Estimation of the frequency component and the orientational angle in texture image based on the QPS filter

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QPS 필터에 의한 질감영상의 주파수성분과 방향각 평가

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ABSTRACT

Several improved quadrature polar separable (QPS) filters have been proposed and applied in texture processing since Knutsson proposed the QPS filter. They include a Knutsson's cosine function or an exponential attenuation function, as the orientational function, and a Knutsson's exponential function or a finite prolate spheroidal sequence (FPSS) or an asymptotic FPSS, as the radial weighting functions. They represent different properties in terms of the generation of texture images, the orientational estimation, and the segmentation of synthetic texture image.

In this paper, we have constructed several kernel functions for the 2-D QPS filter and analyzed their properties. A series of experiments have been carried out in order to estimate the frequency components and orientational angles of a local texture in Fourier domain. Finally some problems encountered in applying QPS filters to feature description and segmentation are considered. Experimental results show that the improved Knutsson's filter and the asymptotic FPSS filter are useful in terms of the orientatioal estimation and the segmentation of synthetic texture image.

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요 약

Knutsson이 QPS(quadrature polar separable) 필터를 제안한 이후 여러가지 개선된 QPS필터가 발표되고 있으며 질감 영상처리에 응용되고 있다. 이들 필터는 방향각 함수로 Knutsson의 cosine 함수나 지수감쇄 함수를 이용하고 있으며, 라디엘웨이팅 함수로 FPSS(finite prolate spheroidal sequence)나 점근적 FPSS를 이용하고 있다. 이들 필터는 질감영상 생성이나 방향각평가, 그리고 합성된 질감영상의 영역분할면에서 다른 특성을 나타낸다.

본 논문에서는 2-D QPS 필터를 위한 여러가지 커넬함수를 분석하여 그들의 특성을 고찰하며 푸리에영역내에서 국부조직의 주파수성분과 방향각을 평가한다. 그리고 이들 필터에 의한 질감 영상의 특성추출과 영역분할에 대한 응용상의 문제점에 대해서 고찰한다. 실험결과는 개선된 Knutsson 필터와 점근적 FPSS 필터함수가 합성 질감영상의 방향각 평가와 영역분할면에서 유용하다는 것을 나타낸다.

I. Introduction

Texture analysis in digital image processing is one of the important steps toward idendifying regions and objects. The design and application of the digital filter in such task are among what to be necessarily considered. In recent years, a lot of digital filters for texture image processing have been developed and their applications have been discussed. The important factors in choosing a good filter for texture image processing include small energy loss, flexibility in filter design, accuracy of local frequency and orientation estimation, and tunable center frequency etc.

Among several filters for texture image processing, Gabor's filter is most commonly used as it can be easily implemented and exhibits an interesting property for achieving optimal joint resolution in space and spatial frequency.[1] However, the shape of the Gabor filter may not be optimal for tessellating the frequency plane when strongly directional textures are considered. In order to achieve effective discrimination of strongly directional textures using Gabor filters, many narrow bandwidth filters are required. Cartesian filter is optimal in terms of energy loss as the filter function is composed of 2 FPSS's (finite prolate spheroidal sequences), while the frequency characteristics are not easy to control.

Knutsson proposed a new 2-D QPS(quadrature polar separable) filter with several benefits in terms of good filter design.[2-4] The frequency response of Knutsson's filter is described on Fourier plane by polar coordinates. And the response can be expressed as the product of two kernel quantities: a radial weighting function and an orientational weighting function. The important feature on the filter function for such a kernel pair is that they should have a smooth variation and unimodal property, and that it is quadrature and polar separable in frequency domain.

However, Knutsson's filter is not optimal in energy loss. Zhao and Park etc. have developed some improved 2-D QPS filters, which consists of a FPSS or an exponential attenuation function.[5-8] They have different properties in terms of symmetry and texture estimation.

In this paper, some design methods and problems of the 2-D QPS filters are discussed.

The kernel pairs for the QPS filters are based on a Knutsson's cosine function or an exponential attenuation function, as the orientational function, and a Knutsson's exponential function or a FPSS or an asymptotic FPSS, as the radial weighting functions. They represent different properties for the generation of texture images, orientation and frequency estimation, and the segmentation.

