpha_g\rho_f})]^{0.5} \tag{33}$$

When Eq.(33) is compared with the existing models shown in Table 1, it can be noticed that the numerical coefficient of the existing models obtained by Wallis and Dobson [2], Mishima and Ishii [5], and Chun et al. [9] varies from 0.47 to 0.5, whereas Eq.(33) has an extra coefficient of $(1+\alpha_f\rho_g/\alpha_g\rho_f)^{0.5}$.

For flow in round pipes and for disturbances of finite amplitude, Taitel and Dukler [3] speculated that the coefficient to be used in their model can be estimated as follows :

$$C = 1 - H_f / D \tag{34}$$

A close examination of Eq.(33) shows that this equation does not explicitly include the effect of the liquid depth (H_f) on the slug flow formation for a given pipe diameter. Therefore, a parameter to be used in Eq.(33) is sought to satisfy the following physical conditions :

(1) When the equilibrium liquid level approaches the top of the pipe (i.e., $H_f \cong D$) slug flow occurs at zero critical relative velocity ($u_{r,crit}=0$).

(2) Conversely, when the equilibrium liquid level approaches zero (i.e., $H_f \cong 0$) the slug flow occurs at the maximum critical relative velocity for given conditions. In fact, slug flows cannot occur in this case.

(3) The wave amplitude (or the critical relative velocity) at which slug flows occur decreases linearly as the equilibrium liquid level (H_f) is increased. For example, when H_f is increased from $H_f=D/2$ to $H_f=3D/4$, the wave amplitude (or $u_{r,crit}$) needed to form a slug flow decreases by one-half.

It can be recognized that the coefficient used by Taitel and Dukler [3], i.e., Eq.(34) satisfies all the physical conditions specified above. To incorporate the above physical conditions into the present model, Eq.(33) is modified as follows :

$$u_{r,crit} = (1-\frac{H_f}{D})[(\frac{\pi}{4}\frac{\Delta\rho\alpha_g gD\cos\theta}{\rho_g \sin\gamma})(1+\frac{\alpha_f\rho_g}{\alpha_g\rho_f})]^{0.5} \tag{35}$$

Equation (35) is the final form of the onset of slug flow criterion. Note that Eq.(35) gives the critical relative velocity at which the transition occurs from a stratified to a slug flow in horizontal or inclined pipes. It may be noted here that for all practical conditions of air-water (or steam-water) two-phase flow $\alpha_f\rho_g/\alpha_g\rho_f \ll 1$. Therefore, Eq.(35) can be further reduced to a simpler form as follows :

$$u_{r,crit} = (1-\frac{H_f}{D})[\frac{\pi}{4}\frac{\Delta\rho\alpha_g gD\cos\theta}{\rho_g \sin\gamma}]^{0.5}, \text{ or}$$

$$u_{r,crit} = (\frac{1+\cos\gamma}{2})[\frac{\pi}{4}\frac{\Delta\rho\alpha_g gD\cos\theta}{\rho_g \sin\gamma}]^{0.5} \tag{35a}$$

5. Results and Discussion

For convenience in comparing between the present model given by Eq.(35) with existing models and experimental data, the slug formation criteria included for comparison are first transformed into dimensionless forms as follows :

Taitel and Dukler's criterion [3] :

$$j_g^* = \alpha_g^{2.5} \tag{36}$$

Mishima and Ishii's criterion for a slug formation [5] :

$$\frac{j_g^*}{\alpha_g} - \sqrt{\frac{\rho_g}{\rho_f}}\frac{j_f^*}{\alpha_f} = 0.487\alpha_g^{0.5} \tag{37}$$

