

On the Negative Drift Force Acting on a Freely Floating Surface-Piercing Cylinder

Do-Chun Hong*

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2차원 浮遊體에 작용하는 陰의 수평방향 漂流力에 대한 고찰

洪 道 天*

Key Words : Drift Force(표류력), Water Wave Radiation Diffraction Problem(수파 발산·산란 문제), Floating Body Motions(부유체 운동)

초 록

수면상에 떠있는 2차원 물체에 작용하는 시간평균 표류력 및 표류모멘트를 비점성 선형 포텐셜 이론을 사용하여 계산하는 방법에 대하여 검토하였다. 부유체 접수면상의 압력을 직접 적분하여 구한 수평방향 표류력이 특징주파수 부근에서 음의 값을 보이고 있다. 이는 무한원방에서의 에너지 보전방법에 의한 표류력이 항상 양의 값을 취한다는 기존 이론과 상이하다. 본 논문에서, 이러한 차이가 부유체의 횡요 및 상하동요에 기인한 복원력의 성분과 횡요와의 연성효과에 의하여 발생하였음을 규명하였다. 이는 횡요가 있는 경우, 표류력을 산출하는 기존의 무한원방 방법에 결함이 있음을 보이고 있다. 이에 반하여 기존의 접수면압력 직접적분 방법은 부유체에 작용하는 시간평균 표류력 및 표류모멘트를 모든 주파수에 대하여 정확하게 산출한다고 결론지을 수 있다.

1. INTRODUCTION

Since Suyehiro reported in 1924, a non-zero steady horizontal force acting on an oscillating vessel in waves, the drift force has been studied by many hydrodynamicists to establish a rigorous formula for its computation¹⁾. Neglecting the effect of viscosity, there are two methods which are well defined; the far-field method developed principally by Maruo and Newman and

the near-field method completed by Pinkster and van Oortmerssen^{2,3,4)}. Other methods which need restrictions in the vessel geometry are not mentioned here since computational techniques for a floating body of arbitrary shape are in common use.

Since the time-mean second-order force and moment can be found from the products of first order quantities, it is essential to solve the first order potential problem properly. In this paper,

* Hyundai Maritime Research Institute, HHI, Ulsan 682-792, Korea

the first-order radiation-diffraction problem for a surface-piercing body freely floating in waves is solved by the improved Green integral equation which makes use of a mixed distribution of sources and normal doublets over the closed surface composed of the wetted surface and the waterplane of the body^(5,6). In the contrast to the fact that the source integral equation where only sources are distributed on the wetted surface of the body fails to solve the problem at irregular frequencies for a surface-piercing body, the former gives a good solution for all frequencies.

According to Maruo's discussion, the horizontal drift force acting on a cylinder, fixed or freely floating, is always positive if no net work is done to the body. It is correct that the net average energy flux through vertical planes surrounding the body at far-field, is zero. But, it was not evident whether the hydrostatic pressure force acting on the body should have no contribution to the mean horizontal force. In this paper, the horizontal drift force computed by the near-field method has been found to be negative at certain frequencies. It seems that the coupling of hydrostatic restoring force component with the roll motion at frequencies where the roll response is dominant, act in favor of the negative force. In fact, the negative drift force has already been reported by Huse in 1977. Although it was an experimental result for a three-dimensional body in real fluid, Huse pointed out that the phase of body motion might have caused the negative drift force⁽⁷⁾.

2. RESOLUTION OF POTENTIAL PROBLEM

The fluid is assumed to occupy a space V bounded by the wetted surface S of a surface-piercing body and by the free surface F of deep water under gravity. Cartesian coordinates (x, y)

attached to the mean position of the body, are employed with the origin o in the waterplane W of the body at its mean position and the y axis vertically upwards. The body performs simple harmonic oscillations of small amplitude about its mean position with circular frequency ω which is equal to that of plane progressive waves incident from $x=-\infty$. With the usual assumptions of an incompressible fluid and irrotational flow without capillarity, the fluid velocity is given by the gradient of a velocity potential. It is assumed that the magnitude of unsteady flow due to the incident wave can be represented by a small parameter ϵ .

