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Multi-Level Skip-Lot Sampling Plan-Average Fraction Inspected Properties

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Abstact

The general formulas of average fraction inspected, average sample number and average outgoing quality in n-level skip-lot sampling plan are derived. Average sample number and average outgoing quality of a reference plan, three-level, five-level and ten-level skip-lot sampling plans are compared.

1. Introduction

The Skip-Lot Sampling Plan(SKSP) is a system of lot by lot inspection plan in which a provision is made for inspecting only some fraction of submitted lots. This plan should be used only when the quality of the submitted product is good as demonstrated by the vender's quality history and applicable to bulk materials or furnished in successive batches or lots.

Dodge (1955) initially presented SKSP as an extension of continuous sampling plan(CSP) type. In effect, a Multi-Level Skip-Lot Sampling Plan(MLSKSP) is the application of multi-level continuous sampling to lots rather than to individual units of production on an assembly line. Perry (1973) developed it to single-level and two-level SKSP of which the latter has no restriction on the fraction of lots inspected. Parker and Kessler (1981) presented a modified single-level SKSP under which at least one unit is always sampled from a lot. Choi (1993) presented a general n-level MLSKSP which has no restriction on the level n and skipping parameters f_k and i_k, where f_k is the fraction of lots inspected on the kth skipping inspection, and i_k is the number of lots consecutively inspected and accepted on the $(k-1)$ th skipping inspection, $k = 1, 2, \cdots, n$. Also, he derived the general formula of the operating characteristic function for the n-level skip-lot sampling plan.

In this paper, the general formula of the average fraction inspected(AFI), average sample number(ASN) and average outgoing quality(AOQ) for the n-level skip-lot sample plan are

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derived. Note that $0 < f_k \leq 1$ and i_k's are natural numbers for $k = 1, 2, \cdots, n$.

2. Procedure of the MLSKSP

MLSKSP uses a specified lot inspection plan by the method of attributes (single sampling, double sampling, etc.) called the "reference-sampling plan" together with the following rules. The following procedure of the MLSKSP is defined in terms of Choi (1993).

(1) Begin with normal inspection, using the reference plan. At this stage of operation, every lot is inspected.

(2) When i_1 consecutive lots are accepted on normal inspection, switch to the first skipping inspection at rate f_1.

(3) During the first skipping inspection ;

- When i_2 consecutively inspected lots are accepted, switch to the second skipping inspection at rate f_2.
- When a lot is rejected, switch to the normal inspection.

(4) During the kth skipping inspection, $k = 2, 3, \cdots, n-1$;

- When i_{k+1} consecutively inspected lots are accepted, switch to the $(k+1)$th skipping inspection at rate f_{k+1}
- When a lot is rejected, switch to the $(k-1)$th skipping inspection.

(5) During the nth skipping inspection at rate f_n ;

- When a lot is rejected, switch to the $(n-1)$th skipping inspection.

Screen each rejected lot and correct or replace all defective units found in inspecting lot by nondefective units.

3. Transition Matrix for the MLSKSP

The Markov chain approach is used to determine AFI, for a given value of process defective fraction. The states of the Markov chain for MLSKSP are defined in terms of Choi (1993) as follows.

NR = state where lot is rejected under normal inspection.

N_j = states under normal inspection representing the number of consecutively accepted lots j, $j = 1, 2, \cdots, i_1$

$SkAj$ = states that the number of consecutively inspected and accepted lots during the kth skipping inspection at rate f_k is j, $k = 1, 2, \cdots, n-1$, $j = 1, 2, \cdots, i_{k+1}$.

$SkNj$ = states that the lot skipped during the kth skipping inspection at rate f_k and previous number of inspected and accepted lots on the kth skipping inspection at rate f_k is j, $k = 1, 2, \cdots, n-1$, $j = 0, 1, \cdots, i_{k+1}-1$.

SkR = states that the lot rejected during the kth skipping inspection at rate f_k, $k = 1, 2, \cdots, n$.

SnA = state that the lot inspected and accepted during the nth skipping inspection at rate f_n.

SnN = state that the lot skipped during the nth skipping inspection at rate f_n.

SnR = state that the lot rejected during the nth skipping inspection at rate f_n.

