Lift of and Wave Breaking behind a Moving Submerged Body with Shallow Submergence

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Abstract

We consider the following two questions mainly in this study. First one is how the free surface waves affect the lift of a shallowly submerged moving body. For this matter, we reinterpret the theoretical results of Kochin(1936), and point out that the high Froude number approximation is not always on the safer side. Second one is what sort of dimensionless parameters determine the occurrence of wave breaking behind a moving submerged body. Temporarily before getting a better answer, we propose that the two-parameter-plane, namely, the plane of the Froude number and the square root of the ratio of the submerged depth and the body length, may be used for predicting the possibility of wave breaking behind the submerged body. A region in the parameter plane is put forth as that of wave breaking, and the validity of this proposal is shown by its agreement with the existing experimental data of Parkin et al(1955) and those of Duncan(1983). Finally, linear and nonlinear numerical results are compared with the existing experimental data to see in what range of the parameters the linear and nonlinear theory can predict the wave field and the pressure on the body with reasonable accuracy. However, since the experimental data, which offer both the pressure and wave elevation for a submerged moving body, are very scarce, much cannot be attained through this comparative study. Hence, it is strongly recommended to carry out well planned experiments to get such data.

1 Introduction

There have been numerous works on the lift, and on the distribution of the pressure acting on the surface, of a hydrofoil moving near the free surface. One of the most extensive and fundamental works was done by Kochin(1936, translated into English in 1951), who assumed that the depth of submergence is large and accordingly that the boundary condition on the free surface can be linearized. Then he obtained various analytical results regarding the lift and the wave drag of the hydrofoil by using the theory of analytic functions.

Another cornerstone was set up by the experimental work of Parkin, Perry and Wu(1955, hereafter will be denoted as PPW). They reported that there are two regimes

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of Froude number[$F_n = \frac{U}{\sqrt{gL}}$, U is the speed of the body, g the gravitational acceleration, L the body length, respectively], namely, 'the high F_n' and 'the low F_n'. They observed that for the high F_n the phenomenon of wave breaking after the hydrofoil did not occur, while for the low F_n it did. They also measured the pressure distribution on the hydrofoil surface for various cases of F_n and the depth of submergence, and showed that there are many aspects of the flow phenomena which are not easy to understand at least from the academic point of view.

Duncan(1983) carried out a nice set of experiments through which he obtained the data of the free surface elevations and the horizontal velocity profile at a vertical plane produced by a moving submerged hydrofoil with an angle of attack. He also showed that by the wake survey measurements the drag associated with wave breaking reached more than three times the maximum wave drag predicted by a theory, which does not include the effect of the wave breaking. He said that the steady breaker following the hydrofoil was produced when the wave slope was 17° or higher, but did not give any further specific criterion for the occurrence of wave breaking in the trailing region. Coleman(1986) numerically simulated the flow field corresponding to the experimental setup of Duncan(1983).

Our interests here are focused upon two questions. One is how the free surface waves affect the lift of the shallowly submerged moving hydrofoil, and the other is what are the physical parameters determining the wave breaking occurrence behind the hydrofoil.

2 Two questions

In order to answer to the first question mentioned in the section 1, let us consider the result of the linear theory obtained by Kochin(1936). When a vortex of strength Γ [(+) when the rotating flow direction is clockwise] is located in a uniform stream of speed U, its lift Y is given by

$$Y = \rho\Gamma[U - \frac{\Gamma}{4\pi h} + \frac{\Gamma K}{\pi} e^{-2Kh} E_{i1}(2Kh)], \tag{1}$$

where ρ is the fluid density, h the submerged depth, and $K = \frac{g}{U^2}$. Here,

$$E_{i1}(z) = \mathcal{R} \int_{-\infty}^{z} \frac{e^{\zeta}}{\zeta} d\zeta, \tag{2}$$

that is the real part of the complex exponential integral, for which the integration path should be in the lower half plane. Since $\rho U\Gamma$ is the Kutta-Joukowski lift, the other terms may be called as the added lift Y_a due to the presence of the free surface. Then we may define the dimensionless added lift Y_A as