Here, the following displacement vector is introduced in order to represent all quantities on S by appropriate power series expansions in ϵ of the quantities evaluated on S_0 which represents S at its mean position:

$$\begin{aligned} \vec{A}(M_0) &= \vec{M}_0 M_1 = a_1 \vec{e}_1 + a_2 \vec{e}_2 + a_3 \vec{e}_3 \\ &\times \vec{OM}_0, \quad M_0 \in S_0, \quad M_1 \in S \end{aligned} \quad (1)$$

where $a_j(j=1,2,3)$ denote respectively sway, heave and roll amplitudes and O the center of rotation of the body at its mean position. Since the first-order problem is linear, the unsteady potential $\Phi(x,y,t)$ can be decomposed into the incident wave potential Φ_0 , the diffraction potential Φ_I and the radiation potential Φ_R .

$$\begin{aligned} \Phi(x, y, t) &= \Phi_0 + \Phi_R + \Phi_I \\ &= \text{Re} \{ (\Psi_0 + \Psi_R + \Psi_I) e^{-i\omega t} \} \end{aligned} \quad (2)$$

$$\Psi_0 = -\frac{a_0 g}{\omega_0} e^{-ik(x+iy)} \quad (3)$$

where a_0 denotes the amplitude of free surface with respect to the plane $y=0$, g the gravitational acceleration and k the wavenumber.

The time-mean hydrodynamic force and moment are second-order quantities and can be found from the products of first-order quantities.

Thus all oscillating quantities expressed with respect to the mean position of the oscillating body should be developed to reveal their quantities of $O(\varepsilon)$ as follows:

$$\Phi|_S = \Phi|_{S_0} + \vec{A} \cdot \nabla \Phi|_{S_0} + O(\varepsilon^3) + \dots \quad (4)$$

$$\vec{n} = \vec{n}_0 + a_3 \vec{e}_3 \times \vec{n}_0 + O(\varepsilon^2) + \dots \quad (5)$$

where \vec{n} denotes a normal vector directed into the fluid region V from S and \vec{n}_0 represent \vec{n} at its mean position. The governing equation and the first-order free surface boundary condition for Φ_R and Φ_I are as follows:

$$\nabla^2 \Phi = 0 \quad \text{in } V \quad (6)$$

$$\frac{\partial^2 \Phi}{\partial t^2} + g \frac{\partial \Phi}{\partial y} = 0 \quad \text{on } y=0 \quad (7)$$

The body boundary condition for Φ_R and Ψ_I are

$$\frac{\partial \Phi_R}{\partial n_0} = [a_1' \vec{e}_1 + a_2' \vec{e}_2 + a_3' \vec{e}_3 \times \vec{OM}] \cdot \vec{n}_0, \quad M \in S_0 \quad (8)$$

and

$$\frac{\partial \Psi_1}{\partial n_0} = - \frac{\partial \Psi_0}{\partial n_0} \quad (9)$$

where the prime denotes the differentiation with respect to the time. The potential must also satisfy the radiation condition at infinity.

Introducing complex amplitude defined as follows,

$$a_k = \text{Re} \{ a_k e^{-i\omega t} \} \quad (10)$$

and considering the body boundary condition given by the Eq(8), it can be found that Ψ_R takes the following form :

$$\Psi_R = -i\omega \sum_{k=1}^3 a_k \Psi_k \quad (11)$$

Then body boundary conditions for Ψ_k can be written as follows:

$$\frac{\partial \Psi_k}{\partial n_0} = \vec{e}_k \cdot \vec{n}_0 \quad \text{on } S_0, \quad k = 1, 2 \quad (12)$$

$$\frac{\partial \Psi_3}{\partial n_0} = (\vec{e}_3 \times \vec{OM}) \cdot \vec{n}_0 \quad \text{on } S_0 \quad (13)$$

By making use of a Green function which satisfies the free surface boundary condition as well as the radiation condition at infinity, Ψ_q ($q=1,2,3,4$) can be found by the following formula :

$$\Psi_q = \int_{S_0} \left(\sigma_q G + \mu_q \frac{\partial G}{\partial n} \right) dS \quad (14)$$

where G is the Green function represented in the complex plane $z = x + iy$:

$$G(z, z') = \frac{1}{2\pi} \text{Re} \left\{ \log \frac{z-z'}{z-\bar{z}'} - 2J [-ik(z-\bar{z}')] \right\} + i \text{Im} \{ -i e^{-ik(z-\bar{z}')} \} \quad (15)$$

with

$$J(W) = e^{i\pi} [E_1(W) + i\pi] \quad (16)$$

where E_1 is the modified complex exponential integral⁽⁸⁾.