Since it can be shown that this Markov chain is finite, recurrent, irreducible and aperiodic, the long run or stationary probabilities, Π_i's, of all the given states are the unique solutions to the following system of equations ;

$$\Pi_i = \sum_{all\ states\ j} \Pi_j P_{ji} \qquad \text{for all states } i,$$

and

$$\sum_{all\ states\ i} \Pi_i = 1,$$

where P_{ji} is the one-step transition probability of going from state j to state i.

Let P denote the probability of accepting a lot according to the reference plan and $Q = 1-P$ throughout this paper. Then the one-step transition probability matrix and operating characteristic curve are given by Choi (1993).

4. Derivation of Average Fraction Inspected (AFI) and Average Sample Number (ASN)

A very important property of skipping lot sampling plan is that of reduced inspection, and it will be investigated here using the average sample number of the plan as a basis. The average sample size or average sample number is defined as the average number of sample units inspected per lot. For the skipping lot plan, lots that are not sampled will have an average sample size of zero. It will be assumed that there is no curtailment in the inspection of a sample ; i.e. even when the acceptance number is exceeded and rejection is certain, the remainder of the sample is nevertheless inspected. For the plan MLSKSP, lots that are not sampled will have an average sample number of zero. For the lots that are sampled, the average sample number will be that of the reference sampling plan. Thus, letting ASN(MLSKSP) be the average sample number of the plan MLSKSP and ASN(R) be that of the reference plan. Perry (1973a) has shown that ASN(SKSP) = ASN(R) × AFI , $0 < AFI \leq 1$, where AFI is the average fraction of lots inspected which are sampled. Thus

$$\text{ASN(MLSKSP)} = \text{ASN(R)} \times \text{AFI}, \quad 0 < \text{AFI} \leq 1,$$

and it is obvious that ASN(MLSKSP) $\leq$ ASN(R) since $0 < AFI \leq 1$. This shows that MLSKSP yields a reduction in inspection as based upon the plan's ASN.

Now, we can derive AFI of MLSKSP as follows. Let $\Pi(k)$ be the long run stationary probabilities of level k, and

$$\Pi(k)' = \Pi(k) - \sum_{l=0}^{i_{k+1}-1} \Pi_{SkNl}, \quad k = 1, 2, \cdots, n.$$

Then

$$\begin{aligned}
\Pi(0)' &= \Pi(0), \\
\Pi(1)' &= \Pi(1) - (\Pi_{S1N0} + \Pi_{S1N1} + \cdots + \Pi_{S1Ni_2-1}) \\
&= \Pi_{S1A1} + \Pi_{S1A2} + \cdots + \Pi_{S1Ai_2} + \Pi_{S1R}, \\
\Pi(2)' &= \Pi(2) - (\Pi_{S2N0} + \Pi_{S2N1} + \cdots + \Pi_{S2Ni_3-1}) \\
&= \Pi_{S2A1} + \Pi_{S2A2} + \cdots + \Pi_{S2Ai_3} + \Pi_{S2R}, \\
&\;\;\vdots \qquad \cdots\cdots\cdots\cdots\cdots\cdots \\
\Pi(n-1)' &= \Pi(n-1) - (\Pi_{Sn-1N0} + \Pi_{Sn-1N1} + \cdots + \Pi_{Sn-1Ni_n-1}) \\
&= \Pi_{Sn-1A1} + \Pi_{Sn-1A2} + \cdots + \Pi_{Sn-1Ai_n} + \Pi_{Sn-1R}, \\
\Pi(n)' &= \Pi_{SnA} + \Pi_{SnR}
\end{aligned}$$

It can be shown that

$$\text{AFI} = \Pi(0)' + \Pi(1)' + \Pi(2)' + \cdots + \Pi(n)' = \frac{V}{W},$$

where

$$V = 1 + \frac{P^{i_1}}{(1-P^{i_1})} + \frac{P^{i_1}P^{i_2}}{(1-P^{i_1})(1-P^{i_2})} + \frac{P^{i_1}P^{i_2}P^{i_3}}{(1-P^{i_1})(1-P^{i_2})(1-P^{i_3})} + \cdots + \frac{P^{i_1}P^{i_2}\cdots P^{i_n}}{(1-P^{i_1})(1-P^{i_2})\cdots(1-P^{i_n})},$$

$$W = 1 + \frac{1}{f_1}\frac{P^{i_1}}{(1-P^{i_1})} + \frac{1}{f_2}\frac{P^{i_1}P^{i_2}}{(1-P^{i_1})(1-P^{i_2})} + \frac{1}{f_3}\frac{P^{i_1}P^{i_2}P^{i_3}}{(1-P^{i_1})(1-P^{i_2})(1-P^{i_3})} + \cdots + \frac{1}{f_n}\frac{P^{i_1}P^{i_2}\cdots P^{i_n}}{(1-P^{i_1})(1-P^{i_2})\cdots(1-P^{i_n})}.$$

