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Checking the Additive Risk Model with Martingale Residuals †

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Abstract

In contrast to the multiplicative risk model, the additive risk model specifies that the hazard function with covariates is the sum of, rather than product of, the baseline hazard function and the regression function of covariates. We, in this paper, propose a method for checking the adequacy of the additive risk model based on partial-sum of matingale residuals. Under the assumed model, the asymptotic properties of the proposed test statistic and approximation method to find the critical values of the limiting distribution are studied. Several real examples are illustrated.

Key Words : Additive risk model; Multiplicative risk model; Proportional hazards model; Martingale residual.

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1. INTRODUCTION

The additive and multiplicative risk models provide the two principal frameworks for studying the association between risk factors and disease occurrence or death.

The multiplicative risk models(Cox,1972) with the partial likelihood principle(Cox,1975) has become exceedingly popular for the analysis of failure time observations. This model specifies that the hazard function for the failure time T associated with a $p \times 1$ vector of covariates $\mathbf{Z}$ is of the form

$$\lambda(t; \mathbf{Z}) = \lambda_0(t) g(\gamma_0' \mathbf{Z}), \tag{1.1}$$

where $\lambda_0(\cdot)$ is unspecified baseline hazard function, γ_0 is a $p \times 1$ vector of unknown regression parameters and $g(\cdot)$ is relative risk function, which must be positive. Taking $g(x) = e^x$ which is the most common form leads to the proportional hazards model(PHM).

In contrast to the multiplicative risk model, the additive risk model specifies that the hazard function associated with covariates is the sum of, rather than the product of, the baseline hazard function and the regression function of $\mathbf{Z}$. Therefore the hazard function is of the form

$$\lambda(t; \mathbf{Z}) = \lambda_0(t) + \beta_0' \mathbf{Z}, \tag{1.2}$$

where β_0 is a $p \times 1$ vector of unknown regression parameters.

Numerous graphical and analytical methods have been suggested for checking the adequacy of proportional hazards model by several authors including Lin and Wei(1991). In paticular, Lin, Wei and Ying(1993) introduced a new class of graphical and numerical methods which are derived from cumulative sums of martingale-based residuals over follow-up time and/or covariate values.

Although additive risk models in various forms have been eloquently advocated and successfully utilized by many authors, no satisfactory semiparametric methods has been developed for model (1.2). Recently, Lin and Ying(1994) introduced a simple semiparametric estimating function for β_0, which mimics the matingale feature of the partial likelihood score function for γ_0 under proportional hazards model. And Kim(1995) provided some test procedures for checking the adequacy of the additive risk model with time-invariant binary covariate.

In this paper, we propose a test statistic for checking the additivity of risks, which extends the Lin *et al.*(1993)'s idea in the proportional hazards model to the additive risk model with time-invariant continuous covariate. In Section 2, we review the inference procedures under additive risk model. Also we propose a test statistic for checking the adequacy of the model and study the asymptotic properties of it. In Section 3, model checking technique is described and two examples are illustrated to apply our results.

2. MODEL AND MARTINGALE RESIDUALS

Let T and C denote the failure time and censoring time, respectively. Assume that the vector of covariates $\mathbf{Z}$ is bounded and T and C are conditionally independent given $\mathbf{Z}$. Suppose that the data consists of n independent replicates of $(X, \Delta, \mathbf{Z})$, where $X = \min(T, C)$, $\Delta = I(T \leq C)$, and $1 - \Delta$ is the censoring indicator function.

Let $N_i(t) = \Delta_i I(X_i \leq t)$, $(i = 1, 2, \cdots, n)$ be a counting process for the i-th subject, which indicates that the true failure time of the i-th subject is observed up to time t.

Under model (1.2), the intensity function for the counting process $N_i(t)$ is given by

$$Y_i(t) d\Lambda(t; \mathbf{Z}_i) = Y_i(t)\{d\Lambda_0(t) + \beta_0' \mathbf{Z}_i(t) dt\},$$

where $Y_i(t)$ is a predictable indicator process indicating whether or not the i-th subject is at risk just before time t, and Λ_0 is the baseline cumulative hazard function.