$$Y_A = \frac{Y_a}{\rho\Gamma^2/4\pi h} = -1 + 4Khe^{-2Kh}E_{i1}(2Kh), \tag{3}$$

which is a function of Kh or the depth-based Froude number $[F_h = \frac{U}{\sqrt{gh}} = \frac{1}{\sqrt{Kh}}, Kh = F_h^{-2}]$ alone. We note that $\rho\Gamma^2/4\pi h$ is a Lagally force due to the image vortex. Y_A vs. F_h is shown in Figure 1. There are 4 points of interests; a) maximum of Y_A is 1.97 at $F_h = 0.81$, b) Y_A is zero at $F_h = 1.57$, c) minimum of Y_A is -1.30 at $F_h = 4.08$, d) the limit of Y_A as $F_h \to \infty$ is -1. Since the added lift is positive when F_h is less than 1.57, we may make use of this fact for the design purpose, when the flow condition allows the use of linear theory.

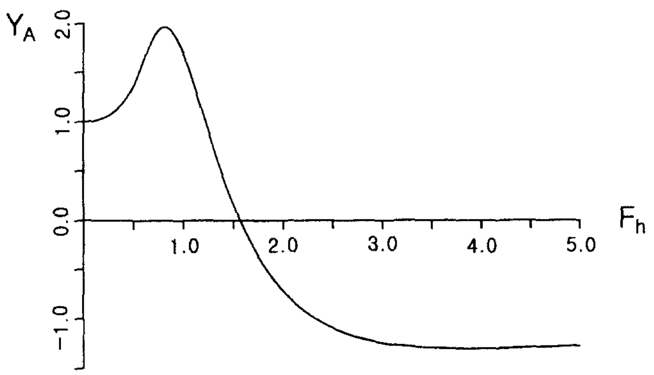


Figure 1: Added lift of a vortex plotted against depth-based Froude number

Another interesting theoretical result is, when the free surface is present, a moving submerged dipole of strength μ is influenced by the lift Y_d given as

$$Y_d = -\frac{\rho\mu^2}{8\pi h^3}[1 + 2Kh + (2Kh)^2 - (2Kh)^3 e^{-2Kh} E_{i1}(2Kh)], \tag{4}$$

which can be rewritten in dimensionless form as

$$Y_D = \frac{Y_d}{\rho\mu^2/8\pi h^3} = -[1 + 2Kh + (2Kh)^2 - (2Kh)^3 e^{-2Kh} E_{i1}(2Kh)], \tag{5}$$

that is again a function of Kh or F_h only. We show the curve of Y_D vs. F_h in Figure 2. The pattern of the curve is similar to that in Figure 1, and the 4 interesting points are as follows; a) maximum of Y_D is 3.04 at $F_h = 0.58$, b) Y_d is zero at $F_h = 0.84$, c) minimum of Y_D is -2.37 at $F_h = 2.39$, d) the limit of Y_D as $F_h \to \infty$ is -1. Considering a circular cylinder, as a first approximation we may take $\mu = \frac{\pi}{2}UL^2$ where now L is the diameter of the circular cylinder, then we see that

$$\frac{\rho\mu^2}{8\pi h^3} = \frac{1}{32}\rho g\pi L^2 {F_n}^2 (\frac{L}{h})^3. \tag{6}$$

From these results, it is clear that there are two important dimensionless parameters for a moving submerged body with circulation around it, namely F_n and $\frac{L}{h}$. The linear theory is based upon the assumption, the ratio $\frac{L}{h} \ll 1$, and for most hydrofoils in application the ratio is less than 0.5. Thus the linear theory is probably useful for practical design problems, but the high F_n approximation may not be on the safer side. For instance, say, when $F_n = 3.0$, $\frac{L}{h} = 0.25$, thus $F_h = 1.5$, for which the value of Y_D is about -2, while that given by the high F_n approximation is -1.