A series of experiments have been carried out in order to analyze and compare their properties in terms of feature description, the orientation estimation,

and segmentation.

Ⅱ. 2-D QPS Filter

As Knutsson is the first to propose a 2-D QPS filter, the 2-D QPS filter can be analyzed with his filter functions. The frequency response of the Knutsson's filter is described in the Fourier plane by polar coordinates; it can be expressed as the product of two kernel quantities: a radial weighting function and an orientational weighting function.

The radial weighting function and the orientational weighting function are given as

$$F_e(\rho, \psi) = V_k(\rho) \cdot V_e(\psi) \tag{1}$$
$$F_o(\rho, \psi) = V_k(\rho) \cdot V_o(\psi) \tag{2}$$

where,

ρ : the polar variation of frequency,

ψ : the angle variation of frequency,

$V_k(\rho)$: a radial weighting function and

V_e and $V_o(\psi)$: even and odd symmetrical orientational
weighting functions.

The radial weighting function $F_k(\rho)$ is given as follows.

$$V_k(\rho) = \exp[1 - \frac{4}{\log_2} \cdot B^{-2} \cdot Ln^2 \frac{\rho}{\rho_i}] \tag{3}$$

where ρ_i, the central frequency, denotes the peak value of the function along the radial axis. And B is the frequency bandwidth of the radial weighting function. $V_e(\psi)$, $V_o(\psi)$, the orientational weighting functions, are expressed as

$$V_e(\psi) = \cos^{2A}(\psi - \psi_k) \tag{4}$$
$$V_o(\psi) = V_e(\psi) \cdot \sin[(\psi - \psi_k)] \tag{5}$$

where A is the frequency bandwidth of the orientational angle, and ψ_k specifies the preferrence orientation that the function has been tuned to.

Thus the frequency response of the QPS filter

proposed by Knutsson is given as

$$F(\rho, \psi) = F_e(\rho, \psi) + F_o(\rho, \psi) \tag{6}$$

An important feature on the filter function for such a kernel pair is that they should have a smooth variation and unimodal property, and that it is quadrature and polar separable in frequency domain.

Ⅲ. Analysis of Radial Weighting Function and Orientational Weighting Function

Several radial weighting functions and orientational weighting functions have been proposed in order to improve the performance of the Knutsson's QPS filter.[5-8]

3.1 Radial Weighting Function

Zhao et. al. have developed a new QPS filter using the zero order FPSS as the radial weighting function. It is based on an eigenvector corresponding to maximum eigenvalue of the matrix E.

$$(E - \lambda_k \cdot I) \cdot h^{(k)} = 0 \tag{7}$$

where $h = [h_0, h_1, \cdots, h_{N-1}]^T$. And E is N x N matrix. After simplification, elements of E are given as follows.

$$e_{mn} = \begin{bmatrix} 1/2 \cdot m(N-m) & N = m-1 \\ ((N-1)/2 - m)^2 \cdot \cos(2\psi) & N = m \\ 1/2 \cdot (m+1)(N-1-m) & N = m+1 \\ 0 & |N-m| \rangle 1 \end{bmatrix} \tag{8}$$

Therefore the radial weighting function of the QPS filter is as follows.

$$V_p(\rho) = F[\psi_0(x)] \tag{9}$$
$$E\psi_0 = \lambda_0 \cdot \psi_0 \tag{10}$$

It should be pointed out that when the FFT of ψ_0 (x) is executing in Zhao's program for filter design,

one sequence is deleted. It distorts the symmetry of the FPSS filter in frequency domain. After modifying his program,[5] the frequency chracteristics of the filter is to be more circularly symmetric than in the filter's.

