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Free Vibration of Marine Riser System with the Inclusion of Internal Flow
내부 유체흐름을 포함한 Riser System의 자유진동

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Abstract □ A mathematical model for the dynamic analysis of the riser system is developed to investigate the effect of internal flow on the free vibration of marine riser system which includes a steady flow inside the pipe. A semi-analytical method using series expansion is employed to derive Eigenvalue problem to facilitate the evaluation of the system frequencies, and its validity is given through the comparison of the solutions with the conventional method using system matrices. The algorithm is implemented to develop computer programs for the estimation of the system frequency. The investigations of the effect of internal flow on system frequency are performed according to the change of parameters such as top tension, internal flow velocity, and so on. It is found that the effect of internal flow can be controlled by the increase of top tension. However, careful consideration has to be given in the design point, particularly for the long riser.

要 旨 : 해저 riser관내에 내부유체 흐름이 발생할 때 그흐름으로 인해 riser system의 자유진동에 미치는 영향을 조사하기 위해 수학적 모델이 유도된다. System의 고유진동수를 계산하기 위해 조화함수 확장을 적용하여 고유치 방정식을 유도하는 부분해석적 방법을 사용하였다. 그리고 이 방법론은 기존 해석론에 의해 비교 검증된다. 컴퓨터 프로그램을 위한 알고리즘이 개발되어 내부유체 흐름이 system 고유진동수에 미치는 영향을 상부인장력, 내부유체흐름 속도등과 같은 인자의 변화에 따라 조사하였다. 분석결과 내부유체흐름의 영향은 상부인장력에 의해 지배되나 심해저 riser와 같은 장대 riser의 경우에 설계시 세심한 주의가 요구된다.

1. INTRODUCTION

The marine riser is a conductor pipe used in floating drilling operations to convey drilling fluid and to guide tools between the drilling vessel and the well head at the ocean floor. Fig. 1 represents a schematic sketch of a marine riser conductor that contains internal fluid flow and is subject to environmental forces. When the internal fluid travels inside the curved path along the deflected riser, it experiences centrifugal and coriolis accelerations due, respectively, to the curvature of the riser and the relative motion of fluid to time dependent riser motion. Those accelerations exert against the riser which, in turn, affect the dynamic behaviour of the riser and cause riser vibrations.

The primary objectives of this study are: 1) to develop a mathematical model for the analysis of the riser system with the inclusion of internal flow, and 2) to examine the effect of the internal flow on the system frequency of riser. In this study, a semi-analytical solution using series expansion is presented for obtaining system frequencies and a conventional numerical method using system matrices is employed to check the results with those from semi-analytical method.

Although several studies have investigated the vibration of a pipeline conveying fluid supported above ground (Ashley and Haviland, 1950), there are only a few papers dealing with the effect of internal flow on

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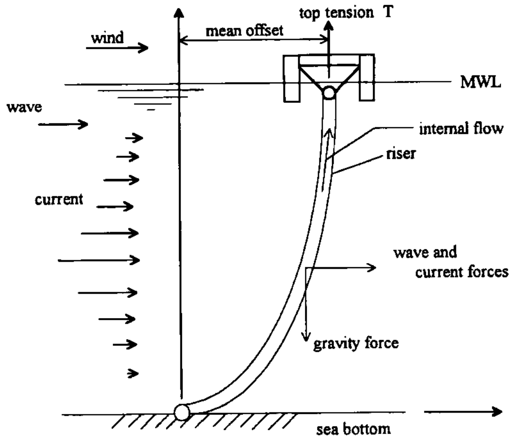

Fig. 1. Configuration of riser system with the inclusion of internal flow.

marine riser dynamics. Moe and Chucheepsakul (1988) was the first who considered the forces due to the internal flowing fluid as a dynamically forcing component acting on the interior wall of the riser and derived the governing equation of motion. Before that, the loading due to the internal fluid was included in the internal tension of the riser as a fluid static force, that is, centrifugal and coriolis acceleration due to the internal flow were neglected. Wu and Lou (1991) developed a mathematical model for the lateral bending vibration of a marine riser and examined the effect of internal flow and bending rigidity of the pipe on the dynamic behavior of the riser. It was found from their results that the internal flow reduced the effect of top tension, but the riser motion was not significantly affected when the top tension of the riser was relatively high. Chen (1992) derived the governing equation with the inclusion of dynamic force for lateral vibration by applying Hamilton's principle. The natural frequencies and the mode shapes were formulated and presented. The critical buckling and significant velocities which associate the internal flowing fluid with system integrity were also presented. The system frequencies and mode shapes of a marine riser with the internal flow are evaluated in this study. First, semi-analytical solutions are obtained using series expansion to the mathematical model and second, numerical solutions are calculated adapting complex eigenvalue method to the matrix equilibrium equation. Finally, the algorithm for the computer program is introduced and the verification of the first method is inspected through the comparison of the system frequencies with the second method. Further, the comparison of semi analytical solutions with results from referenced papers is given.