The counting process $N_i(\cdot)$ can be uniquely decomposed so that for every i and t,

$$N_i(t) = M_i(t) + \int_0^t Y_i(s) d\Lambda(s; \mathbf{Z}_i),$$

where $M_i(\cdot)$ is a local square integrable martingale.

Therefore, the natural estimator of Λ_0 is given by

$$\widehat{\Lambda}_0(\widehat{\beta}, t) = \int_0^t \frac{\sum_{i=1}^n \{dN_i(s) - Y_i(s)\widehat{\beta}' \mathbf{Z}_i(s) ds\}}{\sum_{j=1}^n Y_j(s)}, \tag{2.1}$$

where $\widehat{\beta}$ is a consistent estimator of β_0.

Then by mimicking the partial likelihood score function in the PHM, Lin and Ying(1994) proposed the following estimating function

$$U(\beta) = \sum_{i=1}^{n} \int_0^\infty \mathbf{Z}_i(t)\{dN_i(t) - Y_i(t)d\widehat{\Lambda}_0(\beta, t) - Y_i(t)\beta'\mathbf{Z}_i(t)dt\} \tag{2.2}$$

and they estimated the regression coefficients by solving the equation $U(\beta) = 0$. The resulting estimator is given by

$$\widehat{\beta} = \left[\sum_{i=1}^{n} \int_0^\infty Y_i(t)\{\mathbf{Z}_i(t) - \overline{\mathbf{Z}}(t)\}^{\otimes 2}dt\right]^{-1} \left[\sum_{i=1}^{n} \int_0^\infty \{\mathbf{Z}_i(t) - \overline{\mathbf{Z}}(t)\}dN_i(t)\right], \tag{2.3}$$

where

$$\overline{\mathbf{Z}}(t) = \frac{\sum_{i=1}^{n} Y_i(t)\mathbf{Z}_i(t)}{\sum_{j=1}^{n} Y_j(t)}. \tag{2.4}$$

Furthermore, they showed the weak convergence results of the random vector $n^{\frac{1}{2}}(\widehat{\beta} - \beta_0)$.

For each i, the martingale residuals are defined as

$$\widehat{M}_i(t) = N_i(t) - \int_0^t Y_i(s)\{d\widehat{\Lambda}_0(\widehat{\beta}, s) + \widehat{\beta}'\mathbf{Z}_i ds\}, \tag{2.5}$$

where $\widehat{\Lambda}_0$ is defined in (2.1) and $\widehat{\beta}$ is defined in (2.3). For each t, the martingale residuals can be interpreted as the difference between the observed and expected numbers of events for the i-th subject over the time interval $[0, t]$. And as ordinary residuals in linear models have, the martingale residuals have the following properties.

Proposition. For any t,
(i) $\sum_{i=1}^{n} \widehat{M}_i(t) = 0$,
(ii) $E(\widehat{M}_i(t)) \longrightarrow 0$, for each i , as $n \to \infty$,
(iii) $Cov(\widehat{M}_i(t), \widehat{M}_j(t)) \longrightarrow 0$, for $i \neq j$, as $n \to \infty$.

Proof. By (2.5),

$$\sum_{i=1}^{n} \widehat{M}_i(t) = \int_0^t \sum_{i=1}^{n} dN_i(s) - \int_0^t \sum_{i=1}^{n} Y_i(s)\{d\widehat{\Lambda}_0(\widehat{\beta}, s) + \widehat{\beta}'\mathbf{Z}_i ds\} ,$$

hence (i) follows from (2.1).