Also, it can be developed as an asymptotic representation of the Fourier transform of the order one prolate spheroidal function as follows.

$$V_a(\rho) = A(\rho) \cdot B(\rho) \tag{11}$$

where,

$$V_1 = \rho \cdot ((2.0 \cdot c^7))^{0.25} \cdot 2.0 \cdot \pi$$
$$A(\rho) = V_1 / ((2.0 \cdot N + 1.0) \cdot \omega^2)$$
$$V_2 = (2.0 \cdot \pi^2) C^{0.25}) \cdot C^3 \cdot \rho^2$$
$$B(\rho) = \exp(-V_2 / ((2.0 \cdot N + 1.0)^2 \cdot \omega^4))$$

As controlling the center frequency of the filter, it is represented as follows.

$$V_s(\rho) = V_s'(\rho - \rho_0) \qquad \rho \geq \rho_0 \tag{12}$$
$$V_s(\rho) = 0 \qquad \rho \langle \rho_0 \tag{13}$$

where s = p or a, $V_s(\rho)$ is the radial weighting function in frequency domain

These radial weighting functions have different, important properties in frequency domain. $V_s(\rho)$ is most circularly symmetric to the center. However, the local frequency region according to the filter length in frequency plane is not greatly changable and this is not flexible to construct the filter with wider bandwidth. Further the frequency curve is not smoothly changed and this is less suitable to the design requirement of 2-D QPS filter justified by Knutsson. $V_a(\rho)$ is based on the asymptotic FPSS which approximate to the optimal filter. It is not circularly symmetric to the center in frequency plane, however, it is flexible to construct the near optimal filters with wider bandwidth.

3.2 Orientational Weighting Function

The orientational weighting function can be cons-

tructed by an exponential function as well as the cosine function. It is obtained by combining two exponential attenuation functions $V_1(\psi)$ and $V_2(\psi)$, and is given as

$$V_{1e}(\psi) = V_1(\psi) + V_2(\psi) \tag{14}$$
$$V_{1o}(\psi) = V_1(\psi) - V_2(\psi) \tag{15}$$

The two exponential attenuation functions, which approximate the Fourier transform of the first order Prolate Spheroidal function, are expressed as follows.

$$V_1(\psi) = e^{(-K_c(\psi - \psi_k)^2)} \quad , \quad \psi \leq 180 + \psi_k \tag{16}$$
$$e^{(-K_c(360 - \psi + \psi_k)^2)}, \quad \psi \rangle 180 + \psi_K$$

$$V_2(\psi) = e^{(-K_c(180 - \psi_k + \psi)^2)}, \quad \psi \leq \psi_k \tag{17}$$
$$e^{(-K_c(\psi - \psi_k - 180))^2)}, \quad \psi \rangle \psi_k$$

where ψ_k is primary phase and K_c is an attenuation coefficient which controls the orientational bandwidth of the filter. The optimal value for the attenuation coefficient K_c is evaluated using a least square method by making functions V_1 and V_2 approximate an asymptotic Fourier transform of the first order prolate spheroidal function. That is, it can be used to construct the filter which approximates to the 2-D Cartesian prolate spheroidal filter, and to design the nearer optimal filter in terms of energy loss.

Therefore, the five kernel pairs for some QPS filters can be constructed by combining these radial weighting and orientational weighting functions. They can be expressed as in the following equations.

Filter kernel Functions;

Method 1 : $F_{1Kj}(\rho, \psi) = V_k(\rho) \cdot V_{1j}(\psi)$ (18)

Method 2 : $F_{A1j}(\rho, \psi) = V_a(\rho) \cdot V_j(\psi)$ (19)

Method 3 : $F_{A2j}(\rho, \psi) = V_a(\rho) \cdot V_{1j}(\psi)$ (20)

Method 4 : $F_{P1j}(\rho, \psi) = V_p(\rho) \cdot V_j(\psi)$ (21)

Method 5 : $F_{P2j}(\rho, \psi) = V_p(\rho) \cdot V_{1j}(\psi)$ (22)

where j = e or o.

