

# Performance Analysis of Convolutional Coded DS/CDMA System in Nakagami Fading Channels

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## 나카가미 페이딩 채널에서 길쌈부호화된 DS/CDMA 시스템의 성능분석

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※본 논문은 한국전자통신연구소와 한양대학교 전자재료 및 부품연구센터의 연구비지원에 의한 결과임.

### ABSTRACT

In this paper, we investigate the performance of convolutional coded DS/CDMA system equipped with a noncoherent  $M$ -ary orthogonal modulation scheme, and operating in multi-user environments over slow and frequency nonselective Nakagami- $m$  fading channels with an additive white Gaussian noise (AWGN). An expression for the pairwise error probability that can be used to compute the upper bound of coded system is first derived. Performances of the DS/CDMA system with and without the convolutional codes are then compared. We have observed that the convolutional codes can compensate the degradation quite well in multi-user situations over the Nakagami fading channels with the AWGN. For the case of an extreme fading, however, it has been seen that the convolutional code reaches its limit to improve the overall system performance as the number of users increase.

### 요 약

본 논문에서는 가산성 백색 Gaussian 잡음이 함께 존재하는 느리고 주파수 비선택적인 나카가미 페이딩 채널의 다중 사용자 환경에서 Noncoherent  $M$ -ary 직교변조를 이용한 길쌈부호화된 DS/CDMA 시스템의 성능을 분석하였다. 이를 위해 먼저 길쌈부호화 과정이 결합된 시스템의 비트오류확률 상한식으로 사용되는 Pairwise 오류확률에 대한 관계식을 유도하였다. 또한 길쌈부호가 사용되었을 때와 사용되지 않았을 때에 대한 DS/CDMA 시스템

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의 성능을 서로 비교하였다. 그 결과, 나카가미 페이딩 채널의 다중 사용자 환경에서 길쌈부호화 과정이 시스템 성능저하를 충분히 보상할 수 있음을 알았다. 그러나 페이딩이 깊어지고 사용자의 수가 증가함에 따라 길쌈부호화에 의한 전체 시스템 성능의 향상에는 한계가 있음을 알 수 있었다.

## I. Introduction

During the last few years, the direct sequence/code division multiple access (DS/CDMA) has been one of the most popular radio access techniques for commercial digital cellular systems or personal communication systems. One of the interesting features of the DS/CDMA system is that in the forward link a pilot signal is used so as to provide a reference phase to a mobile station, while in the reverse link the pilot signal is normally not used since it requires the inclusion of the pilot signal for each user [1], [2]. The transmitter of the reverse link employs an  $M$ -ary orthogonal modulation technique instead, so that a noncoherent receiver structure is feasible [3]-[5]. A fading effect is one of the most significant factors that degrade the overall performance of the system. To compensate this performance degradation, a convolutional code can be utilized additionally with the noncoherent  $M$ -ary orthogonal modulation.

The literatures on the fading problem frequently use the Rayleigh, Rician, and log-normal distribution models for representing the strength of a received signal. Another fading model, so called the Nakagami- $m$  distribution [6], fits certain urban experimental data better than the Rayleigh, Rician, or lognormal distributions [7]. While the Rician distribution spans the range of the fading distributions from the Rayleigh fading to nonfading, the Nakagami- $m$  distribution spans the range from the one-sided Gaussian fading to nonfading. Moreover, the Nakagami- $m$  distribution is much more convenient for the mathematical analysis comparing to the Rician distribution because the Nakagami- $m$  distribution is a central distribution [8].

The performance analysis of DS/CDMA system with noncoherent  $M$ -ary orthogonal modulation in an

additive white Gaussian noise (AWGN) channel were presented by Kim [3] and Bi [5]. They derived the closed form equations for the bit error probability and verified the results by simulations. Jalloul and Holtzman [9] extended the work in [3] by analyzing the performance of the DS/CDMA system in multipath fading channels. Bi [10] extended his previous work in [5] by analyzing the performance of the DS/CDMA system in the fading channel that has three different kinds of multipathes. All these results were, however, for the DS/CDMA system without the convolutional codes.

Chang and Sollenberger [11] presented the performance analysis of the convolutional coded CDMA system with the noncoherent  $M$ -ary orthogonal modulation in a single-user environment. Ling and Falconer [12] considered a family of the orthogonal and convolutional codes in the single-user case, and discussed the design and implementational aspects. They also showed that it is desirable to use a code with higher convolutional code rate and a larger orthogonal code size for the noncoherent DS/CDMA system. Unfortunately, however, the closed form or the upper bound equations for the bit error probability were not given.

In this paper, we investigate the performance of convolutional coded DS/CDMA system equipped with the noncoherent  $M$ -ary orthogonal modulation, and operating in multi-user environments over a slow nonselective Nakagami- $m$  fading channel with an AWGN, all in the unified fashion. In particular, we derive expressions that can be used to compute the upper bound of the bit error probability. Performances of the DS/CDMA system with and without the convolutional codes are also compared. In our analysis, the perfect interleaving and no time dispersion are assumed so that each symbol fades independently.