By Taylor series expansion of $\widehat{M}_i$ at β_0, we get, for each t,

$$\widehat{M}_i(t) = M_i(t) - \int_0^t Y_i(s)\frac{\sum_{k=1}^{n} dM_k(s)}{\sum_{k=1}^{n} Y_k(s)} + R(t, \widehat{\beta}) .$$

Since $\widehat{\beta}$ is a consistent estimator of β_0, the remainder term, $R(t,\widehat{\beta})$ converges to zero. Thus (ii) follows from the fact that for each i, $E[M_i(t)] = 0$.

Now for each $i \neq j$,

$$\begin{aligned} E(\widehat{M}_i(t)\cdot\widehat{M}_j(t)) \approx & - E\int_0^t \frac{Y_i(s)}{\sum_{k=1}^n Y_k(s)} d\langle M_j, M_j\rangle(s) \\ & - E\int_0^t \frac{Y_j(s)}{\sum_{k=1}^n Y_k(s)} d\langle M_i, M_i\rangle(s) \\ & + E\int_0^t \frac{Y_i(s)Y_j(s)}{(\sum_{k=1}^n Y_k(s))^2} \sum_{k=1}^n d\langle M_k, M_k\rangle(s) \\ = & \; E\int_0^t -\frac{Y_i(s)Y_j(s)}{\sum_{k=1}^n Y_k(s)} d\Lambda_0(s) \\ \longrightarrow & \; 0 \quad \text{as } n \rightarrow 0 . \end{aligned}$$

Here $\approx$ stands for aysmptotic equality and thus the result (iii) follows. □

The estimating function (2.2) is equivalent to

$$U(\beta) = \sum_{i=1}^n \int_0^\infty \{\mathbf{Z}_i - \overline{\mathbf{Z}}(t)\}\{dN_i(t) - Y_i(t)\beta'\mathbf{Z}_i dt\},$$

and can be written as $U(\beta,\infty)$, where

$$U(\beta,t) = \sum_{i=1}^n \int_0^t \{\mathbf{Z}_i - \overline{\mathbf{Z}}(s)\}\{dN_i(s) - Y_i(s)\beta'\mathbf{Z}_i ds\}.$$

Note that $U(\widehat{\beta},\infty) = 0$ and $U(\widehat{\beta},t) = \sum_{i=1}^n \mathbf{Z}_i\widehat{M}_i(t)$, which is a function of the martingale residuals.

Now we propose the following test statistic for the additive risk model based on partial-sum of martingale residuals with respect to follow-up time and covariate values

$$W(t,\mathbf{z}) = \sum_{i=1}^n I(\mathbf{Z}_i \leq \mathbf{z})\widehat{M}_i(t),$$

where $\mathbf{z} = (z_1, z_2, \cdots, z_p)' \in R^p$, and the event $\{\mathbf{Z}_i \leq \mathbf{z}\}$ means that all the p components of $\mathbf{Z}_i$ are not larger than the corresponding components of $\mathbf{z}$. If the additive risk model holds, this process will fluctuate randomly around zero. But the asymptotic distribution of the test statistic $W(t,\mathbf{z})$ under the additive risk model can not be derived directly, so the following theorem is useful.

Theorem 1. Let

$$\bar{g}(t,\mathbf{z}) = \frac{\sum_{k=1}^{n} I(\mathbf{Z}_k \le \mathbf{z}) Y_k(t)}{\sum_{k=1}^{n} Y_k(t)},$$

and $\tilde{\mathbf{Z}}(t)$ and $\tilde{g}(t,\mathbf{z})$ be the limit of $\overline{\mathbf{Z}}(t)$ and $\bar{g}(t,\mathbf{z})$, respectively. Then the process $n^{-\frac{1}{2}} W(t,\mathbf{z})$ is asymptotically equivalent to the process $n^{-\frac{1}{2}} \widetilde{W}(t,\mathbf{z})$, where