2. GOVERNING EQUATION OF MOTION

2.1 Force Due to Internal Flow

The force acting on the internal wall of the riser is derived in two ways: by obtaining the acceleration of a fluid particle, and by using the concept of Hamilton's principle.

In all two way, no small-scale motions such as turbulence or secondary flow are assumed to be absent. And also, the plug-flow model with no radial variation of velocity is utilized as a fluid model for the internal flow.

First, we obtain the acceleration of a fluid particle by differentiating the fluid velocity $\mathbf{V}_I$,

$$A_I = \frac{d\mathbf{V}_I}{dt} = \frac{\partial \mathbf{V}_I}{\partial s_1}\frac{ds}{dt} + \frac{\partial \mathbf{V}_I}{\partial t} \quad (1)$$

where

$$\mathbf{V}_I = \frac{\partial x_1}{\partial t}\hat{i} + \frac{\partial x_2}{\partial t}\hat{j} + \frac{\partial x_3}{\partial t}\hat{k} + V_I\hat{t} = \dot{\mathbf{r}} + V_I\mathbf{r}' \quad (2)$$

Performing the operations in (1) with $s_1 \approx s$ and $ds/dt = V_I$, we obtain the fluid acceleration in the deformed direction

$$\begin{aligned} \mathbf{A}_I &= (\dot{\mathbf{r}} + V_I\mathbf{r}')'\,\dot{V}_I + V_I \\ &= \ddot{\mathbf{r}} + \dot{V}_I\mathbf{r}' + 2V_I\dot{\mathbf{r}}' + V_I V_I \mathbf{r}' + V_I^2\mathbf{r}'' \end{aligned} \quad (3)$$

Assuming steady flow and no convective acceleration along the riser curve, the fluid force acting on the internal wall of riser becomes

$$\mathbf{F}_I = -m_f\mathbf{A}_I = -m_f(\ddot{\mathbf{r}} + 2V_I\dot{\mathbf{r}}' + V_I^2\mathbf{r}'') \quad (4)$$

where m_f is mass of drilling mud and sea water.

Second, we apply the concept of Hamilton's principle just considering the finite part comprising the pipe and the enclosed volume of fluid. Hamilton's principle in our problem states that

$$\delta\int_{t_1}^{t_2}(T_r + T_f - V_r - V_f)\,dt = 0 \tag{5}$$

where, T_r and V_r are the kinetic and potential energies associated with the tube, and T_f and V_f are the corresponding quantities for the fluid. Using the velocity of the internal fluid obtained from neglecting the stretching strain in (2), we have kinetic energy of the enclosed volume of fluid

$$T_f = \frac{mf}{2}\int_{l_1}^{l_2}\left[V_I^2 + \left(\frac{\partial x_1}{\partial t} + V_I\frac{\partial x_1}{\partial s_1}\right)^2 + \left(\frac{\partial x_2}{\partial t} + V_I\frac{\partial x_2}{\partial s_1}\right)^2 + \left(\frac{\partial x_3}{\partial t} + V_I\frac{\partial x_3}{\partial s_1} + 1\right)^2\right] ds_1 \tag{6}$$

and the potential energy of the fluid is zero because the fluid is assumed to be incompressible, i.e.,

$$V_f = 0 \tag{7}$$

Performing the variation after the substitution of (6) and (7) into (5), we obtain the following integration

$$\int_{t_1}^{t_2}\int_{l_1}^{l_2}[m_f\{(\dot{x}_1 + V_I x'_1)(\delta\dot{x}_1 + V_I\delta x'_1) + (\dot{x}_2 + V_I x'_2)(\delta\dot{x}_2 + V_I\delta x'_2) + (\dot{x}_3 + V_I x'_3 + 1)(\delta\dot{x}_3 + V_I\delta x'_3)\} + \delta T_r - \delta V_r]\,ds_1\,dt = 0 \tag{8}$$

Since $\delta\dot{x}_i = \frac{\partial}{\partial t}(\delta x_i)$, $\delta x'_i = \frac{\partial}{\partial x_i}(\delta x_i)$ $(i = 1, 2, 3)$

Each term may be integrated by parts so as to eliminate the various derivatives of δx_i. When this is done, there is obtained

$$\int_{t_1}^{t_2}\int_{l_1}^{l_2}[\sum_{i=1}^{3}(m_f\ddot{x}_i + 2m_f V_I\dot{x}'_i + m_f V_I x''_i)\,\delta x_i + \delta T_r - \delta V_r]\,ds_1\,dt = 0 \tag{9}$$

where all the integrated terms have disappeared because of the boundary conditions. From the concept of the resulting Euler-Lagrange equations, we can recognize that the expression in round bracket represents the forcing components of a fluid particle inside the pipe due to internal flow. Thus, the fluid force acting on the internal wall of the pipe can be written in vector form.