This paper is organized as follows: In the next section, the statements for the system configurations are made. In Section III, we give the bit error probability of the uncoded system in the multi-user environments over Nakagami- $m$  fading channels with AWGN. The pairwise error probability of the convolutional coded system is then derived in Section IV. Numerical results are provided in Section V. Finally, concluding remarks are made in Section VI.

## II. System Configurations

In the transmitter part of the uncoded DS/CDMA system, every  $\log_2 M$  information bits are first modulated by the  $M$ -ary Walsh orthogonal modulator to obtain the Walsh symbols. Each Walsh symbol is then spread by the  $L$  pseudo-noise (PN) chips that are obtained from the long code generator. The resultant signal goes into the in-phase ( $I$ ) and quadrature ( $Q$ ) channels simultaneously, and the signal on each channel is spread once again by the short PN sequence. The final spread sequences are modulated

by the quadrature phase shift keying (QPSK) procedure.

The block diagram of the receiver of this system is shown in Figure 1. The received signal from all different users is first down-converted to the baseband. They are then despread and detected by a bank of the noncoherent  $M$  matched filters.

Now, the transmitted signal of the  $i$ -th user is given by

$$s_i(t) = \sqrt{P} W^j(t) c_i(t) p_Q(t) \sin \omega_c t + \sqrt{P} W^j(t) c_i(t) p_I(t) \cos \omega_c t, 0 \leq t \leq T_w \quad (1)$$

where  $P$  is the transmitted power per a Walsh symbol,  $\omega_c$  is the carrier frequency, and  $W^j(t)$  is the transmitted  $j$ -th  $M$ -ary Walsh symbol,  $j = 1, 2, \dots, M$ . In (1),  $c_i(t)$  is the spreading waveform of the long PN sequence for the  $i$ -th user, and  $p_I(t)$  and  $p_Q(t)$  are the spreading waveforms of the short PN sequences for the  $I$ - and  $Q$ -channels, respectively. The spreading signals  $c_i(t)$ ,  $p_I(t)$  and  $p_Q(t)$  are the polar rectangular pulses (chips) of the duration  $T_c$ . The chip amplitudes are assumed to be independent, and identically distributed.

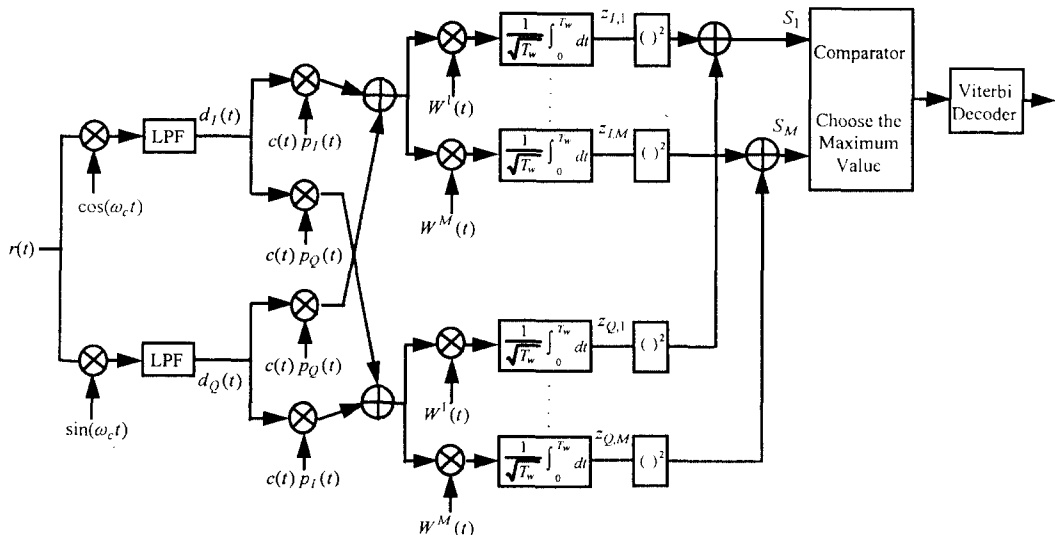


Fig 1. The block diagram of the receiver of the DS/CDMA system with noncoherent detection

buted random variables with probability 1/2 of being  $\pm 1$ .

Let  $a$  is a Nakagami- $m$  random variable whose probability density function (pdf)  $f(a)$  with parameters  $m$  and  $\Omega$  is given by [6]

$$f(a) = \frac{2m^m a^{2m-1}}{\Gamma(m)\Omega^m} \exp\left[-\frac{m a^2}{\Omega}\right], a \geq 0, m \geq 0.5 \quad (2)$$

where  $\Gamma(\cdot)$  is Gamma function,  $\Omega = E[a^2]$  and  $m = \Omega^2 / \text{Var}[a^2]$  are the scale and shape parameters, respectively. For  $m=1$ , we have a Rayleigh fading channel, and as  $m$  goes to infinity, we have a channel of nonfading. Values of  $m$  between 0.5 and 1 correspond to deeper fading than the Rayleigh fading. At the extreme for which  $m=0.5$ , the Nakagami- $m$  distribution becomes a one-sided Gaussian fading distribution.