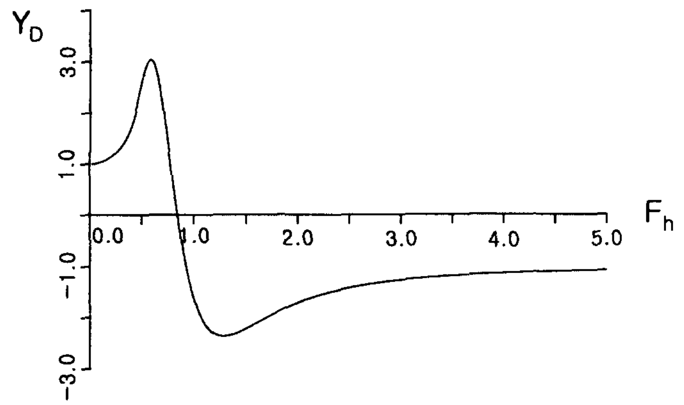


Figure 2: Added lift of a dipole plotted against depth-based Froude number

Related to the second question, we discussed in the previous section two major experimental works, i.e., PPW and Duncan(1983). PPW pointed out that F_n and $\frac{L}{h}$ are important parameters. Duncan also reported that for a given body if the speed of the body is kept constant the wave breaking begins to occur only when the submerged depth decreases to a critical value. We show in the Figure 3 the cases treated by PPW and by Duncan on the plane of two parameters, $\sqrt{\frac{h}{L}} - F_n$. Here h denotes the submerged depth measured from the undisturbed free surface to the trailing edge of the hydrofoil and the points corresponding to those for which the wave breaking in the trailing zone was reported are filled. Now, to make hypotheses on the criteria of the wave breaking, let's consider the following. If h is greater than a half of λ, which is the wavelength of the characteristic wave given by $2\pi F_n^2 L$, the effect of the body on the free surface should not be of significance. Thus, for $F_n < 0.564\sqrt{\frac{h}{L}}$, wave breaking may signify little.

On the other hand, if h is less than $\frac{\lambda}{20}$, corresponding to $F_n > 1.784\sqrt{\frac{h}{L}}$, we may use the assumption of the shallow water. Supposing that it is justifiable to take h as the channel water depth, we may regard the flow supercritical in the sense of open channel flow, since $F_n > \sqrt{\frac{h}{L}}$ is satisfied in the cases under consideration. Then we may not expect much wave action on the free surface for these cases, and in fact this claim is in accordance with the observation of PPW. When these two straight lines are added in Figure 3, we see that the cases encircled almost lie between them. Another part of the criteria can be obtained by considering that when λ is greater than $5L (F_n > 0.892)$ and h is greater than L, the first trailing wave is so far from the body that the possibility of wave breaking there should be close to nil.

All the conditions considered above are not the criteria for the wave breaking itself but those for the heavy or weak influence of the body presence on the free surface waves

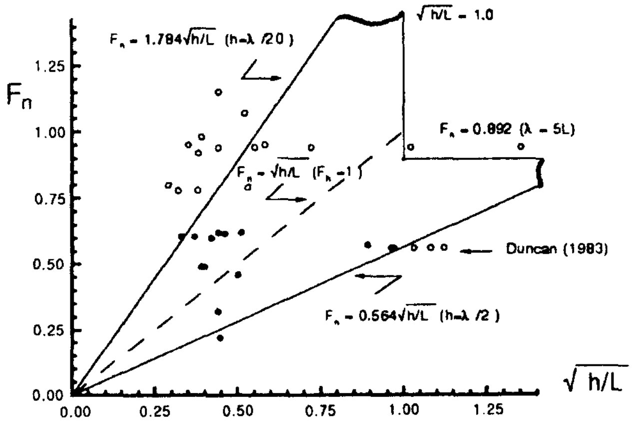


Figure 3: Experimented cases on $\sqrt{\frac{h}{L}} - F_n$ plane

in the trailing zone. Hence, the conditions stated above are not supposed sharply applicable, but replaceable by more sound and accurate ones, and at the moment they are hoped to be used as a quideline for the wave breaking.

3 Example calculations

Although Coleman(1986) and others have computed numerically the surface waves generated by moving submerged hydrofoils, it is not clear yet how the wave breaking phenomena show up in the numerical computations and what the effect of the wave breaking upon the pressure distribution on the hydrofoil is. Also the question that in what range of parameters the linearization is valid does not seem answered fully. To shed some light on these questions, numerical computations were performed for the cases which there exist experimental measurements, and the results are shown in Figures 4 $\sim$ 7.