From above formula, if we let $f_1 = f,\ f_2 = f_3 = \cdots = f_n = 1,\ i_1 = i,\ i_2 = i_3 = \cdots = i_n = \infty$, then

$$\text{AFI} = \frac{f}{(1-f)P^i + f},$$

which is exactly Perry's (1973a) formula for the single level skip-lot sampling plan SkSP-2 .

5. Average Outgoing Quality (AOQ)

AOQ is widely used for the evaluation of a rectifying sampling plan. The AOQ is the quality in the lot that results from the application of rectifying inspection. It is the average value of lot quality that would be obtained over a long sequence of lots from a process with defective fraction p_{dl}. Since AOQ of reference plan is

$$\text{AOQ(R)} = \frac{P \cdot p_{dl}(N-n^*)}{N},$$

The AOQ of MLSKSP can be easily shown that

$$\text{AOQ(MLSKSP)} = \text{AFI} \cdot \text{AOQ(R)} + (1-\text{AFI}) \cdot p_{dl},$$

where N is lot size, n^* is sample size, p_{dl} is lot quality and P is probability of accepting a lot according to the reference plan. Note that as the lot size N becomes large relative to the sample size n^*, we may write AOQ(R) and AOQ(MLSKSP) as

$$\text{AOQ(R)} \simeq P \cdot p_{dl},$$

and

$$\text{AOQ(MLSKSP)} \simeq \text{AFI} \cdot P \cdot p_{dl} + (1-\text{AFI}) \cdot p_{dl}.$$

6. Numerical results

To illustrate the properties of ASN(MLSKSP) and AOQ(MLSKSP), under the single sampling reference plan of sample size $n^* = 50$, $c=2$, we have the following

$$\text{ASN(MLSKSP)} = n^* \cdot \text{AFI},$$

and Figure 1 and Figure 2 represent the ASN and AOQ curves for the above reference plan, following 3-level, 5-level and 10-level MLSKSPs ;

(1) $n=3$,

$$f_1 = \frac{1}{2}, \quad f_2 = \frac{1}{4}, \quad f_3 = \frac{1}{6},$$

$$i_1 = 2, \quad i_2 = 4, \quad i_3 = 6.$$

(2) $n=5$,

$$f_1 = \frac{1}{2}, \quad f_2 = \frac{1}{4}, \quad f_3 = \frac{1}{6}, \quad f_4 = \frac{1}{8}, \quad f_5 = \frac{1}{10},$$

$$i_1 = 2, \quad i_2 = 4, \quad i_3 = 6, \quad i_4 = 8, \quad i_5 = 10.$$

(3) n=10,

$$f_1 = \frac{1}{2}, \quad f_2 = \frac{1}{4}, \quad f_3 = \frac{1}{6}, \quad f_4 = \frac{1}{8}, \quad f_5 = \frac{1}{10},$$

$$f_6 = \frac{1}{12}, \quad f_7 = \frac{1}{14}, \quad f_8 = \frac{1}{16}, \quad f_9 = \frac{1}{18}, \quad f_{10} = \frac{1}{20},$$

$$i_1 = 2, \quad i_2 = 4, \quad i_3 = 6, \quad i_4 = 8, \quad i_5 = 10,$$

$$i_6 = 12, \quad i_7 = 14, \quad i_8 = 16, \quad i_9 = 18, \quad i_{10} = 20.$$

From both figures, we can see that (1) skip-lot sampling plans are more desirable than the reference plan, and (2) ten-level skip-lot sampling plans seems most resonable among the plans considered because ASNs and AOQs are smallest. It seems that the higher-level skip-lot sampling plan seems more resonable than the lower-level skip-lot sampling plans considered in the aspect that the ASNs and AOQs are the smallest for all defective rates.

References

[1] Choi, Byoung-Chul (1993). Multi-Level Skip-Lot Sampling Plan, *The Korean Journal of Applied Statistics*, 6, 277-287.

[2] Dodge, H. F. (1943). A Sampling Inspection Plan for Continuous Production, *Annals of Mathematical Statistics*, 14, 264-279.