Fig. 1 shows the frequency responses of five QPS

filters with zero-phase. The filter functions F_{P1j} and F_{P2j} are circularly symmetric and the filter function F_{P1j} and F_{P2j} show wider bandwidth properties in frequency domain with the same bandwidth condition. The filter function F_{IKj} shows the medium symmetry property and wide bandwidth property.
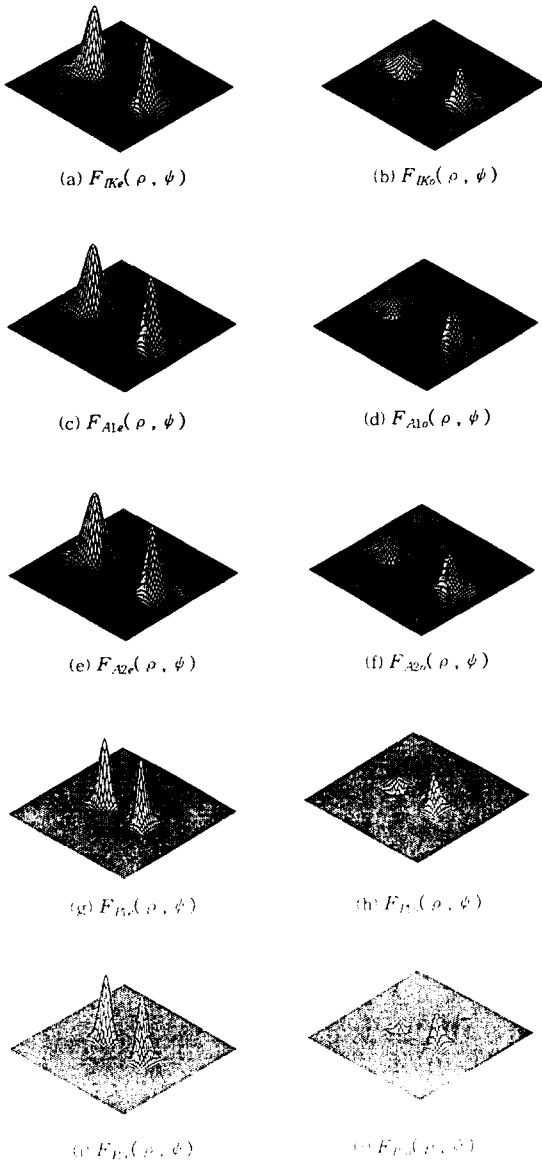

(a) $F_{IKe}(\rho,\psi)$ (b) $F_{IKo}(\rho,\psi)$

(c) $F_{A1e}(\rho,\psi)$ (d) $F_{A1o}(\rho,\psi)$

(e) $F_{A2e}(\rho,\psi)$ (f) $F_{A2o}(\rho,\psi)$

(g) $F_{P1e}(\rho,\psi)$ (h) $F_{P1o}(\rho,\psi)$

(i) $F_{P2e}(\rho,\psi)$ (j) $F_{P2o}(\rho,\psi)$

Fig 1. Frequency responses of the QPS filters

Ⅳ. Experiment

In order to estimate the orientation and frequency components of a local texture in Fourier domain, we can form a set of the narrow bandwidth QPS filters. When a random noise image is passed through each filter, the textures with the corresponding orientation angles are produced. In our experiments we have drived the narrow bandwidth filters which have orientation angle $\psi_k = 0°$, 22.5°, 45°, and 67.5°, 90°, 112.5°, 135°, and 157.5°. And a synthetic texture consists of a set of the generated textures.

As the QPS filter can extract information on texture orientation and frequency, it may be used for feature description and segmentation following the steps, described below.

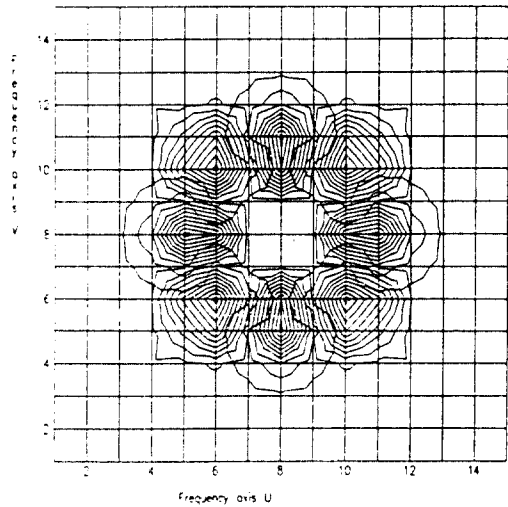

Fig 2. Contour property of the orientational filter in frequency domain

*Consist of an orientational filter by combining four bandpass QPS filters with the same bandwidth but different orientations. In our experiment, these filters are wider than the filters used to create each test texture images and the four orientation angles are $\psi_k = 0°$, 45°, 90°, and 135°. Fig. 2 shows the contour

property of the orientational filter in frequency domain.