The signal arriving at the receiver of the base station in Nakagami- $m$  fading channels has the form

$$r(t) = \sum_{i=1}^N a_i s_i(t - \tau_i) + n(t) \quad (3)$$

where  $N$  is the number of users,  $\tau_i$  is a random delay of user  $i$ ,  $n(t)$  is a narrowband noise obtained from the zero-mean AWGN with the two-sided power spectral density  $N_0/2$ , and  $a_i$  is the Nakagami- $m$  random variable of the  $i$ -th user. After the bandpass filter of the bandwidth  $W$ , the AWGN becomes a narrowband noise  $n(t)$  that can be represented as

$$n(t) = n_c(t) \cos \omega_c t + n_s(t) \sin \omega_c t, \quad (4)$$

where  $n_c(t)$  and  $n_s(t)$  are zero-mean and lowpass Gaussian random processes with the variance  $N_0 W$ .

### III. Performance Analysis of the Uncoded System

The outputs of the lowpass filters on the  $I$ - and the  $Q$ -channels are given by

$$d_I(t) = \sum_{i=1}^N a_i \sqrt{P} W^j(t - \tau_i) c_i(t - \tau_i) \left[ p_I(t - \tau_i) \frac{\cos \theta_i}{2} + p_Q(t - \tau_i) \frac{\sin \theta_i}{2} \right] + \frac{n_c(t)}{2}, \quad (5)$$

and

$$d_Q(t) = \sum_{i=1}^N a_i \sqrt{P} W^j(t - \tau_i) c_i(t - \tau_i) \left[ p_I(t - \tau_i) \frac{\sin \theta_i}{2} + p_Q(t - \tau_i) \frac{\cos \theta_i}{2} \right] + \frac{n_s(t)}{2}, \quad (6)$$

respectively, where  $\theta_i = \omega_c \tau_i$ .

Let the  $k$ -th user be the user of interest. For convenience, we assume the perfect synchronism for the PN chips. Define

$$z_I^k(t) = d_I(t) c_k(t - \tau_k) p_I(t - \tau_k) + d_Q(t) c_k(t - \tau_k) p_Q(t - \tau_k). \quad (7)$$

Substituting (5) and (6) into (7), we obtain

$$\begin{aligned} z_I^k(t) = & a_k \sqrt{P} W^j(t - \tau_k) \left[ \cos \theta_k + p_I(t - \tau_k) p_Q(t - \tau_k) \sin \theta_k \right] \\ & + \sum_{i=1, i \neq k}^N a_i \sqrt{P} W^j(t - \tau_k) c_k(t - \tau_k) c_i(t - \tau_i) \\ & \left\{ \left[ p_I(t - \tau_k) p_I(t - \tau_i) + p_Q(t - \tau_k) p_Q(t - \tau_i) \right] \frac{\cos \theta_i}{2} \right. \\ & \left. + \left[ p_I(t - \tau_k) p_Q(t - \tau_k) + p_Q(t - \tau_i) p_I(t - \tau_i) \right] \frac{\sin \theta_i}{2} \right\} \\ & + c_k(t - \tau_k) \left[ p_I(t - \tau_k) \frac{n_c(t)}{2} + p_Q(t - \tau_k) \frac{n_s(t)}{2} \right]. \end{aligned} \quad (8)$$

The output of the  $m$ -th correlator on the  $I$ -channel is then given by

$$\begin{aligned} z_{I,m}^k &= \frac{1}{\sqrt{T_w}} \int_0^{T_w} z_I^k(t) W^m(t - \tau_k) dt \\ &= \int_0^{T_w} a_k \sqrt{\frac{P}{T_w}} W^j(t - \tau_k) W^m(t - \tau_k) \cos \theta_k dt \\ &+ I_I^{k,k} + I_I^{k,j} + N_I^k \end{aligned} \quad (9)$$

where  $I_I^{k,k}$  is the self-interference due to the  $Q$ -channel short PN sequence,  $I_I^{k,i}$  is the multiple access interference from other users, and  $N_I^k$  is the term due to the narrowband Gaussian noise. These are computed

respectively as

$$I_I^{k,k} = a_k \sqrt{\frac{P}{T_w}} \int_0^{T_w} W^j(t-\tau_k) W^m(t-\tau_k) p_I(t-\tau_k) p_Q(t-\tau_k) \sin \theta_k dt, \quad (10)$$

$$I_I^{k,j} = \sum_{i=1, i \neq k}^N a_i \sqrt{\frac{P}{T_w}} \int_0^{T_w} W^j(t-\tau_i) W^m(t-\tau_k) c_i(t-\tau_i) c_i(t-\tau_k) \times \left\{ [p_I(t-\tau_k) p_I(t-\tau_i) + p_Q(t-\tau_k) p_Q(t-\tau_i)] \frac{\cos \theta_i}{2} + [p_I(t-\tau_k) p_Q(t-\tau_k) + p_Q(t-\tau_i) p_I(t-\tau_i)] \frac{\sin \theta_i}{2} \right\} dt, \quad (11)$$

and

$$N_I^k = \frac{1}{2\sqrt{T_w}} \int_0^{T_w} c_k(t-\tau_k) W^m(t-\tau_k) [n_c(t) p_I(t-\tau_k) + n_s(t) p_Q(t-\tau_k)] dt. \quad (12)$$