$$\mathbf{F}_I = -m_f(\ddot{\mathbf{r}} + 2V_I\dot{\mathbf{r}}' + V_I^2\mathbf{r}'') \tag{10}$$

We can easily see that (4) and (10) are identical. The first term on the right represents the inertia force associated with the riser acceleration, the second term is the inertia force associated with the coriolis acceleration which arise because the fluid is flowing with velocity V_I relative to the riser, while the riser itself has an angular velocity at any point along its length, and the last term represents the inertia force associated with the change in direction of the flow velocity, owing to the curvature of the riser.

2.2 Linear Governing Equation of Motion

Restricting attention to risers whose deformation lies wholly in a single plane, the governing equation of motion (Hong, 1994) can be derived as

$$m_t\ddot{x}_1 + 2m_f V_I\dot{x}'_1 + m_f V_I^2 x''_1 + (EIx''_1)'' - (T_e x'_1)' = q_1 \tag{11}$$

$$m_t\ddot{x}_3 - T'_e = q_3 \tag{12}$$

where m_t is the total mass including riser itself.

Most riser problems in intermediate water depth, longitudinal vibration is unimportant and we may neglect the longitudinal inertia term in (12). Further, it is convenient to include the hydrostatic effects of internal and external fluid pressures by defining effective weight per unit length, w and effective tension, T_e as

$$w = w_r + \gamma_i A_i - \gamma_o A_o \tag{13}$$

$$T_e = T - p_i A_i = p_o A_o \tag{14}$$

where w_r=riser weight per unit length, γ_i, γ_o=specific weight of internal and external fluid, p_i, p_o=internal and external static pressure of riser, and A_i, A_o=internal and external area of riser.

Noting that q_3=-w, (12) can then be integrated to give

$$T_e = TTR - \int_0^{d} w\, ds \qquad (15)$$

or

$$T_e = TTB + \int_0^{d} w\, ds \qquad (16)$$

where TTR and TTB are, respectively, the top tension and the bottom reaction tension.

Thus, the effective tension is independent of the horizontal deflections and then, with no torsion, (11) becomes

$$m_t\ddot{x}_1 + 2m_f V_f \dot{x}'_2 + m_f V_f^2 x''_1 + (EIx''_1)'' - wx'_1 - T_e x''_1 = q_1 \qquad (17)$$

$$m_t\ddot{x}_2 + 2m_f V_f \dot{x}'_2 + m_f V_f^2 x''_2 + (EIx''_2)'' - wx'_2 - T_e x''_2 = q_2 \qquad (18)$$

Equations (17) and (18) represent the linear model.

3. SOLUTION OF THE FREE VIBRATION OF THE SYSTEM

3.1 Semi-Analytical Solution

The linear governing equation for lateral vibration of a marine riser system with the internal flow and linear tension has been derived in section 2.2. The free vibration equation of the system can be derived simply by setting the forcing term in the right hand side of (17) to zero and dropping the subscript 1 of variable x indicating the lateral deformation. The resulting equation represents the free vibration of the system in the lateral direction

$$m_t\ddot{x} + 2m_f V_f \dot{x}' + m_f V_f^2 x'' + (EIx'')'' - wx' - T_e x'' = 0 \qquad (19)$$

where the unknown variable x represents the displacement in lateral direction and prime denotes the derivative with respect to a vertical coordinate, z.

Providing that pipe has an uniform section of constant EI and selecting T_e as the mean value of the tension force acting along the length of riser, (19) becomes

$$m_t\ddot{x} + 2m_f V_f \dot{x}' + m_f V_f^2 x'' + EIx'''' - wx' - \bar{T}_e x'' = 0 \qquad (20)$$

where $\bar{T}_e$ represents mean tension and is given as TTB+wL/2 or TTR-wL/2.