Chun et al.'s criterion [9] :

$$j_g^* = 0.470\alpha_g^{1.5} \tag{38}$$

Present criterion given by Eq.(35) :

$$\frac{j_g^*}{\alpha_g} - \sqrt{\frac{\rho_g}{\rho_f}}\frac{j_f^*}{\alpha_f} = (1-\frac{H_f}{D})[(\frac{\pi}{4}\frac{\alpha_g\cos\theta}{\sin\gamma})(1+\frac{\rho_g}{\rho_f}\frac{\alpha_f}{\alpha_g})]^{0.5}, \text{ or}$$

$$\frac{j_g^*}{\alpha_g} - \sqrt{\frac{\rho_g}{\rho_f}}\frac{j_f^*}{\alpha_f} = (\frac{1+\cos\gamma}{2})[(\frac{\pi}{4}\frac{\alpha_g\cos\theta}{\sin\gamma})(1+\frac{\rho_g}{\rho_f}\frac{\alpha_f}{\alpha_g})]^{0.5} \tag{39}$$

In the above equations, j_f^* and j_g^* and are defined by

$$j_f^* = j_f[\frac{\rho_f}{\Delta\rho g D}]^{0.5} \tag{40}$$

$$j_g^* = j_g[\frac{\rho_g}{\Delta\rho g D}]^{0.5} \tag{41}$$

In Fig. 2, j_g^* versus α_g curves obtained by various models expressed in dimensionless forms as above are compared with existing data. It can be seen from Fig. 2 that for the onset of slugging in horizontal pipes Eq.(35) (or Eq.(39)) agrees very well with Taitel and Dukler's model [3], in particular, and also with existing data for a broad range of α_g values.

In Fig. 3, on the other hand, j_g^* versus α_g curves obtained by various models for an inclined pipe (θ =50°) are shown along with experimental data reported by Ohnuki et al. [15]. This figure shows that for α_g between 0.5 and 0.8 (i.e., within the range of experimental data given), prediction of the present model agrees more closely with the experimental data of Ohnuki et al. [15] than those of existing models.

6. Conclusions

1. A more general expression than the existing models for the onset of slug flow criterion, represented by Eq.(31), has been derived from singular points and neutral stability conditions of the transient one-dimensional two-phase flow equations of

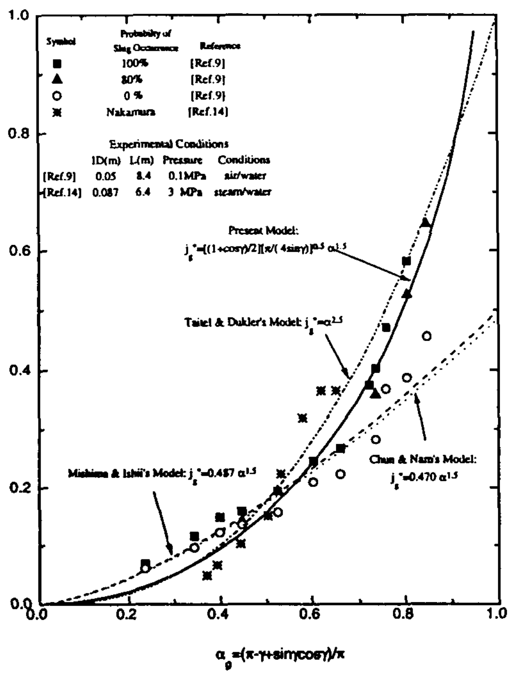

Fig. 2. Comparison of Present and Existing Model Predictions with Experimental Data for the Onset of Slugging in Horizontal Pipes

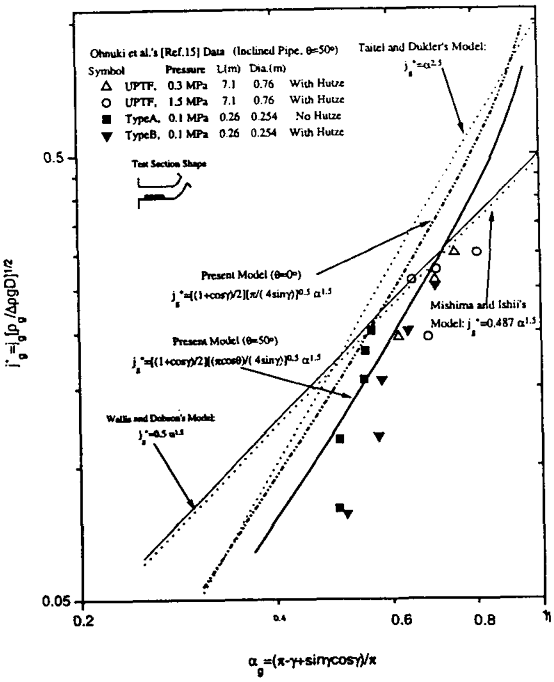

Fig. 3. Comparison of Present and Existing Model Predictions with Experimental Data for the Onset of Slugging in an Inclined Pipe(θ=50°)