Due to the quadrature nature of the short PN sequence, the self-interference  $I_I^{k,k}$  can be negligible as in [3], [5], [9], and [10]. Since we have assumed that the  $j$ -th Walsh symbol is transmitted, the output of the  $m$ -th correlator on the  $I$ -channel becomes

$$z_{I,m}^k = \begin{cases} a_k \sqrt{E_w} \cos \theta_k + I_I^{k,j} + N_I^k, & m = j \\ \sum_{i=1, i \neq k}^N I_I^{k,j} + N_I^k, & m \neq j \end{cases} \quad (13)$$

where  $E_w = PT_w$  is the Walsh symbol energy. Similarly, repeating the analysis for the  $Q$ -channel, we obtain

$$z_{Q,m}^k = \begin{cases} a_k \sqrt{E_w} \sin \theta_k + I_Q^{k,j} + N_Q^k, & m = j \\ \sum_{i=1, i \neq k}^N I_Q^{k,j} + N_Q^k, & m \neq j \end{cases} \quad (14)$$

We now form the decision variables  $S_m^k (m=1, 2, \dots, M)$  of the  $k$ -th user as

$$S_m^k = (z_{I,m}^k)^2 + (z_{Q,m}^k)^2. \quad (15)$$

The receiver makes use of the maximum likelihood decision rule to select the maximum value among the decision variables  $S_m^k$ . The index of the largest decision

variables will represent the transmitted Walsh symbol. For the case of a single user without the AWGN and the interference from other users, the detector output is  $a^2 E_w$  when the  $M$ -ary Walsh symbol of the receiver is the same as that of the transmitter, and is zero otherwise.

Note that the noise terms  $N_I^k$  and  $N_Q^k$  are Gaussian random variables with zero mean and equal variance  $N_o/2$  [3]. Now, for the statistics of multiple access interferences  $I_I^{k,i}$  and  $I_Q^{k,j}$ , it is easy to see that  $I_I^{k,i}$  and  $I_Q^{k,j}$  are also Gaussian random variables with zero mean and the same variances. The variance of  $I_I^{k,j}$  or  $I_Q^{k,i}$  can be expressed from the results in [3] as

$$Var[I_I^{k,j}] = Var[I_Q^{k,i}] = \frac{E_w}{2L} \sum_{i=1, j \neq k}^N E[a_i^2]. \quad (16)$$

Let us restrict our discussion to the case of a slow nonselective fading. Without loss of generality, we choose  $\Omega=1$  for convenience so that the received energy in the fading channel,  $a_i^2 E_w$ , has the average value  $E[a_i^2 | E_w] = \Omega E_w$ . Therefore, the variances of  $I_I^{k,i}$  and  $I_Q^{k,i}$  become equally  $E_w (N-1)/2L$ . Also, assuming that the Gaussian noise terms and the multiple access interference terms are mutually uncorrelated,  $N_I^k + I_I^{k,i}$  and  $N_Q^k + I_Q^{k,i}$  become Gaussian random variables having the same variance such that

$$\sigma^2 = \frac{E_w}{2L} (N-1) + \frac{N_o}{2}. \quad (17)$$

Following Crepeau [8], we find the symbol error probability for the Nakagami- $m$  channels by computing the average of the nonfading symbol error probability over the underlying fading random variable. Without employing the convolutional codes in the DS/CDMA system, the bit error probability  $P_b$  on the Nakagami- $m$  channels can be expressed based on the result in [3] as

$$P_b = \frac{M}{2(M-1)} \int_0^\infty \sum_{n=1}^{M-1} \frac{(-1)^{n+1}}{n+1} \binom{M-1}{n} \exp\left[-\frac{na^2 \gamma_w}{2(n+1)}\right] p(a) da, \quad (18)$$

that leads to

$$P_b = \frac{M}{2(M-1)} \sum_{n=1}^{M-1} \frac{(-1)^{n+1}}{n+1} \binom{M-1}{n} \left[ \frac{m}{m + \frac{n}{2(n+1)} \gamma_w} \right]^m \quad (19)$$

Here, is given by

$$\gamma_w = \frac{E_w}{\sigma^2} = \left( \frac{N-1}{2L} + \frac{N_o}{2E_w} \right)^{-1}, \quad (20)$$

and  $E_w = \log_2(M) E_b$  with  $E_b$  being denoted as the information bit energy. Recall that as  $m$  changes from 1 to infinity, we get the Rayleigh fading to nonfading (that is, just AWGN only).