[3] Dodge, H. F. (1955). Skip-Lot Sampling Plan, *Industrial Quality Control*, 11, 3-5.

[4] Lieberman, G. J. and Soloman, H. (1955). Multi-Level Continuous Sampling Plans, *Annals of Mathematical Statistics*, 26, 686-704.

[5] Parker, R. D. and Kessler, L. (1981). A Modified Skip-Lot Sampling Plan, *Journal of Quality Technology*, 13, 31-35.

[6] Perry, R. L. (1973a). Skip-Lot Sampling Plans, *Journal of Quality Technology*, 5, 123-130.

[7] Perry, R. L. (1973b). Two-Level Skip-Lot Sampling Plans-Operating Characteristic properties, *Journal of Quality Technology*, 5, 160-166.

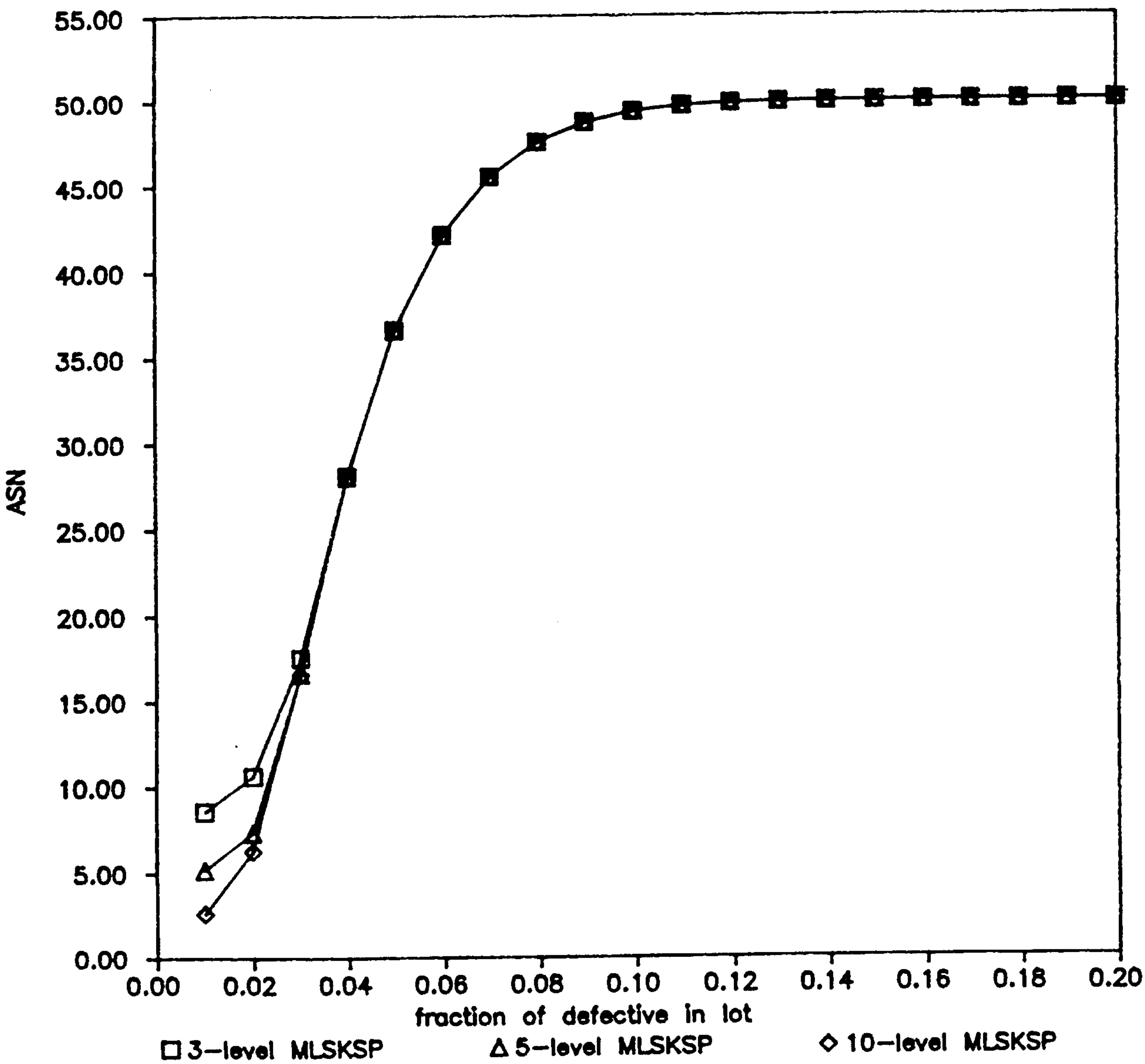

Figure 1. Average Sample Number Curve

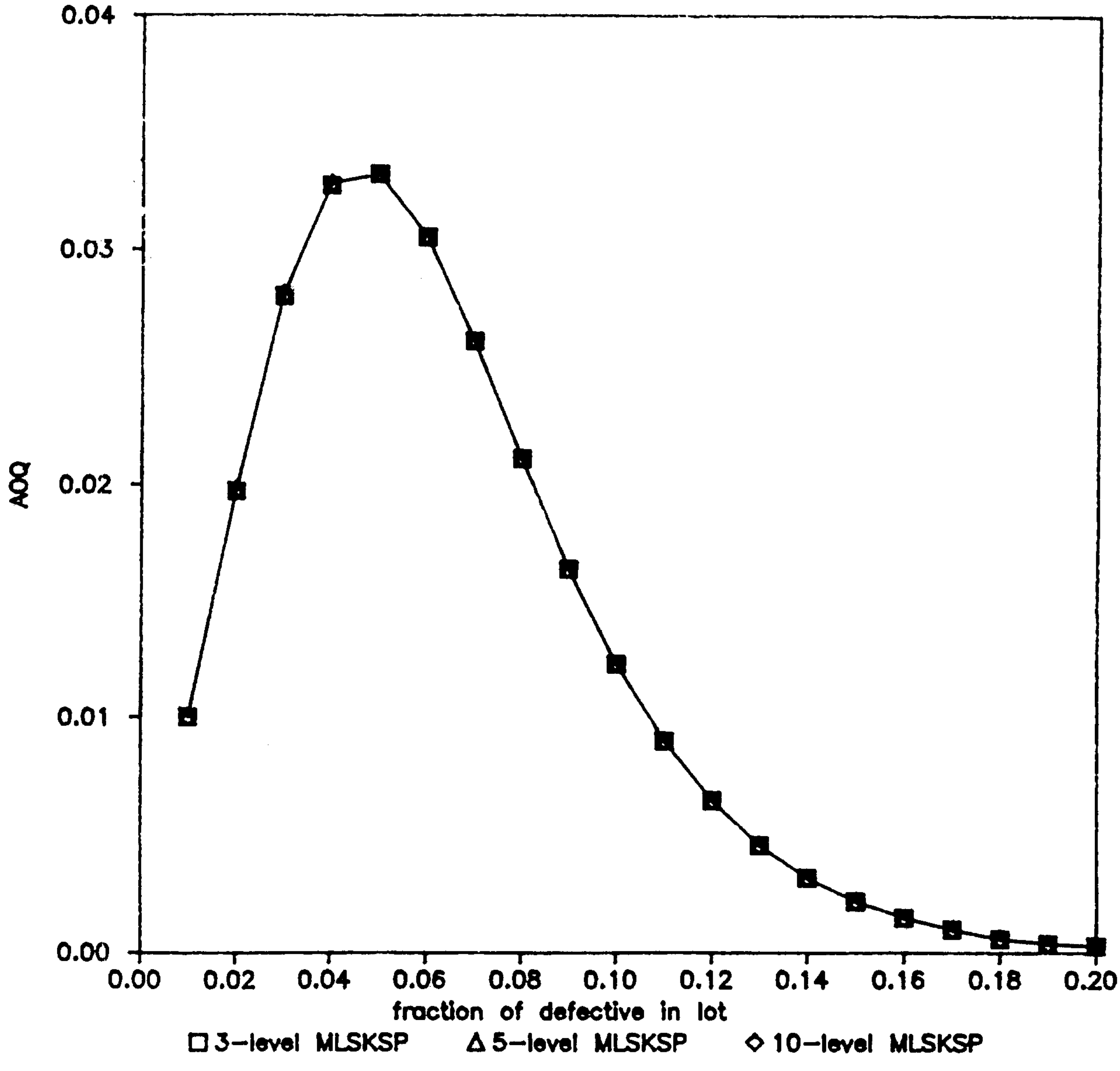

Figure 2. Average Outgoing Quality Curve