In the Eq.(14), σ_q is the density of sources distributed over S which is given by the following formula:

$$\sigma_q = \frac{\partial \Psi_q}{\partial n}, \quad q = 1, 2, 3, 4 \quad (17)$$

Then the unknown surface distribution of normal doublet, μ can be found from the solution of the following integral equation:

$$\begin{aligned} \frac{\mu(M)}{2} + \oint_{S_0, U, W} \mu(M) \frac{\partial G(P, M)}{\partial n(M)} dS(M) \\ = - \int_{S_0} \sigma(M) G(P, M) dS(M), \quad P \text{ on } S_0, U, W \end{aligned} \quad (18)$$

The above integral equation is the improved Green integral equation which can uniquely be solved by a linear system solver after discretization.

3. FIRST-ORDER HYDRODYNAMIC LOADS AND MOTION RESPONSES

The motion responses of the body can be obtained by the following equation of motion in the frequency domain:

$$\begin{aligned} \rho v I_{ik} a_k'' &= - \int p_M \vec{n} \cdot \vec{e}_i dS \\ &\quad - \int p_E \vec{n} \cdot \vec{e}_i dS - \rho L R_{ik} \cdot a_k, \end{aligned} \quad (19)$$

$i = 1, 2$

$$\begin{aligned} \rho v I_{3k} a_k'' &= - \int p_M \left(\frac{\overline{OM}}{L} \times \vec{n} \right) \cdot \vec{e}_3 dS \\ &\quad - \int p_E \left(\frac{\overline{OM}}{L} \times \vec{n} \right) \cdot \vec{e}_3 dS \\ &\quad - \rho g L R_{3k} \cdot a_k \end{aligned} \quad (20)$$

with

$$p_M = -\rho \frac{\partial}{\partial t} \Phi_R \quad (21)$$

$$p_E = -\rho \frac{\partial}{\partial t} (\Phi_0 + \Phi_1) \quad (22)$$

Here, I_{ik} and R_{ik} are respectively the inertia coefficients and the hydrostatic restoring forces and moments coefficients, L the characteristic length and ρv the mass of the body.

By making use of the added-mass and wave-damping coefficients, M_{ik} and B_{ik} as well as the wave-exciting force coefficients F_i , equations (19) and (20) become the following complex-valued linear system for a_k ($k=1,2,3$):

$$\begin{aligned} \left\{ \frac{v}{L^3} I_{ik} + M_{ik} + iB_{ik} - \frac{g}{\omega^2 L} R_{ik} \right\} \cdot a_k \\ = -a_0 F_i, \quad i = 1, 2, 3 \end{aligned} \quad (23)$$

4. TIME-MEAN SECOND-ORDER HYDRODYNAMIC FORCES AND MOMENTS

The time-mean second-order hydrodynamic force and moment can be found from the following formula:

$$\begin{aligned} \vec{F} = \rho \int_{S_0} \left\{ \frac{\partial}{\partial t} \Phi + \vec{A} \cdot \nabla \left(-\frac{\partial}{\partial t} \Phi \right) + \frac{1}{2} |\nabla \Phi|^2 \right. \\ \left. + g(y + \vec{A} \cdot \nabla y) \right\} (\vec{n}_0 + a_3 \vec{e}_3 \times \vec{n}_0) dS \end{aligned} \quad (24)$$

$$\begin{aligned} \vec{M} = \rho \int_{S_0} \left[\frac{\partial}{\partial t} \Phi + \vec{A} \cdot \nabla \left(\frac{\partial}{\partial t} \Phi \right) \right. \\ \left. + \frac{1}{2} |\nabla \Phi|^2 + g(y + \vec{A} \cdot \nabla y) \right] \\ (\overline{OM} + \vec{A}) \times (\vec{n}_0 + a_3 \vec{e}_3 \times \vec{n}_0) dS \end{aligned} \quad (25)$$

It is the near-field formulation.