#### IV. Performance Analysis of Coded System

In this section, based on the results in the previous section, an analytical expression for the pairwise error probability of the coded DS/CDMA system in Nakagami- $m$  fading channels with AWGN will be derived. The performances of the DS/CDMA system with and without the convolutional codes are also compared.

The information bits are first passed through the convolutional encoder of the code rate  $R$  and the constraint length  $K$  to generate the code symbols. Every  $\log_2 M$  code symbol is then modulated by the  $M$ -ary Walsh orthogonal modulator to obtain the Walsh symbols. The detector output  $S_m^k$  is used as a metric for the soft decision of the Viterbi algorithm.

Let  $y_n^{(r)}$  ( $n=1, 2, \dots$ ) represent the decision variables for the  $n$ -th branch of the  $r$ -th path through the trellis. Suppose that the first Walsh symbol, i.e., the all-zero code word, is transmitted. Let the case  $r=1$  denote the first  $B$ -branch all-zero path, and let the case  $r=2$  denote the second  $B$ -branch path that begins in the initial state (i.e., all-zero state) and remerges with the all-zero path after  $B$  transitions.

Then by the decision variables given in (15), the two input variables to the decoder become

$$y_n^{(1)} = S_{m=1}^k(n), \quad (21)$$

$$y_n^{(2)} = \begin{cases} S_{m=1}^k(n), & \text{if the Walsh symbol of the } n\text{-th branch} \\ & \text{of the second path} \\ & = \text{the Walsh symbol of the first path} \\ S_{m \neq 1}^k(n), & \text{otherwise} \end{cases} \quad (22)$$

Let  $y_n^{(2)'} = S_{m \neq 1}^k(n)$ , and define the decision variable for the  $r$ -th path consisting of  $B$  branches through the trellis as

$$U^{(r)} = \sum_{n=1}^B y_n^{(r)}. \quad (23)$$

The decision variables for the first and the second paths consisting of  $B$  branches are given by

$$U^{(1)} = \sum_{n=1}^B y_n^{(1)}, \quad (24)$$

and

$$U^{(2)} = \sum_{n=1}^d y_n^{(2)'} + \sum_{n=d+1}^B y_n^{(1)}, \quad (25)$$

respectively, where  $d$  is the hamming distance between the two path. The difference between  $U^{(1)}$  and  $U^{(2)}$  is then

$$U^{(1)} - U^{(2)} = \sum_{n=1}^d (y_n^{(1)} - y_n^{(2)'}). \quad (26)$$

For convenience, we now redefine the two decision

$$U^{(1)} = \sum_{n=1}^d y_n^{(1)} = \sum_{n=1}^d S_{m=1}^k(n), \quad (27)$$

and

$$U^{(2)} = \sum_{n=1}^d y_n^{(2)'} = \sum_{n=1}^d S_{m \neq 1}^k(n). \quad (28)$$

Note that  $U^{(1)}$  is described statistically as a noncentral chi-square random variable with  $2d$  degrees of freedom

and noncentrality parameter

$$z = \sum_{n=1}^d (a_n)^2 E_w = E_w \sum_{n=1}^d (a_n)^2, \quad (29)$$

where  $a_n$  is the signal strength of the received signal of the  $n$ -th branch, and  $E_w = R \log_2(M)E_b$  with  $R$  being denoted as the convolutional code rate. Note also that unlike the uncoded DS/CDMA system described in the last part of the previous section, the convolutional code rate  $R$  is included in the expression of  $E_w$ . The conditional pdf of  $U^{(1)}$  can now be written as

$$f\{u^{(1)}|z\} = \frac{1}{2\sigma^2} \left\{ \frac{u^{(1)}}{z} \right\}^{(d-1)/2} \exp\left\{-\frac{z+u^{(1)}}{2\sigma^2}\right\} I_{d-1}\left\{\frac{\sqrt{z u^{(1)}}}{\sigma^2}\right\}, u^{(1)} \geq 0 \quad (30)$$

where  $\sigma^2$  is given in (17). On the other hand,  $U^{(2)}$  is statistically a central chi-square random variable, having  $2d$  degrees of freedom with the pdf as

$$f(u^{(2)}) = \frac{1}{(2\sigma^2)^d (d-1)!} u^{(2)^{d-1}} \exp\left\{-\frac{u^{(2)}}{2\sigma^2}\right\}, u^{(2)} \geq 0 \quad (31)$$

Using the results in [19], the probability of error in pairwise comparison of the  $U^{(1)}$  and  $U^{(2)}$  leads to

$$P_2(d|z) = 2^{-d} \exp\left\{-\frac{z}{2\sigma^2}\right\} \sum_{l=0}^{d-1} \binom{d+l-1}{d-1} {}_1F_1\left[d+l, d; \frac{z}{4\sigma^2}\right] 2^{-l}, \quad (32)$$

where  ${}_1F_1(\alpha, \beta; x)$  is defined as

$${}_1F_1(\alpha, \beta; x) = \sum_{k=0}^{\infty} \frac{\Gamma(\alpha+k)\Gamma(\beta)x^k}{\Gamma(\alpha)\Gamma(\beta+k)k!}, \beta \neq 0, -1, -2, \dots \quad (33)$$