$$\begin{aligned}\widetilde{W}(t,\mathbf{z}) = &\sum_{i=1}^{n} \int_0^t \{I(\mathbf{Z}_i \le \mathbf{z}) - \tilde{g}(s,\mathbf{z})\}\, dM_i(s) \\ &- \sum_{i=1}^{n} I(\mathbf{Z}_i \le \mathbf{z}) \int_0^t Y_i(s)\Big(\mathbf{Z}_i - \tilde{\mathbf{Z}}(s)\Big)' ds \\ &\times \left[\sum_{i=1}^{n} \int_0^\infty \{\mathbf{Z}_i - \tilde{\mathbf{Z}}(s)\} Y_i(s) \mathbf{Z}_i'\, ds\right]^{-1} \\ &\times \sum_{i=1}^{n} \int_0^\infty \{\mathbf{Z}_i - \tilde{\mathbf{Z}}(s)\}\, dM_i(s).\end{aligned}$$

Proof. By the Taylor series expansion of $W(t,\mathbf{z})$ at β_0, $W(t,\mathbf{z})$ can be rewritten as

$$\begin{aligned}W(t,\mathbf{z}) = &\sum_{i=1}^{n} I(\mathbf{Z}_i \le \mathbf{z}) \left[M_i(t) - \int_0^t Y_i(s) \frac{\sum_{k=1}^{n} dM_k(s)}{\sum_{k=1}^{n} Y_k(s)}\right] \\ &- \sum_{i=1}^{n} I(\mathbf{Z}_i \le \mathbf{z}) \int_0^t Y_i(s)\{\mathbf{Z}_i - \overline{\mathbf{Z}}(s)\}' ds \; (\widehat{\beta} - \beta_0).\end{aligned}$$

The equality holds because the second derivative of $W(t,\mathbf{z})$ is zero. The first term of the right-hand side of the equality is rewritten as

$$\sum_{i=1}^{n} \int_0^t \Big[I(\mathbf{Z}_i \le \mathbf{z}) - \bar{g}(s,\mathbf{z})\Big] dM_i(s).$$

and by the Taylor series expansion of $U(\widehat{\beta})$ at β_0, $(\widehat{\beta} - \beta_0)$ in the second term of the right-hand side of the equality, is of the form

$$\begin{aligned}(\widehat{\beta} - \beta_0) = &\left[\sum_{i=1}^{n} \int_0^\infty \{\mathbf{Z}_i - \overline{\mathbf{Z}}(s)\} Y_i(s) \mathbf{Z}_i' ds\right]^{-1} \\ &\times \sum_{i=1}^{n} \int_0^\infty \{\mathbf{Z}_i - \overline{\mathbf{Z}}(s)\}\{dN_i(s) - Y_i(s)\beta_0' \mathbf{Z}_i ds\}.\end{aligned}$$

Thus by the above assumptions of this theorem, the result follows. □

In order to derive the asymptotic distribution of $n^{-\frac{1}{2}}W(t,\mathbf{z})$, let us consider the asymptotic distribution of $n^{-\frac{1}{2}}\widetilde{W}(t,\mathbf{z})$.

Theorem 2. Under the additive risk model, the process $n^{-\frac{1}{2}}\widetilde{W}(t,\mathbf{z})$ converges weakly to a mean zero Gaussian process.

Proof. Let

$$\tilde{A}_n(t,\mathbf{z}) = \left[\sum_{i=1}^{n}\int_0^\infty \{\mathbf{Z}_i - \tilde{\mathbf{Z}}(s)\}Y_i(s)\mathbf{Z}_i'\,ds\right]^{-1} \times \sum_{i=1}^{n} I(\mathbf{Z}_i \leq \mathbf{z}) \int_0^t Y_i(s)\Big(\mathbf{Z}_i - \tilde{\mathbf{Z}}(s)\Big)\,ds.$$

Then

$$n^{-\frac{1}{2}}\widetilde{W}(t,\mathbf{z}) = n^{-\frac{1}{2}}\sum_{l=1}^{n}\int_0^t \{I(\mathbf{Z}_l \leq \mathbf{z}) - \tilde{g}(s,\mathbf{z})\}\,dM_l(s) - \tilde{A}_n'(t,\mathbf{z})n^{-\frac{1}{2}}\sum_{i=1}^{n}\int_0^\infty \{\mathbf{Z}_i - \tilde{\mathbf{Z}}(s)\}\,dM_i(s).$$

