Graphical Approach to Access Weak Population
- 불량품 적출을 위한 그래프 기법 -

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Abstract

CMOS부품에는 양품과 불량품이 혼재되어 있는 경우가 많다. 이 경우, 부품의 신뢰성을 향상 시키기 위해서는 불량품을 제거하여야 한다.

이와 관련하여 불량품과 양품의 모수를 구하기 위해 많은 연구가 있었다.

본 논문에서는 Jasen & Petersen 기법을 Bayesian 기법의 실제 상황에 대한 적용에 대하여 연구하였다.

Introduction

Electrical or micro-electrical components are subjected to stress testing in order to "burn-in" the components prior to futher assembly. A two-mixed Weibull distribution is a good model to describe the time-to-failures of electrical components. Kao[4] introduced a two-mixed Weibull distribution to describe the failure time of electronic tubes. stitch [5] found that the failure time of microcircuits follows a mixed distribution. Reynolds & Stevens [2] also found that two-mixed Weibull distribution describe the time-to-failure patterns of electronic components. For mixed population, we have two representative cases; well separated and well mixed. Well separated distribution has a flat portion between weak and main populations; e. g., the Weibull probability plot shows S shape curve with truly flat area between two populations. In this case, there is no difficulty in finding the proportion of weak population using Jensen & Petersen method. On the other hands, if the mixed population has no flat portion on its Weibull probability plot, Bayesian approach [1] is recommended to estimate the proportion of weak population and the parameter values of weak and main population.

In this case study, we show that the mixed Weibull distribution in time-to-failure patterns is not the special case in practice and the graphical analyses to estimate the proportion of weak population and the parameter values of the mixed population.

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Two-Mixed Weibull Distribution

A two-mixed Weibull distribution is composed of two pdfs. Let $F_1(t)$ be the CDF (cumulative density function) of weak population, $F_2(t)$ be the CDF of main population (the rest of the components), and F(t) be the total CDF for the entire population. Then, $F(t)$ is constructed by taking a weighted average of the CDFs for the weak and main subpopulation has p proportion, and the main population has (1-p) proportion, then

$$F(t) = p\,F_1(t) + (1-p)\;F_2(t) \qquad (1)$$

Typically, $F_1(t)$ has a high early failure rate while $F_2(t)$ has a low early failure rate that either stays useful or weariest region in life.

The failure rate of the two-mixed distribution is expressed as

$$h(t) = \frac{p\,f_1(t) + (1-p)\;f_2(t)}{1-\{p\,F_1(t) + (1-p)\;F_2(t)\}} \qquad (2)$$

Now, let us consider the two-mixed Weibull distribution. From (1), the CDF of a two-mixed Weibull distribution is as follow:

$$\begin{aligned} F(t) &= p\,F_1(t) + (1-p)\;F_2(t) \\ &= 1 - p(\exp[-\;(t/\;\eta_1)^{\beta_1}] - (1-p)(\exp[-\;(t/\;\eta_2)^{\beta_2}]) \qquad (3) \end{aligned}$$

where

β_1 = shape parameter of weak population,

β_2 = shape parameter of main population,

η_1 = shape parameter of weak population,

η_2 = shape parameter of main population,

p = proportion of weak population.

The scale parameter is also called as a characteristic life at which 63.2% of units will have failed.

Jensen & Petersen method

The method is :

1. Plot the sample data on Weibull Probability Paper and fit a smooth curve by inspection.

2. Determine the place with the smallest slope on the CDF curve (where the curve levels off), and read the corresponding p value from the Y-axis (percentage failures). p represents the mixing weight of the subpopulation (weak population) located at the left.

3. Determine $\widehat{\eta_1}$ & $\widehat{\eta_2}$ by entering the Y-axis at 0.632 $\hat{p}$ and $\hat{p}$ + 0.632(1- $\hat{p}$) horizontally; intersecting the CDF curve and dropping down,

then $\widehat{\eta_1}$ & $\widehat{\eta_2}$ can be read from the X-axis (time-to-failure)

4. Determine $\widehat{\beta_1}$ & $\widehat{\beta_2}$ from the slopes of the tangent lines which are drawn at each end of the CDF curve.

Bayesian Approach

1. Calculate the probability of failure i belonging to the weak and main subpopulations as follow:

Weak population :

$$\text{weak population: } \hat{P}_1^i = \frac{(f_1 \mid t_i)}{(f_1 \mid t_i) + (f_2 \mid t_i)}, \quad \text{Main population: } \hat{P}_2^i = 1 - \hat{P}_1^i$$

where

$$(f_1 \mid t_i) = \frac{\widehat{\beta_1}}{\widehat{\eta_1}} \exp\left[-(t_i/\ \hat{\eta}_1)^{\widehat{\beta_1}}\right]\left(\frac{t_i}{\hat{\eta}_1}\right)^{\widehat{\beta_1}-1} \text{ and } (f_2 \mid t_i) = \frac{\widehat{\beta_2}}{\hat{\eta}_2} \exp\left[-(t_i/\ \hat{\eta}_2)^{\widehat{\beta_2}}\right]\left(\frac{t_i}{\hat{\eta}_2}\right)^{\widehat{\beta_2}-1}$$

In the above equations, $\hat{\beta}_1$ and $\hat{\eta}_1$ are estimated parameters of the weak population and $\hat{\beta}_2$ and $\hat{\eta}_2$ are estimated parameters of the main population

2. Calculate the proportion of weak population as follow:

$$\hat{P} = \frac{\sum_{i=1}^{r} \hat{P}_1^i}{N} \tag{4}$$

where r is the number of failures and N is the sample size.