two-fluid model.

2. The final form of the present model, Eq.(35), has been obtained by introducing simplifications and incorporating a parameter into the general form to explicitly include the effect of liquid depth (H_f) on the slug flow formation. The parameter, which has the same form as the coefficient of Taitel and Dukler's model [3], has been chosen to satisfy three physical conditions a priori specified.
3. Comparisons between the models transformed into dimensionless forms and experimental data show that the present model agrees very closely with Taitel and Dukler's model [3] and experimental data for the onset of slug flow in horizontal pipes. In an inclined pipe ($\theta=50°$), however, the difference between the predictions of the present model and those of existing models including Taitel and Dukler's is appreciably large and the present model gives the best agreement with Ohnuki et al. 's data [15].

Acknowledgment

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Nomenclature

A	area, m^2
A_{ij}	matrix
B_{ij}	matrix
C_i	column vector
D	pipe diameter, m
f	friction factor
g	acceleration due to gravity, $9.806 m/s^2$
H	total channel height, m
H_f	liquid level, m
i_{fg}	latent heat of evaporation or condensation, J/kg
j	volumetric flux, m/s
j^*	dimensionless volumetric flux, $j_k[\frac{\rho_k}{\Delta\rho\, gD}]^{0.5}$
M	momentum per unit volume, N/m^2
N_j	determinant
P	pressure, N/m^2
ΔP_i	interfacial pressure difference, $P_{ig}-P_{if}$, N/m^2
Q_w	heat source, J = $kg\, m^2/s^2$
R_1, R_2	principal curvatures, m
t	time, sec
u	velocity, m/s
u_r	relative velocity, u_g-u_f, m/s
X_j	column vector
z	spatial coordinate

Greek Letters

α	volume fraction (or void fraction)
Γ	mass exchange rate, kg.
γ	stratification angle, radian or degree
θ	angle of inclination, radian or degree
λ	characteristics
ξ	wave coordinate
ρ	density, kg/m^3
$\Delta\rho$	density difference, $\rho_f-\rho_g$, kg/m^3
σ	surface tension
τ	shear stress, Pa = N/m^2.

Subscripts

crit	critical value
f	liquid phase
g	gas phase
i	interface
r	relative value

Superscripts

*	dimensionless value

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Appendix I. Derivation of Variables in the Pipe Geometry

Considering the stratified flow in pipe show in Fig. 1, the area values (A_f, A_g, A) can be expressed as follows :

$$A_f = [D^2(\gamma - \sin\gamma\cos\gamma)]/4 \quad \text{(I-1)}$$

$$A_g = [D^2(\pi - \gamma + \sin\gamma\cos\gamma)]/4 \quad \text{(I-2)}$$

$$A = [\pi D^2]/4 \quad \text{(I-3)}$$

The water depth can be written as :

$$H_f = [D(1-\cos\gamma)]/2 \quad \text{(I-4)}$$

And wetted perimeters (S_i, S_f, S_g) can be :

$$S_i = D\sin\gamma \quad \text{(I-5)}$$

$$S_f = D\gamma \quad \text{(I-6)}$$

$$S_g = D(\pi - \gamma) \quad \text{(I-7)}$$

Volume fraction can be :

$$\alpha_f = \frac{A_f}{A} = \frac{1}{\pi}(\gamma - \sin\gamma\cos\gamma) = \frac{1}{\pi}(\gamma - \frac{\sin 2\gamma}{2}) \quad \text{(I-8)}$$

$$\alpha_g = \frac{A_g}{A} = \frac{1}{\pi}(\pi - \gamma + \sin\gamma\cos\gamma) = \frac{1}{\pi}(\pi - \gamma + \frac{\sin 2\gamma}{2}) \quad \text{(I-9)}$$