3.1.1 Quasi mode shape

Equation (20) differs from the usual vibration equations in that it contains a mixed derivative term with respect to time and space which is the coriolis force, the force required to rotate fluid elements with local pipe rotation. Its mixed derivative causes an asymmetric distortion of classical mode shapes and could lead to a flutterlike instability. Thus, Eq. (20) does not possess the classical normal modes. Its solution can not be separated simply into time and space components. For example, if a trial solution of the form

$$x(z,t) = \tilde{x}(z)\sin\omega t \qquad (21)$$

is substituted into (20), it can be seen that the coriolis force term varies as cos ωt, while the remainder of the terms have a sin ωt term. This suggests that solution should be written as

$$x(z,t) = a_1\tilde{x}(z)\sin\omega t + a_2\tilde{x}(z)\cos\omega t \qquad (22)$$

The boundary conditions for a pinned-pinned span are given by

$$x(0,t) = x(L,t) = \partial^2 x(0,t)/\partial z^2 = \partial^2 x(L,t)/\partial z^2 = 0 \qquad (23)$$

and those boundary conditions are satisfied by the set of sinusoidal mode shapes,

$$\tilde{x}(z) = \sin n\pi z/L,\quad n = 1, 2, 3\cdots \qquad (24)$$

These mode shapes make the first, third, fourth, and sixth terms in (20) unaltered, but the mixed derivative in the coriolis force term generates spatially asymmetric terms for a symmetric mode shape ($n = 1, 3, 5, \ldots$) and spatially symmetric terms for an asymmetric mode shape ($n = 2, 4, 6, \ldots$) These considerations imply that the solution of (20) with pinned end conditions should be the sum of symmetric and asymmetric spatial modes with sine and cosine components (Housner, 1952; Blevins, 1990).

$$x_j(z,t) = \sum_{n=1,3,5,\cdots} a_n \sin(n\pi z/L)\sin\omega_j t$$

$$+ \sum_{n=2,4,6,\cdots} a_1 \sin(n\pi z/L) \cos\omega_j t \quad j = 1, 2, 3,.. \tag{25}$$

where ω_j is the natural frequency of the j-th vibration mode and the coefficient equation will be derived in next section.

Eq. (25) shows that the classical mode shape does not exist in our system as stated early. Moe and Chucheepsakul (1988) has shown that there is a phase shift due to the internally flowing fluid. Chen (1992) has made a heuristic assumption that the time components have been dropped out of (25) at the end of his derivation and showed through his numerical example that the error due to the assumption was negligible as compared to other author's results from different approaches. According to his assumption, the 1st quasi mode shape can be formed by dropping the time components from (25) and given by

$$x_1(z) = \sum_{n=1,3,5,\cdots} a_n \sin(n\pi z/L) + \sum_{n=2,4,6,\cdots} a_n \sin(n\pi z/L) \tag{26}$$

where the coefficients were determined from the equation to the 1st natural frequency ω_1.

In this paper, the quasi mode shape, which is not from Chen's assumption but by including time dependent component in (25), is implemented for the determination of system frequency.

3.1.2 Modified eigenvalue problem

In order to obtain the coefficient equations, let's begin with the substitution of trial solution (25) into (20). By this substitution, the coriolis force and the fifth term in (20) produce terms containing $\cos(n\pi z/L)$. These cosine terms can be expanded in a Fourier half-range series of sine functions

$$\cos(n\pi z/L) = \sum_{p=1,2,3,\cdots} b_{np} \sin(p\pi z/L), \quad n = 1, 2, 3, \tag{27}$$

where

$$b_{np} = \begin{cases} 0 & n+p = \text{even} \\ 4p/\{(p^2-n^2)\} & n+p = \text{odd} \end{cases} \tag{28}$$

This Fourier series converges relatively slowly over the span and not at all at the ends, but it does allow the spatial dependence to be factored out of the solution. With the substitution of the series expansion of cosine function, the terms in (20) can be grouped according to whether they contain $\sin\omega t$, $\cos\omega t$. For a neutrally buoyant case (w=0),

$$\begin{aligned} &\sum_{n=1,3,5,\ldots} a_n \{-m_t\omega_j^2 - m_f V_I^2(n\pi/L)^2 + \bar{T}_e(n\pi/L)^2 \\ &+ EI(n\pi/L)^4\} \sin(n\pi z/L)\sin(\omega_j t) \\ &+ \sum_{n=1,3,5,\ldots} (-8m_f V_I\omega_j/L) \left\{ \sum_{p=2,4,6,\ldots} (a_p pn)/(n^2-p^2) \right\} \\ &\sin(n\pi z/L)\sin(\omega_j t) \\ &+ \sum_{n=2,4,6,\ldots} a_n\{-m_t\omega_j^2 - m_f V_I^2(n\pi/L)^2 + \bar{T}_e(n\pi/L)^2 \\ &+ EI(n\pi/L)^4\} \sin(n\pi z/L)\cos(\omega_j t) \\ &+ \sum_{n=2,4,6,\ldots} (8m_f V_I\omega_j/L) \left\{ \sum_{p=1,3,5,\ldots} (a_p pn)/(n^2-p^2) \right\} \\ &\sin(n\pi z/L)\cos(\omega_j t) = 0 \end{aligned} \tag{29}$$