* Filter the synthetic texture image using the orientational filter.

* For each pixel form a vector whose components are energy values computed from the filter outputs. i.e.

$$f(x, y) = [f_0(x, y), f_1(x, y), f_2(x, y), f_3(x, y)]^t \quad (24)$$
with $f_j(x, y) = ((g(x, y) * F_{ei}(x, y))^2 + (g(x, y) * F_{oi}((x, y))^2)^{0.5}$

where $i = 0, 1, 2, 3$. $g(x, y)$ is input image and the asterisk (*) denotes convolution. $F_{ji}(x, y)$ functions

denote impulse responses of the new filter after changing into the Cartesian coordinates.

* Segment the synthetic image using the non-parametric classification method proposed by Spann and Wilson.[10] This includes quadtree smoothing, local centroid clustering, and boundary estimation.

Knutsson have announced that orientation estimates are produced having an expected deviation of less than 7 degrees.[2] In our experiments, orientation estimates show different angles according to each method and parameter. Table 1-Table 5 show the maximum and rms errors of the estimated orientation angles to

Table 1. Estimated orientation angles by method 1

Para. True ang.	$B = 1.4$ $\rho_o = 1$ $K_c = 0.0026$	$B = 1.2$ $\rho_o = -1$ $K_c = 0.0026$	$B = 1.0$ $\rho_o = -1$ $K_c = 0.05$	$B = 1.4$ $\rho_o = 0$ $K_c = 0.05$	$B = 1.0$ $\rho_o = 0$ $K_c = 0.003$	$B = 1.0$ $\rho_o = -1$ $K_c = 0.008$
0	176(4)	176(4)	178(2)	2	2	177(3)
22.5	20	19	13	21	28	16
45	45	45	44	44	44	45
67.5	66	65	68	71	64	70
90	94	93	92	88	87	94
112.5	110	113	112	98	99	118
135	134	134	135	140	135	135
157.5	154	154	160	168	166	157
RMS err.	2.74	2.67	3.64	6.79	6.23	3.60

Table 2. Estimated orientation angles by method 2

Para. True ang.	$HBW = \pi/2$ $A = 5$ $\rho_o = 1$	$HBW = \pi/2$ $A = 10$ $\rho_o = 1$	$HBW = \pi/2$ $A = 15$ $\rho_o = 1$	$HBW = \pi/2$ $A = 20$ $\rho_o = 1$	$HBW = \pi/2$ $A = 10$ $\rho_o = 1$	$HBW = \pi/2$ $A = 15$ $\rho_o = 1$
177(3)	178(2)	178(2)	179(1)	172(8)	172(8)	
22.5	20	19	18	16	27	19
45	44	44	44	44	44	44
67.5	67	70	70	74	67	67
90	91	92	92	92	92	92
112.5	118	115	115	114	126	123
135	132	132	143	133	134	134
157.5	150	153	154	153	143	129
RMS err.	3.76	3.82	2.81	3.84	7.77	12.17

Table 3. Estimated orientation angles by method 3

Para. True ang.	HBW $= \pi/2$ $K_c = 0.003$ $\rho_o = 1$	HBW $= \pi/2$ $K_c = 0.006$ $\rho_o = 1$	HBW $= \pi/2$ $K_c = 0.008$ $\rho_o = 1$	HBW $= \pi/2$ $K_c = 0.003$ $\rho_o = 0$	HBW $= \pi/2$ $K_c = 0.004$ $\rho_o = 0$	HBW $= \pi/2$ $K_c = 0.005$ $\rho_o = 0$
0	177(3)	178(2)	177(3)	172(8)	172(8)	172(8)
22.5	21	18	16	27	25	19
45	45	45	45	44	44	44
67.5	68	72	74	67	67	70
90	92	93	94	92	92	92
112.5	116	114	113	125	126	123
135	135	134	134	134	134	134
157.5	154	155	155	143	141	127
RMS err.	2.24	2.81	3.82	7.56	8.15	11.87