The first term of the right-hand side of the equality is tight because the two moment inequalities hold in Lemma 1(Lin *et al.*(1993)). And by the law of large numbers, $\tilde{A}_n$ converges to some nonrandom function and

$$n^{-\frac{1}{2}}\sum_{i=1}^{n}\int_0^\infty \{\mathbf{Z}_i - \tilde{\mathbf{Z}}(s)\}\,dM_i(s)$$

is also converge in distribution, so the second term is also tight.

Let

$$\tilde{h}_i(t,\mathbf{z},s) = I(s \leq t)\{I(\mathbf{Z}_i \leq \mathbf{z}) - \tilde{g}(s,\mathbf{z})\} - \tilde{A}_n'(t,\mathbf{z})\{\mathbf{Z}_i - \tilde{\mathbf{Z}}(s)\}.$$

Then $\widetilde{W}(t,\mathbf{z})$ is rewritten as

$$\widetilde{W}(t,\mathbf{z}) = \sum_{i=1}^{n}\int_0^\infty \tilde{h}_i(t,\mathbf{z},s)\,dM_i(s).$$

Since $\widetilde{W}(t,\mathbf{z})$ is simply a sum of independent random variables, it follows from the central limit theorem and the above tightness result that the process $n^{-\frac{1}{2}}\widetilde{W}(\cdot,\cdot)$ converges to a mean zero Gaussian process. The covariance function for $n^{-\frac{1}{2}}\widetilde{W}$ is given by

$$Cov\Big(n^{-\frac{1}{2}}\widetilde{W}(t_1,\mathbf{z}_1), n^{-\frac{1}{2}}\widetilde{W}(t_2,\mathbf{z}_2)\Big)$$
$$= E\left[\int_0^\infty \tilde{h}_i(t_1,\mathbf{z}_1,s)\tilde{h}_i(t_2,\mathbf{z}_2,s)Y_i(s)\{d\Lambda_0(s) + \beta_0'\mathbf{Z}_i ds\}\right]. \quad (2.6)$$

□

3. TESTING THE ADDITIVE RISK MODEL

We now need to show how to approximate the limiting distribution of the process $n^{-\frac{1}{2}}\widetilde{W}(t,\mathbf{z})$. If we know the stochastic structure of the martingale process $M_l(s)$, we could easily simulate $\widetilde{W}$. But the distributional form of $M_l(s)$ is unknown, so one way is to replace $M_l(s)$, which has a known distribution. Since for any t, $E[M_l(t)] = 0$, $Var[M_l(t)] = E[N_l(t)]$ (Fleming and Harrington(1991)), a natural candidate for $M_l(s)$ is $N_l(s)G_l$, with the same first and second moments, where $N_l(s)$ is the observed counting process and $\{G_l\ ;\ l = 1,\cdots,n\}$ denotes a random sample of standard normal variables. Therefore we obtain the following theorem.

Theorem 3. Let

$$\widehat{W}(t,\mathbf{z}) = \sum_{l=1}^{n} I(\mathbf{Z}_l \le t)\Delta_l\{I(\mathbf{Z}_l \le \mathbf{z}) - \overline{g}(X_l,\mathbf{z})\}G_l$$
$$-\sum_{i=1}^{n} I(\mathbf{Z}_i \le \mathbf{z})\int_0^t Y_i(s)\Big(\mathbf{Z}_i - \overline{\mathbf{Z}}(s)\Big)' ds$$
$$\times\left[\sum_{i=1}^{n}\int_0^\infty \{\mathbf{Z}_i - \overline{\mathbf{Z}}(s)\}Y_i(s)\mathbf{Z}_i' ds\right]^{-1} \times \sum_{l=1}^{n}\Delta_l\{\mathbf{Z}_l - \overline{\mathbf{Z}}(X_l)\}G_l.$$

Then the conditional distribution of $n^{-\frac{1}{2}}\widehat{W}(t,\mathbf{z})$ given $\{X_i,\Delta_i,\mathbf{Z}_i\}$ is the same in the limit as the unconditional distribution of $n^{-\frac{1}{2}}\widetilde{W}(t,\mathbf{z})$.