For a surface-piercing body, the integration over S_0 should cover the area between the actual position of the waterline and the instantaneous position of the water surface, ζ_R :

$$\zeta_R(M) = \zeta_M - A_M, \quad M = C, D \quad (26)$$

with

$$\zeta_M = -\frac{1}{g} \frac{\partial \Phi(M)}{\partial t}, \quad M = C, D \quad (27)$$

$$A_M = \vec{A}(M) \cdot \vec{e}_2, \quad M = C, D \quad (28)$$

where ζ indicates the first-order wave elevation and the subscripts C and D denote respectively left and right intersecting points of the plane $y=0$ and the body surface S_0 . Thus the following integrals should respectively be added to F and M :

$$\begin{aligned} \vec{F}_\zeta = \rho \int_0^{\zeta_R} [gy + \vec{A} \cdot \nabla (gy) - g\zeta_C] n_0 dy \\ + \rho \int_0^{\zeta_R} [gy + \vec{A} \cdot \nabla (gy) - g\zeta_D] n_0 dy \end{aligned} \quad (29)$$

$$\begin{aligned} \vec{M}_\zeta = \rho \int_0^{\zeta_R} [gy + \vec{A} \cdot \nabla (gy) - g\zeta_C] (\overline{OM} \times \vec{n}_0) dy \\ + \rho \int_0^{\zeta_R} [gy + \vec{A} \cdot \nabla (gy) - g\zeta_D] (\overline{OM} \times \vec{n}_0) dy \end{aligned} \quad (30)$$

Taking the time-mean of the integrals, the drift force and moment can be found. In particular, the drift force component in x -direction, after some manipulation, is given by

the following formula :

$$F_x = f_s + f_\zeta + f_g \quad (31)$$

$$f_s = \rho \int_{S_n} \overline{\frac{\partial \Phi}{\partial t} \cdot a^3} [(\vec{e}_3 \times \vec{n}_0 \cdot \vec{e}_1)] dS \\ + \rho \int_{S_n} \overline{\vec{A} \cdot \nabla \frac{\partial \Phi}{\partial t}} (\vec{n}_0 \cdot \vec{e}_1) dS \quad (32) \\ + \frac{\rho}{2} \int_{S_n} \overline{|\nabla \Phi|^2} (\vec{n}_0 \cdot \vec{e}_1) dS$$

$$f_\zeta = \frac{1}{2} \rho g [\overline{\zeta_R^2(C)}' - \overline{A_C^2} \\ - (\overline{\zeta_R^2(D)}' - \overline{A_D^2})] \quad (33)$$

$$f_g = \rho g \left(\overline{a_2 \cdot a_3} \overline{CD} + \overline{a_3^2} \int_C^D x dx \right) \quad (34)$$

Pinkster represented F_x as the sum of the following four components :

- F_I contribution due to the relative wave elevation
- F_{II} -contribution due to the square of the velocity
- F_{III} -contribution due to the gradient of first order pressure multiplied by the first order displacement
- F_{IV} -contribution due to the inertia force and moment coupled with first order angular motions

It can be seen that f_ζ is F_I and the sum of f_s and f_g is equal to the sum of F_{II} , F_{III} and F_{IV} since the total first order force and moment are equated to the inertia force and moment of the body by Eq.(23). So, Eq.(31) comprises exactly Pinkster's four components and no other components are involved.

5. NUMERICAL RESULTS AND DISCUSSION

The non-dimensional drift force coefficient in x-direction, $F_1 = -F_x / (0.5 \rho g a_0^2)$ acting on el-

liptical or rectangular half immersed cylinders of various drafts T as shown in Fig. 1 are computed. They are all plotted as functions of $\kappa = \omega \sqrt{B/g}$. The moduli of non-dimensional complex amplitudes of sway, heave and roll for the cylinders, denoted respectively by $a_1 = |a_1|/a_0$, $a_2 = |a_2|/a_0$ and $a_3 = |a_3|g/(a_0 \omega^2)$ are also presented.