The coefficients of each group are set to zero to give the following equations

$$\begin{aligned} &a_n [EI(n\pi/L)^4 - m_f V_I^2(n\pi/L)^2 + \bar{T}_e(n\pi/L) - m_t\omega_j^2] \\ &= (8m_f V_I\omega_j/L) \sum_{p=2,4,6,\cdots} a_p pn/(n^2-p^2) \\ &n = 1, 3, 5,... \end{aligned} \tag{30}$$

$$\begin{aligned} &a_n [EI(n\pi/L)^4 - m_f V_I^2(n\pi/L)^2 + \bar{T}_e(n\pi/L) - m_t\omega_j^2] \\ &= (8m_f V_I\omega_j/L) \sum_{p=1,3,5,\cdots} a_p pn/(n^2-p^2) \\ &n = 2, 4, 6,... \end{aligned} \tag{31}$$

For a negatively or positively buoyant case ($w \neq 0$),

$$\begin{aligned} &\sum_{n=1,3,5,\cdots} a_n\{-m_f\omega_j^2 - m_f V_I^2(n\pi/L)^2 + \bar{T}_e(n\pi/L)^2 \\ &EI(n\pi/L)^4\} \sin(n\pi z/L)\sin(\omega_j t) \\ &+ \sum_{n=1,3,5,\cdots} (-8m_f V_I\omega_j/L) \left\{ \sum p = 2,4,6,\cdots (a_p pn)/ \right. \\ &\left. (n^2-p^2) \right\} \sin(n\pi z/L)\sin(\omega_j t) \\ &+ \sum_{n=1,3,5,\cdots} -4w/L \left(\sum_{p=2,4,6,\cdots} a_p pn/(n^2-p^2) \right) \end{aligned}$$

$$\sin(n\pi z/L)\sin(\omega_j t) + \sum_{n=2,4,6,\cdots} a_n\{-m_t\omega_j^2 - m_f V_i^2(n\pi/L^2 + \bar{T}_e(n\pi/L)^2 + EI(n\pi/L)^4\}\sin(n\pi z/L)\cos(\omega_j^t) + \sum_{n=2,4,6,\cdots}(8m_f V_i\omega_j/L)\left\{\sum_{n=1,3,5,\cdots}(a_p pn)/(n^2-p^2)\right\} \sin(n\pi z/L)\cos(\omega_j t) + \sum_{n=2,4,6,\cdots} -4w/L\left(\sum_{p=1,3,5,\cdots} a_p pn/(n^2-p^2)\right) \sin(n\pi z/L)\cos(\omega_j t) \quad (32)$$

Collecting (32) into groups according to $\sin\omega_j t$ or $\cos\omega_j t$ and setting the coefficient to zero, the following set of coefficient equations can be obtained:

$$a_n[EI(n\pi/L)^4 - m_f V_i^2(n\pi/L)^2 + \bar{T}_e(n\pi/L) - m_t\omega_j^2] = (8m_f V_i\omega_j + 4w)\sum_{p=2,4,6,\cdots} a_p pn/(n^2-p^2) \quad n=1,3,5,\cdots \quad (33)$$

$$a_n[EI(n\pi/L)^4 - m_f V_i^2(n\pi/L)^2 + \bar{T}_e(n\pi/L) - m_t\omega_j^2] = (-8m_f V_i\omega_j + 4w)\sum_{p=1,3,5,\cdots} a_p pn/(n^2-p^2) \quad n=2,4,6,\cdots \quad (34)$$

These equations can be put in matrix form:

$$\{[K_{np}] - \omega_j[C_{np}] - \omega^2 m_t[I]\}\{a\} = 0 \quad (35)$$

where

$$K_{np} = \begin{cases} EI(n\pi/L)^4 + \bar{T}_e(n\pi/L)^2 - m_f V_i^2(n\pi/L)^2, & n=p \\ (-4w/L)\{pn/(n^2-p^2)\}, & n=\text{odd}, p=\text{even} \\ (-4w/L)\{pn/(n^2-p^2)\}, & n=\text{even}, p=\text{odd} \\ 0, & n\neq p, n+p=\text{even} \end{cases} \quad (36)$$

$$C_{np} = \begin{cases} 0, & n=p \\ (8m_f V_i/L)\{pn/(n^2-p^2)\}, & n=\text{odd}, p=\text{even} \\ (-8m_f V_i/L)\{pn/(n^2-p^2)\}, & n=\text{even}, p=\text{odd} \\ 0, & n\neq p, n+p=\text{even} \end{cases} \quad (37)$$

$\{a\} = \{a_1, a_2, a_3, \ldots\}^T$, and $[I]$ = the identity matrix with value of one on the diagonal and all other equal to zero.

