OPTIMAL BURN-IN FOR MULTIOBJECTIVES
다목적 경우의 최적 Burn-In 방법

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Abstract

최적 Burn-In 방법은 신뢰성이나 평균수명을 극대화 하고 위험율과 Cost를 극소화 하는 것이다. 기존의 연구는 하나의 목적을 대상으로 Burn-In 방법에 대하여 연구 하였으나 상호 상충되는 목표에 대해 의사 결정을 하는 복잡하고 어려운 상황을 고려하여야 한다. 그러므로 둘 이상의 목표에 대한 최적의 Burn-In 방법에 대하여 연구 되어야 한다.

본 논문에서는 이를 위해 Surrogate Worth Trade Off 기법을 사용하여 실제 최적의 Burn-In 방법을 구하고자 하는 경우에 대하여 연구하였다.

1. INTRODUCTION

Washburn (1970) presented a mathematical model for optimal burn-in procedure with minimum burn-in cost for the first time. Plesser and Field (1977) presented a cost-optimized burn-in model. Chandrasekaran (1977) determinated the optimal burn-in procedure to maximize mean residual life. Since then, most works on optimal burn-in procedure have been treated to optimized single objective such as total burn-in cost, minimum failure rate, maximum mean residual life, or maximize reliability. Haimes (1975) introduces "surrogate worth trade-off method" for solving multiple objective problems. For multiobjective burn-in problem, the method is very powerful. In practice, we usually have two objectives for burn-in. For example, if we consider the best reliability at time x, then $R(x \mid t)$ should be maximized with minimizing C_{AV}. The optimal burn-in procedure should be as following:

$$\begin{aligned}
&\text{MIN Total average cost } (C_{AV})\\
&\text{MIN } -R(x \mid t)\\
&\text{s.t. } R(x \mid t) \geq R_{\min}(x)\\
&\qquad C_{AV} \leq C_{(\min)AV}
\end{aligned}$$

If we need a longer average life, then the goal should be maximizing $m(t)$. Then, the optimal burn-in should be as following:

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MIN Total average cost (C_{AV})
MIN - $m(t)$
s.t. $m(t) \geq MTTF$
$C_{AV} \leq C_{(\min)AV}$

where

$R_{\min}(x)$ = the required reliability at time x by customers or manufacturers
$MTTF$ = the required mean time to fail,
$C_{(\min)AV}$ = the required total average cost.

2. OPTIMIZATION FOR SERIES SYSTEMS

Notation

- $R_s(x \mid t)$ = system conditional reliability
$R_j(x \mid t)$ = subsystem conditional reliability
$r_j(x \mid t)$ = Conditional reliability of each component in subsystem j
$R_{S,\min}(x)$ = required system reliability at time x
$R_{j,\min}(x \mid t)$ = required subsystem reliability at time x
$C_{S,AV}$ = system average cost
$C_{S,\min}$ = required system average cost
n_j = total number of redundant components in subsystem j
N = number of subsystems in series

2.1 Model Assumptions

1. There are N Independent subsystems in series.
2. there are n_j i.i.d. components in each subsystem; n_j is fixed.
3. Each subsystem is 1 out of n_j. A subsystem is good until all n_j components fail
The system conditional reliability is as follow

$$R_s(x \mid t) = \prod_j R_j(x \mid t) \qquad where \ \ R_j(x \mid t) = 1 - [1 - r_j(x \mid t)]^{n_j} \tag{1}$$

4. The system average cost is follow;

$$C_{S,AV} = \sum_j C_{j,AV}(r_j) n_j \tag{2}$$

where $C_{j,AV}(r_j)$ is the average cost for component j

Then, we have the following multi-objective problem for series systems;

$$MIN \quad -[R_s(x \mid t) = \prod_j R_j(x \mid t)] \tag{3}$$

$$MIN \quad C_{S,AV} = \sum_j C_{j,AV}(r_j)n_j$$

s.t

$$R_s(x \mid t) \geq R_{S,\min}(x)$$

$$R_j(x \mid t) \geq R_{j,\min}(x)$$

$$C_{S,AV} \leq C_{S,\min}$$

Procedure

Since we have two objective functions we must compromise in our preferred solution. We apply Surrogate Worth Trade-off Method to find the preferred optimal solution for the above problem.

3. Illustrative Example

Suppose that we defined the following values of the constraints and parameters for the two-mixed Weibull distribution in time-to-failure pattern for a particular product.

$$p_1 = 0.05 \quad p_2 = 0.2 \quad \beta_{11} = \beta_{12} = \beta_{21} = \beta_{22} = 1$$

$$\eta_{11} = 3,000 \quad \eta_{21} = 20,000 \quad \eta_{12} = 1,000 \quad \eta_{22} = 18,000$$

$$C_{S,\min} = 4,500 \quad R_{S,\min}(x) = 0.85 \quad R_{j,\min}(x) = 0.9 \quad x = 25,000 \quad j = 1,2$$

$$n_1 = 8 \quad n_2 = 10$$

If our goal is the maximizing conditional reliability at mission time $x = 25,000$ hours while minimizing the total burn-in cost, we apply the static 2-objective multiplier algorithm of the SWT method (Haimes, 1975) to solve the two objective problem.