Letting $\phi(x, y)$ as the velocity potential, we used the following free surface boundary condition(FSBC)

$$(1 + 2\hat{\phi}_x)\phi_{xx} + (K + 2\hat{\phi}_{xy})\phi_y = 0, \text{ on } y = 0, \tag{7}$$

which was first suggested by Lee(1994), and called as the improved Poisson FSBC. Here, $\hat{\phi}$ satisfies

$$\hat{\phi}_{xx} + K\hat{\phi}_y = 0, \text{ on } y = 0, \tag{8}$$

that is linear and called as the Poisson FSBC. As the Kutta condition a variation of the Morino-Kuo type was used, and a potential-based panel method was employed. For the description of the present method we refer to Lee et al.(1993).

In Figure 4 we compare the surface elevations obtained by the present method with the two different FSBC's given in (7) and (8) with the experimental data of Duncan's for $F_n = 0.567$ and $\sqrt{\frac{h}{L}} = 1.153, 1.098, 1.038, 0.997$, respectively. Hereafter, in presenting the wave elevations on the free surface as well as the pressure distributions on the foil surface, all the legth scales are normalized by the chord legth L. In Duncan's experiment, when F_n was kept constant surface waves in the trailing zone started to break when $\sqrt{\frac{h}{L}} = 0.997$. The numerical results with the improved Piosson FSBC also showed wild behavior for the corresponding case as presented in the Figure 4(d). For other cases the improved Poisson solutions (hereafter the solid curves in Figures) agree better with the experimental data than the Poisson solutions(the dotted curves).

In Figure 5, the computed pressure distributions on the upper surface of the hydrofoil are compared with the experimental data of PPW for $\sqrt{\frac{h}{L}} = 0.5$ and $F_n = 1.072, 0.604$, respectively. According to our criteria on the wave breaking, the breaking is expected to occur when $F_n = 0.604$. For this case, the improved Poisson solution got unbounded, but a bounded linear solution with the Poisson FSBC could be obtained and gave reasonable values. For $F_n = 1.072$, for which the wave breaking is not expected, the latter solution agrees better with the experimental data.

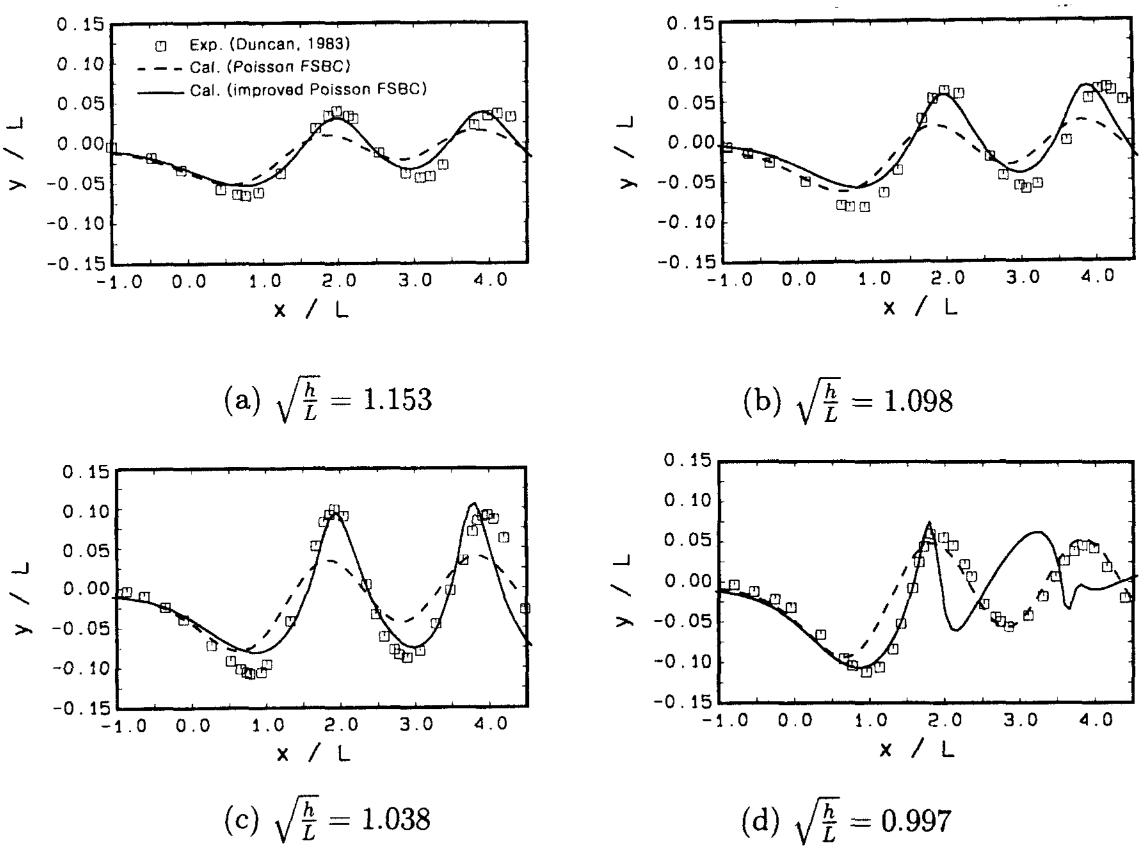
In Figure 6, we show the comparison for $F_n = 0.95$, and $\sqrt{\frac{h}{L}} = 1.34, 0.707, 0.447$, respectively. Again, the wave breaking is expected only when $\sqrt{\frac{h}{L}} = 0.707$, but obtained was a bounded improved Poisson solution, which shows more discrepancy with the experimental data than the linear solution. And we could not get a bounded solution for $\sqrt{\frac{h}{L}} = 0.447$, for which the linear solution agrees with the experiment reasonably well. Furthermore, when $\sqrt{\frac{h}{L}} = 1.34$, the linear solution agrees better with the experiment than the improved Poisson solution.