We can be obtained spatial derivative terms as follows :

$$\frac{\partial H_f}{\partial \gamma} = \frac{D}{2}\sin\gamma \qquad \text{(I-10)}$$

$$\frac{\partial \alpha_s}{\partial \gamma} = -\frac{2\sin^2\gamma}{\pi} \qquad \text{(I-11)}$$

$$\frac{\partial \alpha_s}{\partial z} = \frac{\partial \alpha_s}{\partial \gamma}\frac{\partial \gamma}{\partial H_f}\frac{\partial H_f}{\partial z} \qquad \text{(I-12)}$$

The substitution of the above relations Eq. (I-10) to Eq. (I-11) into Eq. (I-12) gives to yield the following equation :

$$\frac{\partial \alpha_s}{\partial z} = -\frac{4}{\pi D}\sin\gamma\frac{\partial H_f}{\partial z} \qquad \text{(I-13)}$$

therefore,

$$\frac{\partial H_f}{\partial z} = -\frac{\pi D}{4\sin\gamma}\frac{\partial \alpha_s}{\partial z} \qquad \text{(I-14)}$$

Appendix Ⅱ. "Three Classes of Points in the Phase Space Ω"

1. Three Classes of Points in the Phase Space Ω [11] :

(1) Regular points : A point (ξ^o, X^o) in phase space is called 'regular' if $\Delta(X_j) \neq 0$.

(2) Turning points : The points (ξ^*, X^*) which belong to *T* but not to *S* are called 'turning points.'

(3) Singular points : The points (ξ^{**}, X^*) which belong to *S* as well as *T* are called 'singular points.'

2. Hypercyliner (T), Hypersurface (Σ), and the Manifold (*S*) :

(1) The condition defines the hypercylinder *T* and each of the conditions defines a hypersurface Σ_j.

(2) The manifold S of dimensions is the intersection of hypercylinder *T* with one hypersurface, say $\Sigma_1(N_1 = 0)$.

It is assumed that the two hypersurfaces intersect transversely. The theorem [11] asserts that all on *T*, and this implies that all remaining hypersurfaces must necessarily intersect *T* at *S*. This situation is illustrated in this figure.

Appendix Ⅲ. "Classification of Topological Patterns for the Linear System"[11, 16, 17]

From the $\Delta(X_j) = 0$, we find following eignvalue equation

$$\lambda^2 + 2r\lambda + q = 0 \qquad \text{(III-1)}$$

which is called the characteristic equation. When this equation has two different roots, λ_1, λ_2, two linearly independent families of solutions are generated by ξ corresponding to and $\lambda = \lambda_1$ and $\lambda = \lambda_2$, respectively.

Let Θ be the discriminant :

$$\Theta = r^2 - q \qquad \text{(III-2)}$$

then the roots of Eq.(Ⅲ-1) are given by

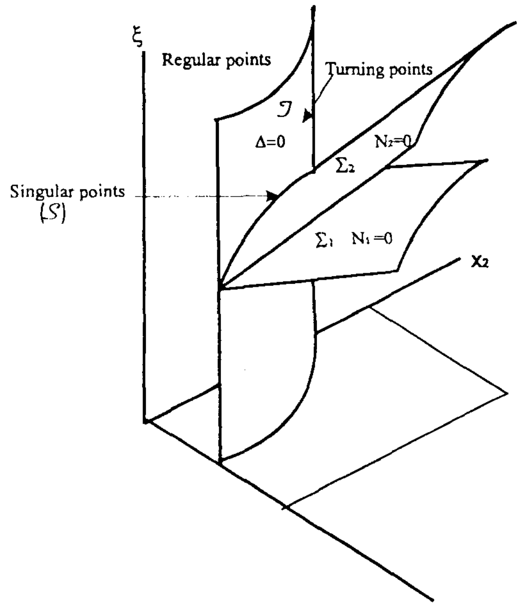

Fig. Ⅱ. Definition of Manifold *S* as the Intersection between Δ(X_j)=0 and All N_j=0 and Illustration of Three Classes of Points in the Phase Space Ω [11].