Proof. Let

$$\overline{A}_n(t,\mathbf{z}) = \left[\sum_{i=1}^{n}\int_0^\infty \{\mathbf{Z}_i - \overline{\mathbf{Z}}(s)\}Y_i(s)\mathbf{Z}_i' ds\right]^{-1}$$
$$\times\sum_{i=1}^{n} I(\mathbf{Z}_i \le \mathbf{z})\int_0^t Y_i(s)\Big(\mathbf{Z}_i - \overline{\mathbf{Z}}(s)\Big) ds,$$

and

$$\overline{h}_i(t, \mathbf{z}, s) = I(s \le t)\{I(\mathbf{Z}_i \le \mathbf{z}) - \overline{g}(s, \mathbf{z})\} - \overline{A}_n'(t, \mathbf{z})\{\mathbf{Z}_i - \overline{\mathbf{Z}}(s)\}.$$

Then $\tilde{h}_l(t, \mathbf{z}, s)$ and $\tilde{A}_n(t, \mathbf{z})$ are the limit of $\overline{h}_l(t, \mathbf{z}, s)$ and $\overline{A}_n(t, \mathbf{z})$, respectively. Thus $\widehat{W}(t, \mathbf{z})$ can be rewritten as

$$\widehat{W}(t, \mathbf{z}) = \sum_{l=1}^{n} \int_0^{\infty} \overline{h}_l(t, \mathbf{z}, s)\, dN_l(s)\, G_l.$$

The only random components in $\widehat{W}(t, \mathbf{z})$ given $\{X_i, \Delta_i, \mathbf{Z}_i\}$ are the independent standard normal variables $\{G_l\}$, and the proof of the tightness of $\widehat{W}(t, \mathbf{z})$ is analogous to that of Theorem 2. Also conditional on $\{X_i, \Delta_i, \mathbf{Z}_i\}$, the process $n^{-\frac{1}{2}}\widehat{W}(t, \mathbf{z})$ is a mean zero Gaussian process, and the covariance structure is given by

$$\begin{aligned} Cov\Big(n^{-\frac{1}{2}}\widehat{W}(t_1, \mathbf{z}_1), n^{-\frac{1}{2}}\widehat{W}(t_2, \mathbf{z}_2) \,\Big|\, \{X_i, \Delta_i, \mathbf{Z}_i\}\; i = 1, 2, \cdots, n\Big) \\ = \frac{1}{n}\sum_{l=1}^{n} \overline{h}_l(t_1, \mathbf{z}_1, s)\overline{h}_l(t_2, \mathbf{z}_2, s)\, dN_l(s), \end{aligned}$$

which converge to (2.6) with probability one by the law of large numbers since $Y_l(s)\{d\Lambda_0(s) + \beta'\mathbf{Z}_l ds\}$ is the intensity function of $N_l(s)$. □

According to the above results, $W(\cdot, \cdot)$ and $\widehat{W}(\cdot, \cdot)$ have the same limiting distribution. Therefore, to approximate the distribution of $W(\cdot, \cdot)$, we simulate a number of realizations from $\widehat{W}(\cdot, \cdot)$ by repeatedly generating normal random samples $\{G_l\}$ while holding the observed data $\{X_i, \Delta_i, \mathbf{Z}_i\}$ fixed.

Now, we develop a model checking technique by considering process W. The following notation will be used : W refer to an original process, w to its observed value, and the corresponding quantities under the Gaussian approximations are indicated by $\widehat{W}$, $\hat{w}$.