To evaluate  $P_2(d)$  from  $P_2(d|z)$ , we still have to know the distribution of  $z$  given in (29). Since the  $a_n$ 's are the independent and identically distributed Nakagami ( $m, \Omega$ ) random variables, it can be shown [20] that  $\sum_{n=1}^d (a_n)^2$  is the square of the Nakagami

( $md, Wd$ ) random variable and thus the pdf of  $z$  is of the form

$$f(z) = \frac{1}{\Gamma(md)} \left\{ \frac{m}{\Omega E_w} \right\}^{md} z^{md-1} \exp\left\{-\frac{m}{\Omega E_w} z\right\}, z \geq 0. \quad (34)$$

Therefore, the pairwise error probability becomes

$$P_2(d) = \int_0^{\infty} P_2(d|z) f(z) dz. \quad (35)$$

It is shown in Appendix A that the pairwise error probability  $P_2(d)$  on the Nakagami- $m$  channels for  $\Omega = 1$  can now be simplified to

$$P_2(d|z) = \left\{ \frac{m}{\gamma+m} \right\} 2^{-d} \sum_{l=0}^{d-1} \binom{d+l-1}{d-1} {}_2F_1\left[d+l, md, d; \frac{\gamma}{2(\gamma+m)}\right] 2^{-l}, \quad (36)$$

where

$$\gamma_w = \left[ \frac{1}{R \log_2(M) \gamma_b} + \frac{N-1}{L} \right]^{-1}, \quad (37)$$

and  $\gamma_b = E_b/N_0$ . Note that  ${}_2F_1(\alpha, \beta, \gamma; x)$  denotes the hypergeometric function.

We are now ready to compute the upper bound for the bit error probability by associating (36) and (37) with the expression for the upper bound given in [14]. We reproduce it here for convenience:

$$P_b < \sum_{d=d_{free}}^{\infty} \beta_d P_2(d). \quad (38)$$

Note that the distance in (38) must be measured by the unit of Walsh symbols. Note also that  $d_{free}$  is the minimum weight (in term of the number of nonzero symbols) of the nonzero path. For the convolutional code of the rate  $R=1/3$  and the Walsh modulation with  $M=8$ , for example,  $d_{free}=9$ . The information weight spectrum  $\beta_d$  is the total number of nonzero information bits for all the inputs that have the path weight equal to  $d$ . Here, a computer search technique is used to find the first few terms of  $\beta_d$ .

### V. Numerical Results

The processing gain for both the uncoded and coded cases is chosen to be 128 so that the chip rate becomes 128 PN chips per an information bit. We first consider the case of the uncoded DS/CDMA system with the following parameters:  $M=8$  (i.e., 8-ary orthogonal modulation) and  $L=384$  (i.e., 384 PN chips per a Walsh symbol). Note that every 3 information bits are modulated by the 8-ary Walsh orthogonal modulator, and each Walsh chip is spread by 48 long code PN chips. Considered next is the case of the convolutional coded DS/CDMA system with the following parameters: the convolutional code rate  $R=1/3$ ; the constraint length  $K=9$ ;  $M=8$ ; and  $L=128$ . Note that every 3 code symbols are modulated by the 8-ary Walsh orthogonal modulator, and each Walsh chip is spread this time by the 16 long code PN chips.

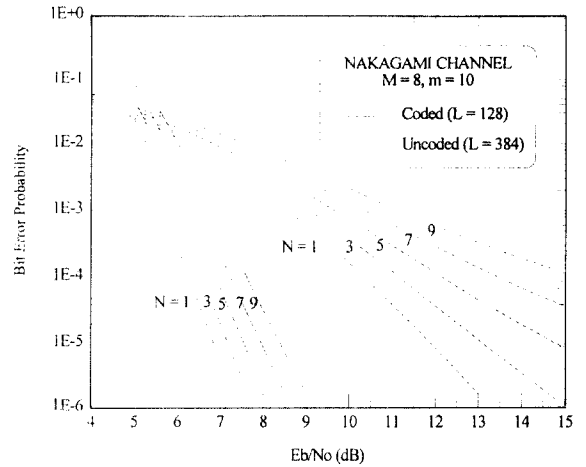


Fig 3. Bit error probability of the coded system versus  $E_b/N_0$ :  $R=1/3$ ,  $K=9$ ,  $M=8$ ,  $N=1, 3, 5, 7, 9$ , and  $m=10$ .

Solid lines are for the theoretical upper bounds of the coded system and dashed lines are for the theoretical curves of the uncoded system.