$$\lambda_{1}, \lambda_{2} = -r \pm \sqrt{\Theta} \qquad \text{(III-3)}$$

The solution of eignvalues (λ_1, λ_2) quoted in $\Delta(X_j)=0$, admit several types of topological patterns, (1) saddle points, (2) nodes, (3) spirals (or foci), (4) inflected node, (5) parallel lines, etc., depending on the relationships obtaining between the constants; theses are illustrated with the aid of Fig. (Ⅲ) which correlates each pattern with the eignvalues. In particular,

(1) Saddle points (λ_1, λ_2 real, with different signs)

- A saddle point there are pass exactly two trajectories
- The discriminant is positive, hyperbolic equations domain
- The equilibrium point is a saddle point, which is always unstable
- Two real characteristic slops. Discontinuities in second derivatives are thus propagated in two directions, and two characteristic curves may pass through every points in that space.

(2) Nodes (λ_1, λ_2 real, same sign, $\lambda_1 \neq \lambda_2$)

- A nodal point there passes an infinity of trajectories
- The discriminant is positive, hyperbolic equations domain
- The equilibrium point is unstable node($r<0$), or stable node($r>0$)
- Two real characteristic slops. Discontinuities in second derivatives are thus propagated in two directions, and two characteristic curves may pass through every points in that space.

(3) Spiral or focus (λ_1, λ_2 complex with non-zero real part)

- A spiral there pass no trajectories
- The discriminant is negative, elliptic equations domain
- The equilibrium point is a stable spiral if $\mathrm{Re}(\lambda_1)<0$ and an unstable spiral if $\mathrm{Re}(\lambda_1)>0$
- Characteristic directions are imaginary. Discontinuities are not propagated in this case, so that discontinuities in sources, sinks and boundaries are not transferred to the integral surface, there being no mechanism for such a transfer.

(4) Inflected nodes ($\lambda_1=\lambda_2$ real)

- A inflected nodal point there passes an infinity of trajectories
- The discriminant is zero, parabolic equations domain
- The equilibrium point is unstable inflected node ($r>0$), or stable inflected node ($r<0$)
- Only one slop. Discontinuities are then propagated in only one direction.

(5) Parallel lines ($\lambda_1 \neq 0$, $\lambda_2=0$ real, $q=0$)

- Parallel lines there passes one trajectory
- $q=0$, hyperbolic equations domain
- The equilibrium point is parallel lines

Fig. (Ⅲ) shows the nature of equilibrium points on the plane. The stable equilibrium points lie in the quadrant $r \geq 0$, $q \geq 0$ except $(r, q)=(0, 0)$.

Therefore, the conditions hyperbolicity breaking are as follows two points.

(a) Parallel lines: This condition is named as a critical condition.

(b) Inflected nodes: This condition is named as a neutral stability condition.

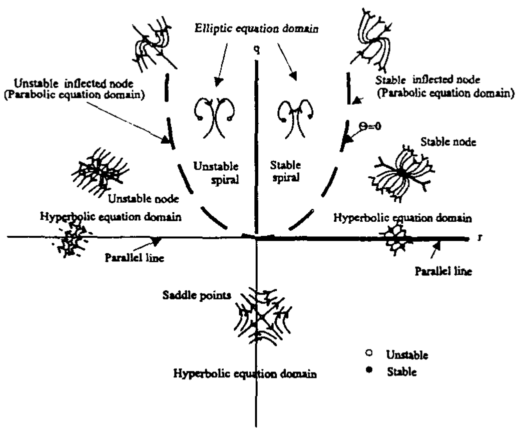

Fig. Ⅲ. General Classification of Topological Patterns for the Linear System