First step is to find the upper bound of λ_{12} by solving the following

$$\text{MIN} \quad -R_s(x \mid t)$$

s.t.

$$C_{S,AV} \leq C_s(\min)$$

$$R_s(x \mid t) \geq R_{S,\min}(x)$$

$$R_1(x \mid t) \geq R_{1,\min}(x)$$

$$R_2(x \mid t) \geq R_{2,\min}(x)$$

where

$C_s(\min)$ is the solution of the following procedure;

$$\begin{aligned}
&\text{MIN } C_{S.AV} \\
&\text{s.t.} \\
&\quad C_{S.AV} \le C_{S,\min} \\
&\quad R_s(x \mid t) \ge R_{S,\min}(x) \\
&\quad R_1(x \mid t) \ge R_{1,\min}(x) \\
&\quad R_2(x \mid t) \ge R_{2,\min}(x)
\end{aligned}$$

The Lagrange multiplier for the $C_{S.AV}$ constraint is the upper bound of λ_{12}. This step is computationally too difficult in most cases. We use the augmented Lagrangian Algorithm (Donald & Michael, 1975) to find the upper bound of λ_{12}. For the example problem, we have the upper bound of λ_{12} equal to 0.00195313.

Different values of λ_{12} are selected from the interval (0, upper bound of λ_{12}) and the corresponding Pareto optimal solutions are calculated. The results are in Table 3. Then, The DM (decision making) is asked for the following questions: "would you be willing to pay an additional one unit of cost in order to improve the system conditional reliability by λ_{12}? "Rate your willingness on a scale from - 10 (totally unwillingness) to +10 (totally willingness) with zero signifying indifference." If the willingness scales are in Table 3.5, then W_{12} (willingness) = 0 is attained at the 5th row. This implies that the DM has reached his compromise solution. Therefore, we have the preferred values of the decision variables as follow:

t_1 (optimal burn-in time for the first component type) = 39.65938

t_2 (optimal burn-in time for the second component type) = 2293.76

$R_1(x \mid t)$ = 0.921402 and $R_2(x \mid t)$ = 0.9337719

Table 3: Pareto optimal solutions and DM responses

Trade off ratio	R(x|t)	System cost	Willingness
1/1000	0.860716	3762.5	+8
1/9000	0.86301	3771.072	+3
1/10000	0.863803	3778.017	+2
1/11000	0.863987	3780.044	+1
1/11500	0.864016	3780.395	0
1/11600	0.864022	3780.462	-1
1/11700	0.864027	3780.528	-2
1/12000	0.85962	3764.456	-10
1/20000	0.85962	3764.456	-10
1/300000	0.85962	3764.456	-10

Conclusion

The optimal burn-in times for three reliability measures do not coincide (Kim and boardman, 1996). In other words, an optimal burn-in procedure that optimizes one reliability measure does not yield an optimal value for another measure. Therefore, the decision of which measure should be considered for a particular product (or component) before planning the burn-in procedure. For example, if the goal is to improve the average life of a particular product, then the mean residual life ($m(t)$) is the right choice. If the goal is to minimize the failure rate ($h(t)$), then the failure rate is the relevant measure. If the goal is to improve reliability at mission time x (fixed length), then the conditional reliability ($R(x \mid t)$) is the relevant measure to be considered for the burn-in procedure.

After making a decision on which measure should be considered for a particular product and formulating the cost model, we need to frame the burn-in problem as a multiobjective optimization problem. In practice, we have at least two objectives for the burn-in problem. If we consider the best reliability at time x, then $R(x \mid t)$ should be maximized while minimizing C_{Av}(total average burn-in cost). On the other hand, if we need a longer average life, then the goal should be maximizing $m(t)$. The Surrogate Worth Trade-Off method is very powerful for solving the multiple objective function burn-in problem.

References

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APPEDIX

Surrogate Worth Trade-Off Method

The n objective multiplier algorithm of the SWT method is follow:

Part 1

1. Choose initial values for $\Lambda > 0$.

2. Solve

$$\min \ f_1(\underline{X}) + \Lambda f_{-2}(\underline{X})$$
$$s.t. \quad \underline{X} \in \overline{X}$$

The solution vector $\underline{X}^*$ is substitute into $f_1(\underline{X})$ and $f_{-2}(\underline{X})$ to find $f_1^*(\Lambda)$ and $f_{-2}^*(\Lambda)$.

3. If enough information has been generated, go on to next step. If not, choose an new value for $\Lambda > 0$ and go back to step 2.

Part 2

4. For each set of values Λ, $f_1^*(\Lambda)$, $f_{-2}^*(\Lambda)$ at which the worth is desired, ask the DM for his assessment of how much λ_{1j} additional units of objective f_1 are worth in relation to one additional unit of objective f_j. The assessment is made on an ordinal scale (from -10 to +10 with zero signifying equivalent worth). The assessment is the value of $W_{1j}(\Lambda)$ (surrogate worth functions) This step is repeated for all $j = 2, 3, \ldots, n$.

5. Repeat step 4 until we find Λ^* such that $W_{1j}(\Lambda^*) = 0$ for $j = 2, 3, \ldots, n$.

6. Find the preferred decision vector X^* by

$$\min \ f_1(\underline{X}) + \Lambda^* f_{-2}(\underline{X})$$
$$\text{s.t.} \quad \underline{X} \in \overline{X}$$

7. A sensitivity analysis could be performed to determine the possible effects of implementing the preferred solution.

8. Stop!

The $W_{1j}(\Lambda)$ (surrogate worth functions) can be defined as following:

a. $W_{1j} > 0$ when λ_{1j} marginal units of $f_1(\underline{X})$ are preferred over one marginal unit of $f_j(\underline{X})$, given the level of achievement of all the objectives.

b. $W_{1j} = 0$ when λ_{1j} marginal units of $f_1(\underline{X})$ are equivalent to one marginal unit of $f_j(\underline{X})$, given the level of achievement of all the objectives.

c. $W_{1j} < 0$ when λ_{1j} marginal units of $f_1(\underline{X})$ are not preferred over one marginal unit of $f_j(\underline{X})$, given the level of achievement of all the objectives.