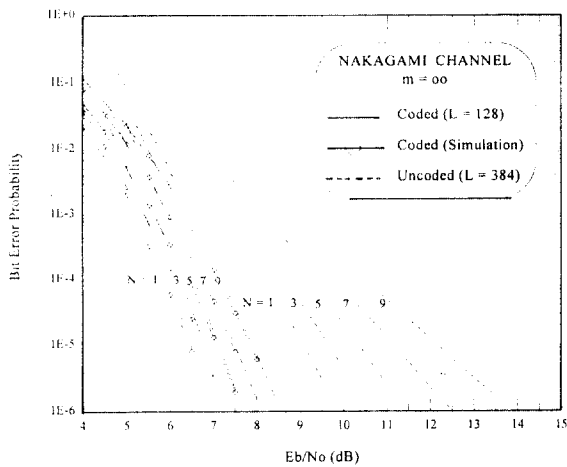


Fig 2. Bit error probability of the coded system versus  $E_b/N_0$ :  $R=1/3$ ,  $K=9$ ,  $M=8$ ,  $N=1, 3, 5, 7, 9$ , and  $m=\infty$ .

Solid lines are for the theoretical upper bounds of the coded system, solid lines with diamonds are for the simulation curves of the coded system, and dashed lines are for the theoretical curves of the uncoded system.

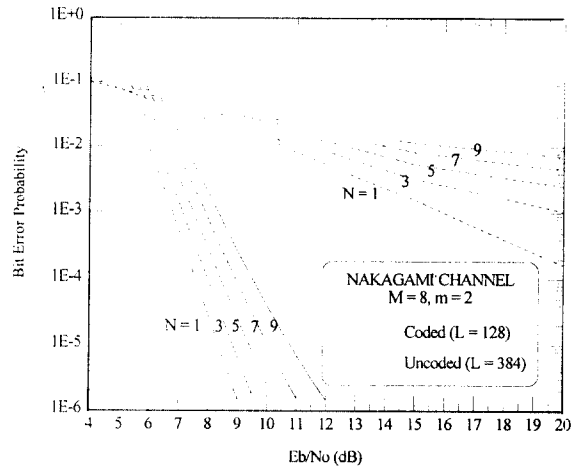


Fig 4. Bit error probability of the coded system versus  $E_b/N_0$ :  $R=1/3$ ,  $K=9$ ,  $M=8$ ,  $N=1, 3, 5, 7, 9$ , and  $m=2$ .



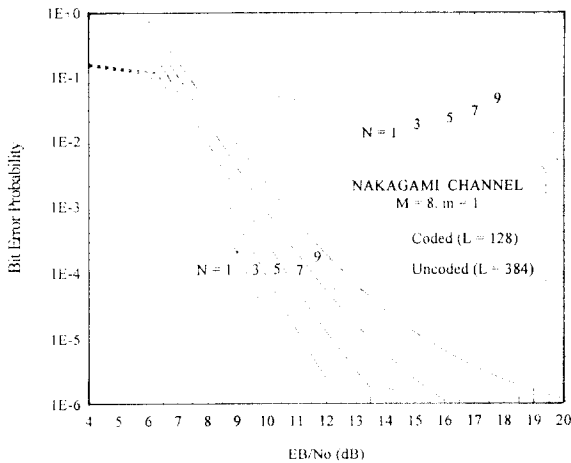


Fig 5. Bit error probability of the coded system versus  $E_b/N_0$ :  $R=1/3$ ,  $K=9$ ,  $M=8$ ,  $N=1, 3, 5, 7, 9$ , and  $m=1$

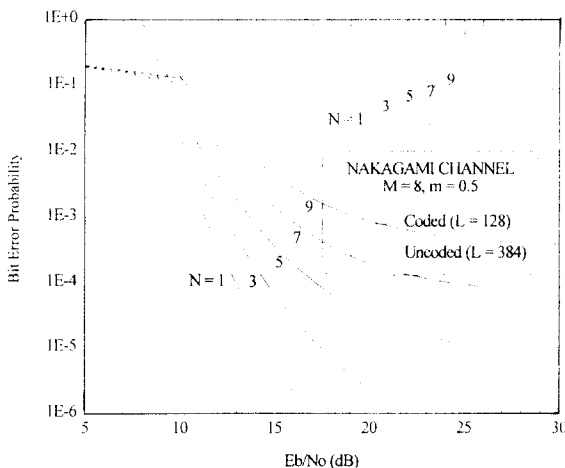


Fig 6. Bit error probability of the coded system versus  $E_b/N_0$ :  $R=1/3$ ,  $K=9$ ,  $M=8$ ,  $N=1, 3, 5, 7, 9$ , and  $m=0.5$ .

For both the cases, the resultant signals are spread once again by the two short PN sequences in both the I-and Q-channels separately. The generator polynomials for the long PN sequence and the two short PN sequences are used as suggested in [15]. We assume the perfect synchronism of the PN chips throughout the experiments.