Nontrivial solution of (35) are sought by setting the determinant of the coe- fficient matrix to zero:

$$|[K] - \omega[C] - \omega^2 m_t[I]| = 0 \quad (38)$$

Because the system has an infinite number of natural modes, we have to consider only the first few modes practically. Blevins (1990) included only the first two modes in his appropriate analysis, then a_3, a_4, a_5,... are set equal to zero. In this paper, the more number of natural modes will be included in practical analysis.

3.2 MDOF System with Coriolis Matrix

Since the semi-analytical method included the assumption in derivation and the simplicity in a practical sense as shown in the previous section, the error resulting from the assumption and the simplicity should be checked. The conventional way for determining the system frequency is to construct eigenvalue problem using the matrix equilibrium equation of free vibration system which is already constructed (Hong, 1994) and given by the following equation:

$$[\tilde{M}]\{\ddot{x}(t)\} + [\tilde{C}_{ij}]\{\dot{x}(t)\}[\tilde{T}_{ij} - \tilde{C}_{eij} + \tilde{K}_{ij}]\{x(t)\} = 0 \quad (39)$$

Rewriting (39) in a usual simple form, we get

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = 0 \quad (40)$$

In above equation, **C** is not a usual damping matrix but a coriolis matrix which is skew-symmetric. As discussed earlier, $\mathbf{C}\dot{\mathbf{U}}$ acts like a dynamic coupling force not a damping force. Thus, it can not be allowed to construct eigenvalue problem simply applying conventional method because the imaginary part of eigenvalue, which indicates a damped frequency, is always zero. The complex value with zero imaginary should be implemented and then, the imaginary part has to be checked whether it is equal to zero. This is verified through the numerical example given in section 3.3.2.

3.3 Numerical Evaluation of System Frequency

3.3.1 Algorithm for computer program

Since the eigenvalue problem in section 3.1 is not a standard form, it may be required to implement a slightly different algorithm based on one for the standard form of eigenvalue problem. As recognized in the algorithm summarized in Fig. 2, the non-standard form of eigenvalue problem has to be resolved for every iteration step. In this paper, the inverse iteration algorithm with shifting value is chosen to solve the given eigensystem. The technique of inverse iteration is very effectively used to calculate an eigenvector, and at the same time the corresponding eigenvalue can also be evaluated. Inverse iteration, like a usual vector iteration method, can be employed even if some diagonal elements of mass matrix has zero value.. Further, the shift of eigenvalue improves the convergence rate in the vector iteration and accelerates the calculation of the eigenvectors. Those techniques are also introduced in this paper and the computer program, of which algorithms are referenced from the text of Bathe and Wilson (1976), has been developed in this study.

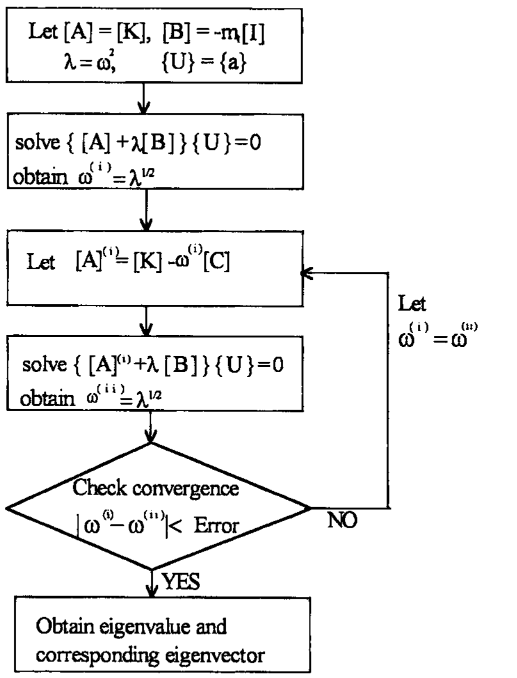

Fig. 2. Algorithm for solving non-standard eigenvalue problem.

3.3.2 Comparison of results

For the verification of the solutions from semi-analytical method, the comparisons are made in two different ways. The first is to calculate the system frequencies of the system with the exclusion of internal flow using semi-analytical method and then compare those with the results from previous investigators. This comparison provides the validity of the representation of solution into the sum of two components as shown in (25) but it does not supply enough evidence for the system including the internal flow. Thus, another one in section 3.2 is implemented to compare with the results from the semi-analytical method. This comparison gives enough evidence even for the system including internal flow. The comparison with Chen's (1992) results may be also given. However, in the expansion of cosine term into Fourier series, he followed the misuse of Fourier coefficients which was also found in Housner's (1952) paper. The wrong expansion of the coriolis force term may lead to the error in the calculation of the system frequency with internal flow.