In Figure 7, the comparison is presented when $\sqrt{\frac{h}{L}} = 0.447$, and $F_n = 1.15, 0.989, 0.617$, respectively. Though the wave breaking is anticipated when $F_n = 0.617$ only, no bounded improved Posson solution was obtained for $F_n = 0.989, 0.617$. And as in the Figure 6 the improved Poisson solution is not much better than the Poisson one.

Since PPW did not say in which case wave breaking occurred, based upon the comparison made above, nothing definite can be said about how the wave breaking in the trailing zone affects the pressure distribution on the hydrofoil, and about the performance of the numerical model we tested.

4 Concluding remarks

We discussed about some analytical results obtained by Kochin, and implied that for most design problems these results can be usefully applied, and also that there are cases when the high F_n approximation is not on the safer side. Then the criteria for the wave breaking in the trailing zone were suggested in terms of two parameters, F_n and $\sqrt{\frac{h}{L}}$, but with the understanding that they can be made much sharper. Finally, some nonlinear and linear numerical results were compared with the existing experimental data, but so



(a) $\sqrt{\frac{h}{L}} = 1.153$ (b) $\sqrt{\frac{h}{L}} = 1.098$

(c) $\sqrt{\frac{h}{L}} = 1.038$ (d) $\sqrt{\frac{h}{L}} = 0.997$

Figure 4. Free surface elevations ($F_n = 0.567$).

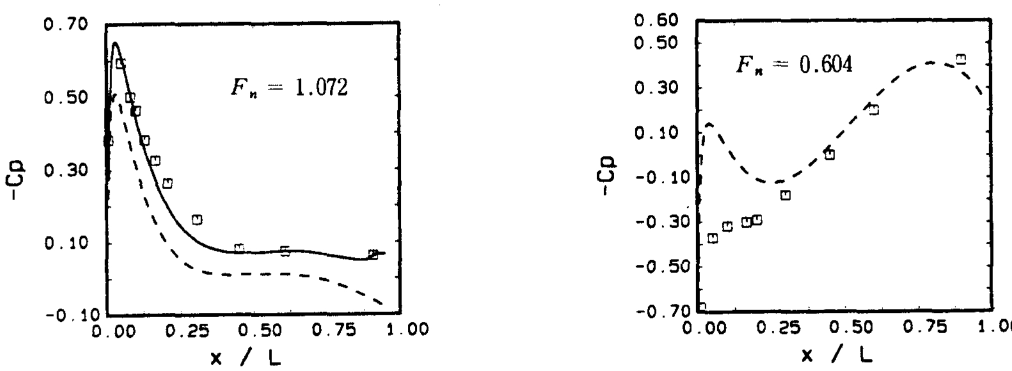


Figure 5: Pressure distribution on the suction side of a Joukowski hydrofoil($\sqrt{h/L} = 0.5$)