In Figures 2 to 6, we configure the overall performances of the coded and uncoded DS/CDMA systems for the AWGN channel with various fading environments having the Nakagami parameter  $m=1, 10, 2, 1$ , and  $0.5$ , respectively. In each figure, the numbers of users are selected to be  $N=1, 3, 5, 7$ , and  $9$ . The dashed lines represent the analytical curves of the uncoded system obtained by using (19), while the solid lines represent the analytical curves for the upper bound of the coded system by using (38). In particular, also plotted in Figure 2 are the simulation results for the coded system in order to check the validity of our analytical results for  $m=1$  and  $N=1, 3, 5, 7$ , and  $9$ . We can see that the theoretical upper bound given in (38) matches quite well with the simulation results.

From Figures 2 to 6, we observe that when the fading gets deeper, the performance of the uncoded DS/CDMA system degrades very sharply as the number of users increases. But we see that the convolutional code with soft decision Viterbi decoder improves the performance of the uncoded system quite successfully. In the coded DS/CDMA case, we can see a dramatic increase in coding gain as the value of  $m$  decreases. For the extreme fading such as the case when  $m=0.5$ , however, the convolutional code seems to reach to the limit in improving the overall system performance as the number of users increase.

## VI. Concluding Remarks

In this paper, we have investigated the performance of convolutional coded and uncoded DS/CDMA system equipped with the noncoherent  $M$ -ary orthogonal modulation, and operating in the multi-user environments over the slow and frequency nonselective Nakagami- $m$  fading and AWGN channels. We first derived expressions for bit error probability of the uncoded system. The pairwise error probability that can be used to compute the upper bound of the coded system was then derived. Performances of the DS/

CDMA system with and without the convolutional codes were also compared. The computer simulations were carried out in order to demonstrate the validity of the theoretical upper bound of the coded system, and found that the two results match quite well.

By employing the convolution code, we have observed a dramatic increase in coding gain when the fading gets deeper. For the extreme fading such as the case when  $m=0.5$ , however, it has been seen that the convolutional code seems to reach to the limit in improving the overall system performance for the large number of users.

Our analytical expressions for the upper bound of the bit error probability of the coded system as well as the bit error probability of the uncoded system are mathematically very simple and practically very useful in evaluating the overall performance of the DS/CDMA system. It is hoped that the results of this work provide additional design criteria for the DS/CDMA system operating in multi-user environments over the slow nonselective Nakagami- $m$  fading and AWGN channels.

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**APPENDIX A**

**Derivations of Equation (36)**

This appendix provides derivations of the expression given in (36) from (35). Rewriting  $P_2(d)$  on the Nakagami- $m$  channels for  $\Omega=1$  gives

$$P_2(d) = \frac{2^{-d}}{\Gamma(md)} \left( \frac{m}{E_w} \right)^{md} \sum_{l=0}^{d-1} 2^{-l} \binom{d+l-1}{d-1} \int_0^\infty \exp \left[ -\frac{z}{2\sigma^2} - \frac{mz}{E_w} \right] z^{md-1} {}_1F_1 \left[ d+l, d; \frac{z}{4\sigma^2} \right] dz. \quad (A1)$$

Using  $E_w/2\sigma^2 = \gamma$  and  $z/E_w = t$ , we obtain

$$P_2(d) = \frac{2^{-d}}{\Gamma(md)} (m)^{md} \sum_{l=0}^{d-1} 2^{-l} \binom{d+l-1}{d-1} \int_0^\infty \exp [ -(\gamma + m)t ] t^{md-1} {}_1F_1 \left[ d+l, d; \frac{\gamma t}{2} \right] dt. \quad (A2)$$

The integral given in (A2) can be solved by making use of the relation as

$$\int_0^\infty x^{\alpha-1} \exp(-\mu x) {}_mF_n(a_1, \dots, a_m; \rho_1, \dots, \rho_n; \lambda x) dx = \Gamma(\sigma) \mu^{-\sigma} {}_{m+1}F_n \left( a_1, \dots, a_m, \sigma; \rho_1, \dots, \rho_n; \frac{\lambda}{\mu} \right). \quad (A3)$$

By using (A3), the integral term of (A2) is given as

$$\int_0^\infty \exp [ -(\gamma + m)t ] t^{md-1} {}_1F_1 \left[ d+l, d; \frac{\gamma t}{2} \right] dt = \Gamma(md) (\gamma + m)^{-md} {}_2F_1 \left[ d+l, md, d; \frac{\gamma}{2(\gamma + m)} \right] \quad (A4)$$

where  ${}_2F_1(\alpha, \beta, \gamma; x)$  denotes the hypergeometric function given by

$${}_2F_1(\alpha, \beta, \gamma; x) = \sum_{k=0}^\infty \frac{\Gamma(\alpha+k)\Gamma(\beta+k)\Gamma(\gamma)x^k}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma+k)k!}, \quad \gamma \neq 0, -1, -2, \dots \quad (A5)$$

Now, equation (36) is obtained by inserting (A4) in (A2).



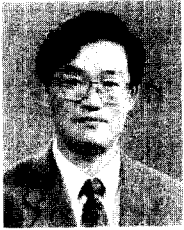
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