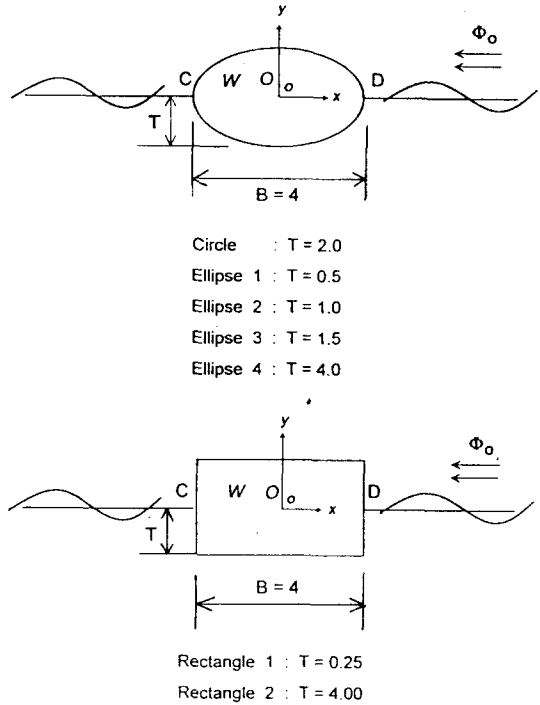


Fig. 1 Coordinate Systems and Numerical Models

The values of F_1 for the half immersed circular cylinder where $B=2T$ are in good agreement with the drift force coefficients F_r , calculated by the far-field method as shown in Fig. 2. It is well known that F_r is the non-dimensional amplitude square of the total reflected waves by the body and is always positive. Detailed computing techniques for F_r can be found, for example, in the reference⁽⁹⁾.

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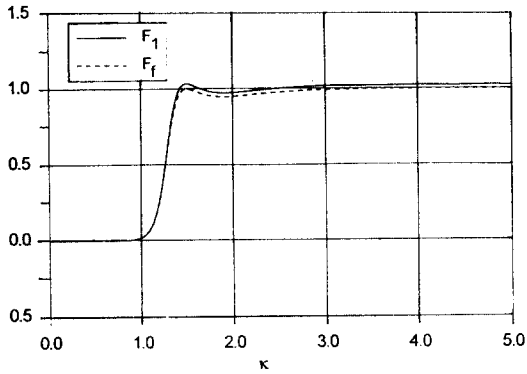