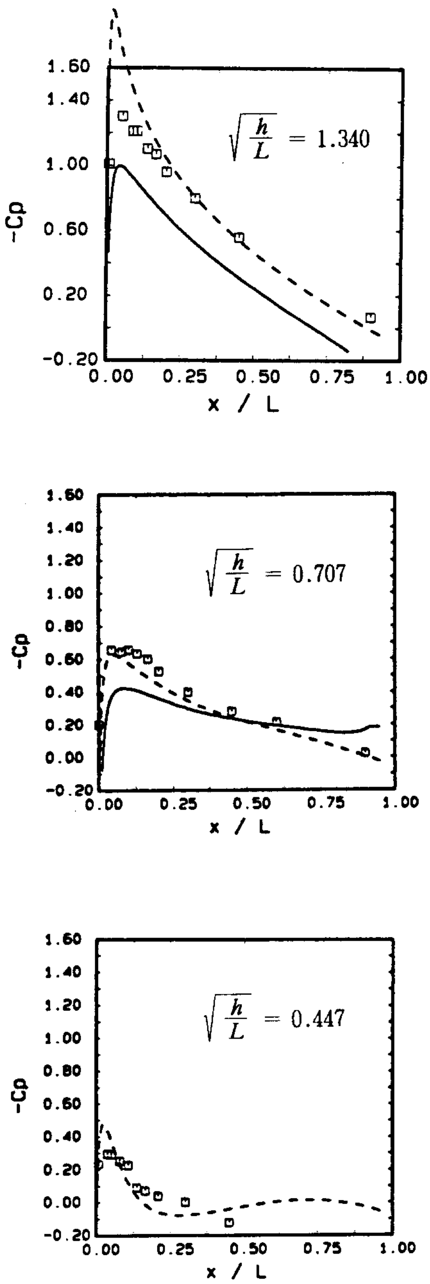


Figure 6: Pressure distribution on the suction side of a Joukowski hydrofoil($F_n = 0.95$)

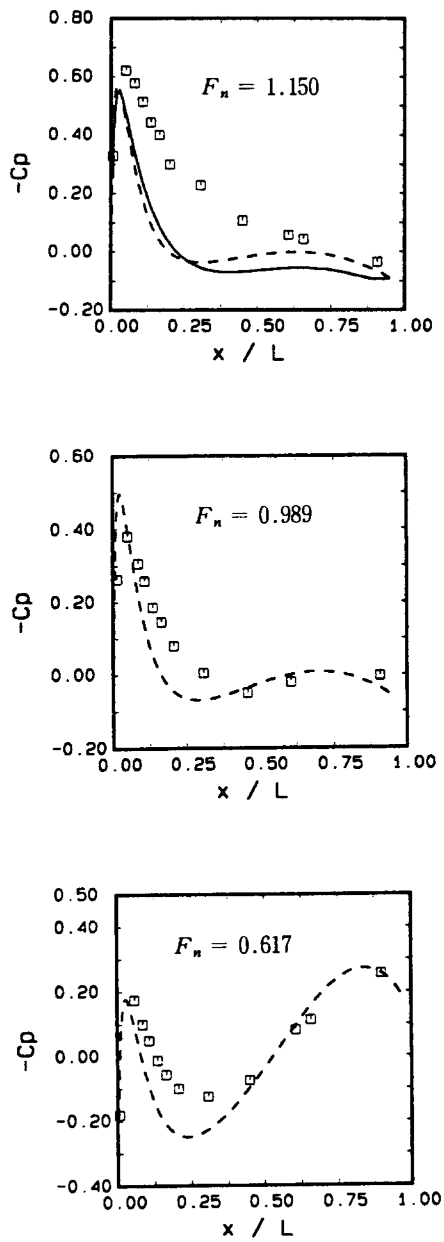


Figure 7: Pressure distribution on the suction side of a Joukowski hydrofoil($\sqrt{h/L} = 0.447$)

far we could not make any definite assessment on the performance of numerical models tested.

To have better and clearer understanding of the wave breaking occurrence and its effects upon the flow field, we think that more experimental data are necessary, especially those of the surface elevations and of the pressure distribution for the same case.

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