Calculations are quite sensitive to variations in $\hat{\beta}_1$ and $\hat{\eta}_1$ However, the calculations are not sensitive to the variations for $\hat{\beta}_2$ and $\hat{\eta}_2$.

Life Testing Procedure

In the micro-electronics field, it is common practice to submit prototype RAM chips to electromigration stress testing for metals in order to study the aspects of this form of burn-in. The author with cooperation from a local firm secured data from the burn-in results of 16 RAM chips is in Table 1. The time-to-failure in hours was recorded One unit had not failed. The stress test on the micro-chips of the lot has been done for the month period. The testing device only can accommodate 16 micro-chips at the same time and testing time for micro-chips vary, The testing time is terminated when most devices out of 16 micro-chips have failed. The test is conducted at the temperature of 200℃. Figure 1 describes the design of the micro-chip. In the diagram, the right square is the metal that is tested for its lifetime.

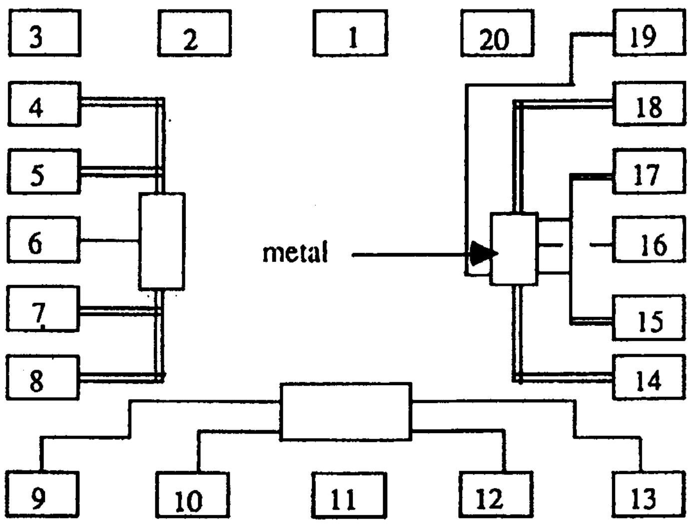

Figure 1 : The design of RAM

Now, Weibull analysis techniques are applied to the life stress testing data in Table 1. First, failure data are taken from the 16 micro-chips in the lot and median ranks for the failures are shown in Table 1.

The following equation for median ranks is used to analyze the data.

$$\text{Median Rank} = \frac{i-0.3}{n+0.4} \times 100 \quad \text{where } i = \text{order and } n = 16.$$

Table 1 : Median Racks for the failures in the lot

Order (i)	Time-To-Failure (hours)	Median Rack (%)
1	215.01	4.27
2	301.09	10.36
3	315.00	16.46
4	319.48	22.56
5	328.321	28.66
6	347.21	34.36
7	425.28	40.85
8	452.57	46.95
9	491.03	53.04
10	706.36	59.15
11	739.30	65.24
12	944.52	71.34
13	1349.29	77.44
14	1482.34	85.54
15	1763.34	89.63
16	not failed	

Next, using time-to-failures and median ranks, a Weibull probability plot is prepared in Figure 2. If the time-to-failure pattern follows S-curve, we could say that the device has weak parts and strong (main) parts (for more detail, see [3]). The advantage of using a Weibull plot is that we can identify the proportion of weak population approximately just by looking at the flat portion between two population on the Weibull plot.

On this type of plot, the vertical axis is the double natural log of percent failures for the sample and the horizontal axis is the natural log of the time-to-failure.