Table 1. The design properties and for a drilling riser system for use in the Northern North Sea.

Outside diameter	D = 60.96 cm, with 15.875 mm wall thickness
Elastic modulus	$E = 2.1 \times 10^6$ kg/cm^2
Sectional moment of inertia	I = 131,018.2 cm^4
Riser length	L = 152.4 m
Riser mass	m = 31.04 kg/m (includes mass of drilling mud and sea water)
Bottom tension	TTB = 1,271.350 kN
Effective weight per unit length	w = 393.89 kg/m
Mean tension	$\bar{T}_e$ = 1,452.8 kN
Density of drilling mud	ρ_m = 1363.6 kg/m^3
Density of sea water	ρ_w = 1039.0 kg/m^3

Table 2. Comparison of system frequencies of the 1st to 10 th modes for a drilling riser system with no internal flow. (System frequency = ω_i (rad/sec), $V_I = 0$ m/s).

Mode i	Semi-Analytical Method	Chen	Dareing	Kim	Spanos
1	0.81813	0.81585	0.81498	0.81484	0.831
2	1.80520	1.80430	1.80362	1.80390	1.831
3	3.08798	3.08585	3.08762	3.08328	3.123
4	4.73703	4.73268	4.73748	4.73328	4.778
5	6.78865	6.78123	6.78901	6.78934	6.832
6	9.26100	9.24969			9.301
7	12.1634	12.14743			12.193
8	15.5008	15.47942			15.506
9	19.2760	19.24850			19.232
10	23.4907	23.45631			23.352

Table 3. Comparison of system frequencies of the 1st to 10 th modes for a drilling riser system with no internal flow (System frequency = ω_i(rad/s), $V_I = 6$ m/s).

Mode i	Semi-Analytical Method	Numerical method described in section 3.3	
		Real value	Imaginary value
1	0.81701	0.81417	0.00231
2	1.80326	1.80030	0.00194
3	3.08550	3.06014	0.00102
4	4.73421	4.73527	0.00032
5	6.78562	6.78776	0.00003
6	9.25782	9.26105	0.000007
7	12.1601	12.1916	0.000001
8	15.4975	15.4908	0.000000
9	19.2727	19.2735	0.000000
10	23.4889	23.4909	0.000000

The example data, which is for a typical drilling riser system for use in the northern North sea and repeated from Chen (1992), is given in Table 1. Using those data, the first comparison is given in Table 2 which presents the comparison of the natural frequencies of the 1st to 10th modes obtained from semi-analytical method, Dareing and Huang (1976), Spanos and Chen (1980), Kim (1988) and Chen (1992). The discrepancy between those methods including semi-analytical method is negligible, that is, the representation of solution such as (25) is acceptable.

For the riser system with internal flow of 6 m/s, the comparisons of the 1st to 10th modal frequencies calculated using both semi-analytical method and numerical method described in section 3.2 are given in Table 3. The system frequencies obtained using second method are complex values which consist of real and imaginary part. The imaginary part represents damping frequency. As shown in Table 3, the imaginary parts have almost zero values, that is, the mixed derivative term in governing equation or the coriolis skew-symmetric term in matrix equilibrium equation does not have the same effect as that of viscous damping force although it involves the time derivatives. In other words, the mixed derivative term or the coriolis matrix indicates the dynamic coupling force as discussed earlier. Further, it can be recognized from the table that the discrepancy in the system frequencies resulting from the two different methods is negligible. Thus, both two methods developed in this chapter are acceptable for the estimation of the system frequency regardless of the existence of internal flowing fluid.

Of two methods, semi-analytical method is more convenient than the other because there is no need to construct the coefficient matrices through the finite element approximation. Thus, semi-analytical method will be adapted for the investigation of the effect of internal flow on system frquency.

4. THE EFFECT OF INTERNAL FLOW ON SYSTEM FREQUENCY

For the deep sea application, the riser system is, in general, designed to be nearly neutrally buoyant. Under such condition, the effective weight, the difference between riser weight and local net buoyancy, which results from the combined fluid action of external and internal hydrostatic pressure, is equal to zero, that is, the balance between riser weight and local net buoyancy is locally maintained for the entire length of the riser.