Fig. 2 Horizontal Drift Forces and Their Components for the Circle

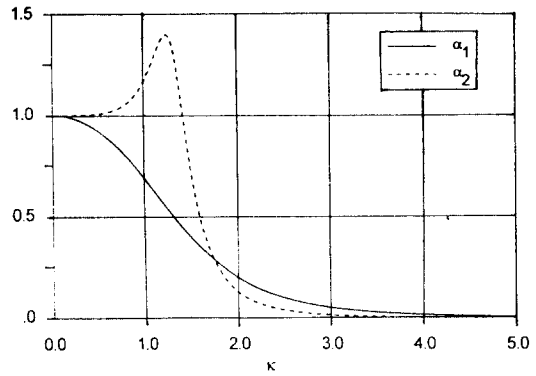


Fig. 3 Motion Responses for the Circle

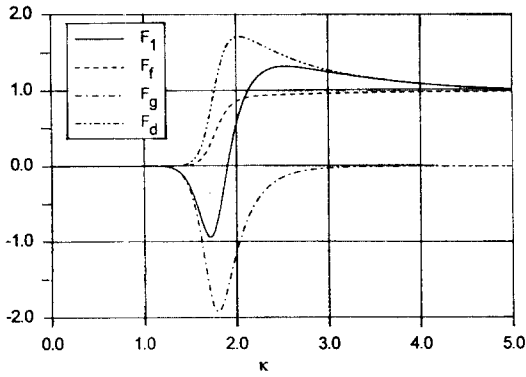


Fig. 4 Horizontal Drift Forces and Their Components for the Rectangle 1

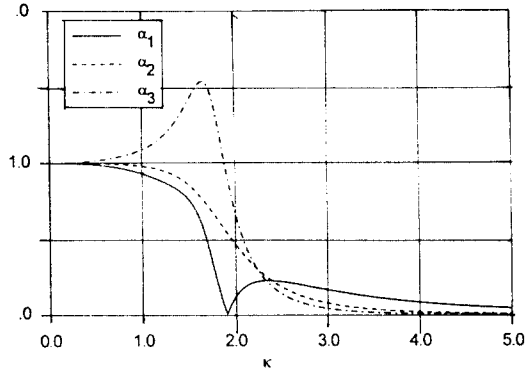


Fig. 5 Motion Responses for the Rectangle 1

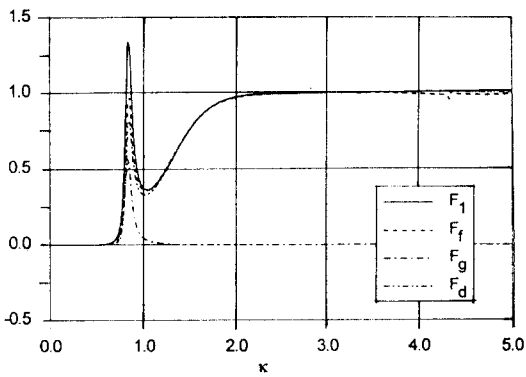


Fig. 6 Horizontal Drift Forces and Their Components for the Rectangle 2

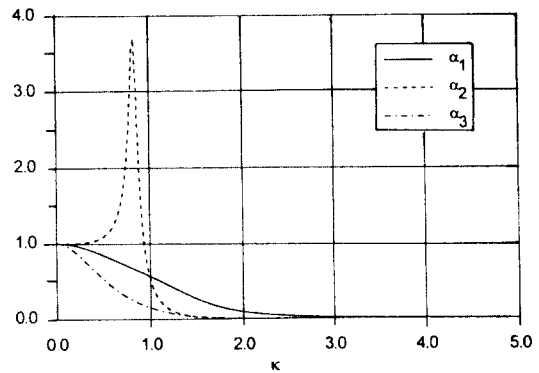


Fig. 7 Motion Response for the Rectangle 2

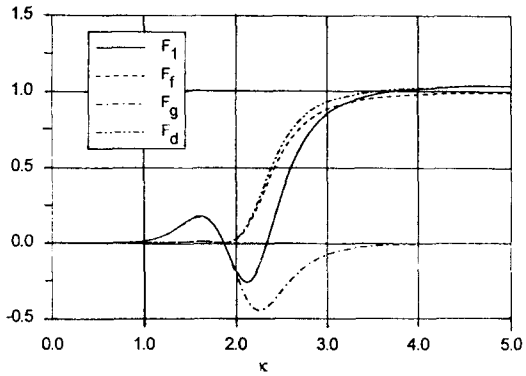


Fig. 8 Horizontal Drift Forces and Their Components for the Ellipse 1

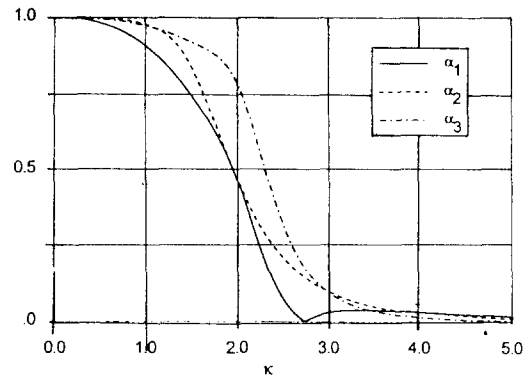


Fig. 9 Motion Responses for the Ellipse 1

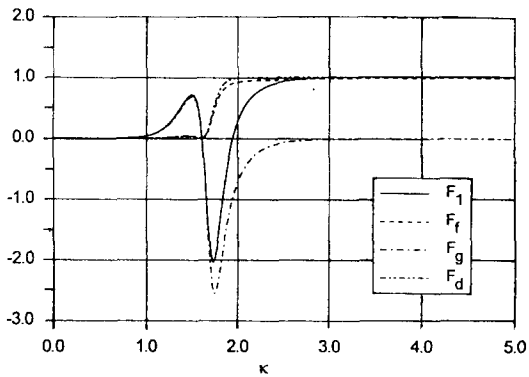


Fig. 10 Horizontal Drift Forces and Their Components for the Ellipse 2

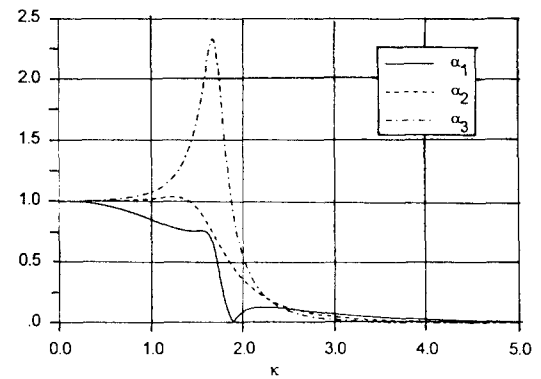


Fig. 11 Motion Responses for the Ellipse 2

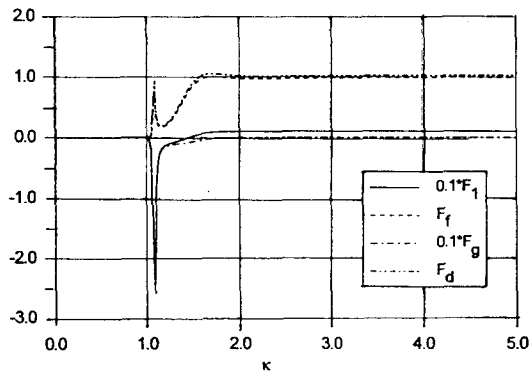


Fig. 12 Horizontal Drift Forces and Their Components for the Ellipse 3

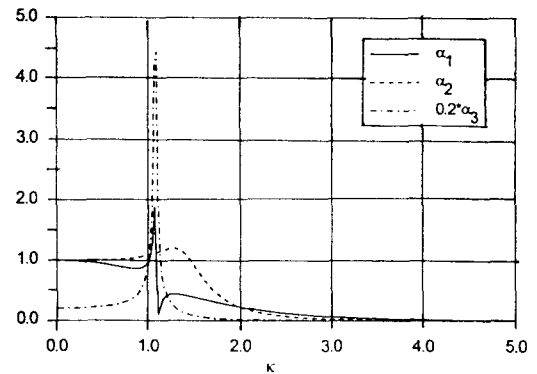


Fig. 13 Motion Responses for the Ellipse 3

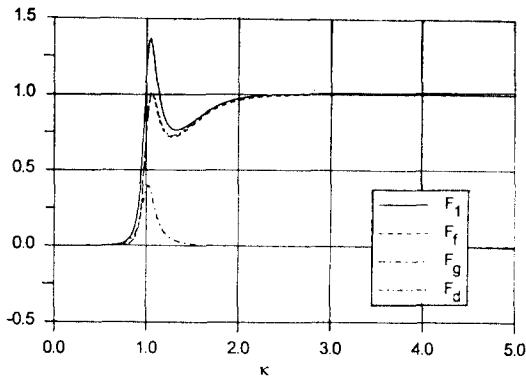


Fig. 14 Horizontal Drift Forces and Their Components for the Ellipse 4

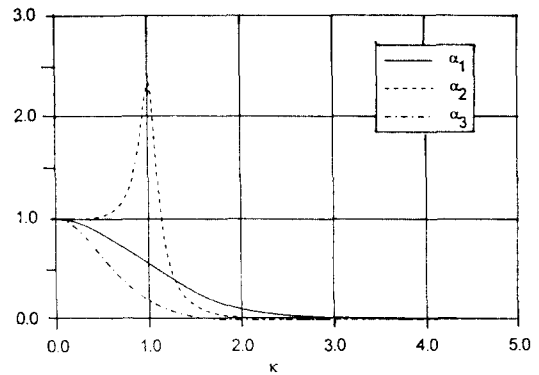


Fig. 15 Motion Responses for the Ellipse 4

As for elliptical or rectangular cylinders which admit roll motions as shown in Fig. 4 through Fig. 15, F_1 are different from F_t . Furthermore, for the ellipses 1, 2 and 3 as well as for the rectangle 1, the values of F_1 become negative at frequencies where roll responses are dominant as shown in Fig. 4 and in Fig. 5 as well as in Fig. 8 through Fig. 13. For the rectangle 2 and for the ellipse 4, the values of F_1 are greater than F_t at frequencies where heave responses are dominant as shown in Fig. 6, in Fig. 7, in Fig. 14 and in Fig. 15.

The drift force components $F_g = -f_{\eta} / (0.5 \rho g a_0^2)$ and $F_d = F_1 - F_g$ are also presented. It is shown that the values of F_t for cylinders except the rectangle 1 are in reasonable agreement with those of F_d . Thus, F_g can be regarded as a missing component in the drift force computed by the far-field method. The great discrepancy between F_d and F_t for the rectangle 1 can not be explained here under the assumption of the first-order wave theory in ideal fluid.

It should be noted that the negative values of F_1 for the ellipse 1 occur where the magnitude of motion responses do not exceed the magnitude of the incident wave amplitude as shown in Fig 8 and in Fig. 9. Thus, the realization of

the the calculated negative drift force can be expected for this special case.

6. CONCLUSIONS

- (1) The drift force and moment computed by the near-field method can provide the actual mean second-order force and moment acting on the wetted surface of a surface-piercing cylinder in case that the viscous damping effect is negligible.
- (2) The far-field method for drift force calculation should be improved to cover the missing component due to the coupling of the hydrostatic restoring force with the roll motion.
- (3) From the solution of the present integral equation, all first-order quantities can successfully be calculated.
- (4) The computed negative values of F_1 are subject to experimental verification.

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