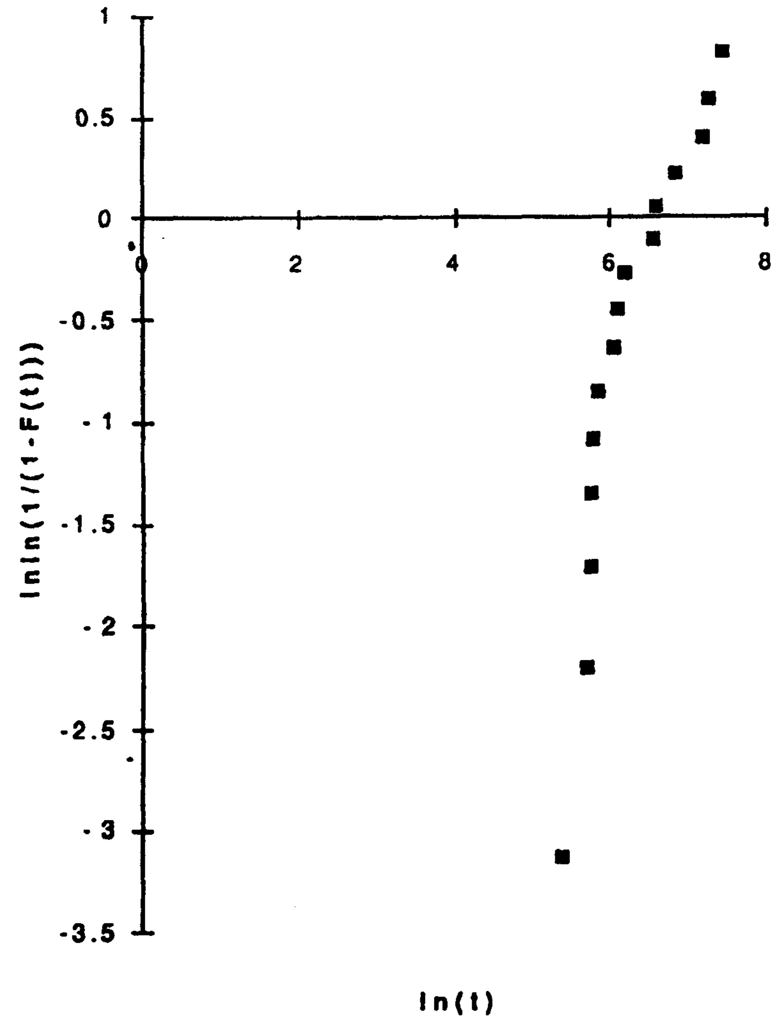

Figure 2: Weibull Plot of the time-to-failures of RAMs in the lot

From Figure 2. we see that the failure points follow a S-curve pattern and there is no flat area between two population. This suggests that the time-to-to-failure pattern follows the well-mixed Weibull distribution law.

There is certainly no truly flat portion in Figure 2. This implies that the mixed population is not well separated. therefore, we apply both Jensen & Petersen method and Bayesian approach to estimate the proportion of the weak population and the parameter values of weak and main populations. First, we apply Jensen & Petersen method to estimate approximate parameter values of mixed populations. The analysis is started by evaluating the main population (distribution). For the main population, we have slope $\hat{\beta}_2 = 1$ from Figure 2. From this line, read off the value of the characteristic lifetime $\hat{\eta}_2 = 820$ hours. For the weak distribution, we first guess of the proportion $\hat{p} = 0.38$ percentage from the Figure 2. Then, the characteristic lifetime of the weak failures is read at the intersection of the Weibull curve with 0.632×0.38 = 0.24 (24%) which gives a value of $\hat{\eta}_1 = 310$ hours.

Bayesian analysis technique is now applied to the data using the following parameters;

$$\hat{\beta}_1 = 5, \qquad \hat{\beta}_2 = 1.0$$

$$\hat{\eta}_1 = 310, \qquad \hat{\eta}_2 = 820$$

In the preceding equations, $\hat{\beta}_1$ and $\hat{\eta}_1$ are estimated Weibull parameters of the weak distribution and $\hat{\beta}_2$ and $\hat{\eta}_2$ are estimated Weibull parameters of main population. The results of the Bayesian analysis are summarized in Table 2.

Table 2 : Bayesian Probabilities for RAMs in the lot

Order (i)	Time-To-Failure (hours)	
1	215.01	0.772
2	301.09	0.877
3	315.00	0.875
4	319.48	0.873
5	328.321	0.867
6	347.21	0.845
7	425.28	0.379
8	452.57	0.121
9	491.03	0.007
10	706.36	0
11	739.30	0
12	944.52	0
13	1349.29	0
14	1482.34	0
15	1763.34	0
16	not failed	0

Then, we have $\hat{p} = (5.61748/16) = 0.351$ from equation (4). Therefore, the proportion of weak failures calculated from this method estimated to be 35%. This implies that all of the weak failures will be eliminated when 35% of the devices have failed. This percent of components have failed at about 347 hours, refer to Figure 1.

Conclusion and Recommendations

The following observations were made based on the results of the evaluation for the above case study.

1. Bayesian approach estimated the proportions of weak failures to be 35%. This suggests that the lot should be burned-in or screened to remove those weak failures. Since the failure rate is very high, going back to design stage is strongly recommended rather than burn-in.

2. From this study, we see that electrical devices such as micro-chips follow the two-mixed Weibull distribution law in time-to-failure patterns. Therefore, the mixed Weibull distribution in time-to-failure patterns may not the special case in practice.

This case study is presented to show that Jensen & Petersen'method and Bayesian approach can be a practical tool to aid in making decisions, even when only a small amount of data is available for analysis. Because failures in electronic components tend to occur in fairly predictable patterns, the techniques described become valuable.

References

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