The system frequencies were computed for different water depths with six different internal flow conditions and three different top tensions. Fig. 3 shows the relationship between water depth and system frequency of a neutrally buoyant riser system, i.e., w=0. The ordinate is the ratio of the 1st mode frequency to the natural frequency of the system with null internal flow. As can be seen in the figure, the system frequency is drastically affected by the internal flow for the null tension case. Finally, riser buckles when the flow velocity approaches a critical velocity. The increase of top tension, on the other hand, counters the internal flow effect as seen on the second and third figures. With adequate top tension, the system frequency approaches constant as water depth increases. This is

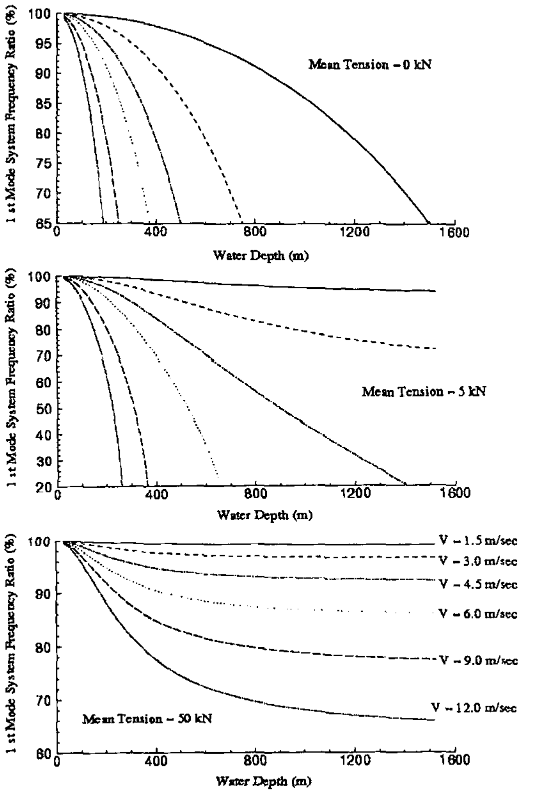

Fig. 3. Internal flowing fluid effect on 1st mode system frequency of a neutrally buoyant riser system. The internal flow velocity changes from 1.5 m/s to 12.0 m/s and mean tension (same as top tension) does from null to 50 kN.

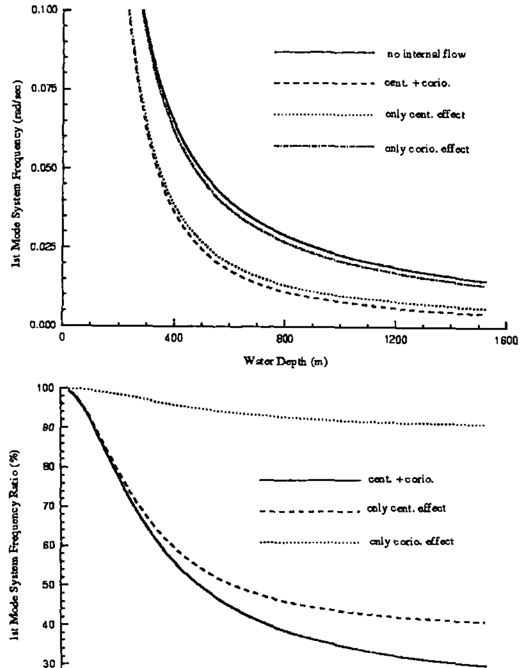

Fig. 4. Comparison between the effects of each forcing terms (centrifugal and coriolis forces) due to internal flow on 1st system frequency. (Mean tension = 50 kN, Internal flow velocity = 12 m/s).

because the influence of the stiffness term becomes negligible. Fig. 4 shows the separate effects of the two flow-induced force components centrifugal and coriolis forces on a riser system. This is one of the cases in Fig. 3, with a mean tension of 50 kN and an internal flow velocity of 12 m/s. From Fig. 4, it can be seen that the main effect on system frequency is due to centrifugal force. However, even though for the case shown the coriolis force seems to have a minor influence on system frequency, it remains unanswered on its effect on system dynamics such as displacement and stress.

Also, as discussed, internal flow, as it travels along the curved path in the riser, generates centrifugal and coriolis force. These dynamic forces exerted against the riser, in turn, affect the dynamic behavior of a riser. The centrifugal force reduces the stiffness of a riser whereas the coriolis force causes the dynamic coupling with other forces.

5. CONCLUSIONS

From the results of sample computations, the effects of internal flow on riser dynamics are examined. The following conclusions are drawn:

1) There are two dynamic forces due to the motion of internal fluid, that is, centrifugal and coriolis force. The centrifugal force, which depends only on the curvature of riser deflection, does not alter displacement shape whereas the coriolis force, which is a term with mixed derivative of time and space, distorts the displacement shape.

2) The system frequency is significantly affected by the internal flow for the null tension case. The riser will buckle finally when flow velocity reaches a critical velocity. The increase of top tension, on the other hand, partially counters the internal flow effects. Further, the main effect on system frequency is due to centrifugal force.

3) It was concluded that the natural frequencies could be reduced drastically by the fluid dynamic force if the tension was insufficient for a neutrally buoyant riser system, and that the dynamic force had less influence on a positively or negatively buoyant riser system.

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