Table 4. Estimated orientation angles by method 4

Para. True ang.	BW $= 3/10$ A $= 5$ $\rho_o = 2$	BW $= 3/10$ A $= 10$ $\rho_o = 2$	BW $= 3/10$ A $= 15$ $\rho_o = 2$	BW $= 2/10$ A $= 15$ $\rho_o = 2$	BW $= 2/10$ A $= 20$ $\rho_o = 2$	BW $= 2/10$ A $= 15$ $\rho_o = 3$
0	179(1)	179(1)	179(1)	179(1)	179(1)	178(2)
22.5	25	24	21	21	19	16
45	44	44	44	44	44	44
67.5	65	65	66	67	70	71
90	96	94	93	93	93	90
112.5	122	118	116	117	117	113
135	136	134	135	136	136	138
157.5	145	149	150	150	150	159
RMS err.	6.10	4.03	3.24	3.55	3.82	2.98

Table 5. Estimated orientation angles by method 5

Para. True ang.	BW $= 2/10$ $K_c = 0.006$ $\rho_o = 2$	BW $= 2/10$ $K_c = 0.007$ $\rho_o = 2$	BW $= 2/10$ $K_c = 0.045$ $\rho_o = 2$	BW $= 2/10$ $K_c = 0.008$ $\rho_o = 2$	BW $= 3/10$ $K_c = 0.006$ $\rho_o = 2$	BW $= 3/10$ $K_c = 0.045$ $\rho_o = 2$
0	0	1	178(2)	2	3	0
22.5	19	19	22	18	18	21
45	44	44	44	44	44	44
67.5	70	70	66	72	70	61
90	93	92	93	92	92	94
112.5	117	116	117	116	116	117
135	136	136	137	136	136	135
157.5	150	150	149	150	150	149
RMS err.	3.64	3.43	3.76	3.86	3.71	3.77

Table 6. Minimum rms errors of estimated orientation angles

error/method	1	2	3	4	5	Knutsson
Min.rms error	2.67	2.81	2.24	2.98	3.43	Max. 7

eight generated textures after they have been analyzed by their orientational filters. These filters have similar bandwidths.

Table 6 show the minimum rms errors of estimated orientation angles by each method. However, we show only maximum error as Knutsson didn´t describe the minimum rms error of estimated orientation angles in his thesis.

The optimal filter in terms of energy loss can be obtained by using FPSS as the radial weighting function. The exact symmetry of the FPSS filter in frequency domain is obtained by modifying the Zhao´s program. However, the frequency region changes according to the filter length is relatively small and the frequency curve is not changed smoothly.

This is less suitable to the Knutsson´s requirement for 2-D QPS filter design, and leads to unexact orientation estimation. The asymptotic FPSS method is flexible to bandwidth changes, and is more suitable to the texture orientation estimation and is easier to control the filter properties in frequency domain. However, it decreases the symmetry to the center.

The improved Knutsson´s filter(method 2) using the exponential attennuation function represents good symmetry as well as the flexibility to the bandwidth change, and show that it is more suitable to orientation estimation and texture segmentation.

Ⅴ. Conclusion

Several 2-D QPS filters for texture processing have been analyzed. They can be constructed by combining the radial weighting function, which consists of the Knutsson´s or the FPSS or the asymptotic FPSS, and the orientational weighting function, which consists of the cosine function or the exponential attenuation function.

The optimal filter can be obtained by the kernel pair using the FPSS as the radial weighting function. However, the frequency bandwidth control according to the filter length is relatively bad and is not smooth. The asymptotic method is flexible to bandwidth changes and the orientation estimation. However, it decreases the symmetry to the center of the filter response. The improved Knutsson´s filter using the exponential attennuation function represents good symmetry as well as the flexibility to the bandwidth change. Finally, the experimental results show that the improved Knutsson´s method and the asymptotic FPSS methods are more suitable to texture analysis.

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