Since under the additive risk model the distribution of $W(\cdot, \cdot)$ process is centered arround zero, it is natural to construct a goodness-of-fit test based on the statistic $S = \sup_{t,\mathbf{z}} |W(t, \mathbf{z})|$. An unusually large value of $s = \sup_{t,\mathbf{z}} |w(t, \mathbf{z})|$ would indicate that the additive risk model may be inappropriate. So the p-value, $P(S \ge s)$, can be approximated by $P(\widehat{S} \ge s)$, where $\widehat{S} = \sup_{t,\mathbf{z}} |\widehat{W}(t, \mathbf{z})|$. Note that $P(\widehat{S} \ge s)$ can be estimated by generating a large number of normal samples $\{G_l\}$, conditional on $\{X_i, \Delta_i, \mathbf{Z}_i\}$, and that $P(\widehat{S} \ge s)$ converges almost surely to $P(S \ge s)$ as $n \to \infty$. Therefore, if p-value is less than a given significance level α, then the assumed additive risk model is not valid.

4. REAL EXAMPLES

We now apply the proposed techniques to the two familiar data sets. In our examples, the p-value is always based on 1000 realizations.

The first one is taken from Embury *et al.*(1977). The clinical trial to evaluate the efficacy of maintenance chemotherapy for acute myelogenous leukemia (AML). The first group(treatment group) received maintenance chemotherapy ; the second group(control group) did not. The objective of the trial was to see if maintenance chemotherapy prolonged the time until relapse, that is, increased the length of remission. The data set consists of the length of complete remission for two groups of leukemia patients. The only covariate is the group indicator, which is coded as 0 or 1. As we know, the assumption of PHM hold for this data set.

Now, we apply this to a additive risk model. Then (1.2) can be rewritten as

$$\lambda(t;1) = \lambda(t;0) + \beta.$$

That is, under additive risk model, β can be interpreted as the difference between the hazard rate of treatment group and control group. In this case, the additive risk assumption is hold (p-value $= 0.627$) and the estimator $\hat{\beta}$ is -0.02685. Figure 1 and Figure 2 display the cumulative hazards function of control and treatment group in the PHM and additive risk model respectively.

The second example applies to Stanford heart transplant data taken from Crowley and Hu(1977). In this data, we only regard age as covariate, then PHM does not fit this data set(Lin *et al.*(1993)). But we can see that the additive risk model fits well(p-value $= 0.238$). Therefore, it seems that applying additive risk model is more appropriate than PHM in this data.

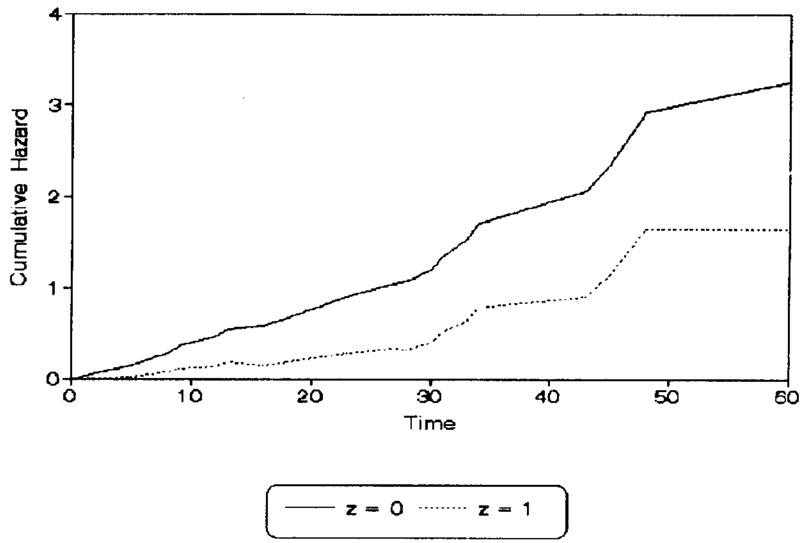

Figure 1. Estimator for $\widehat{\Lambda}(t;0)$, $\widehat{\Lambda}(t;1)$ in Additive risk model

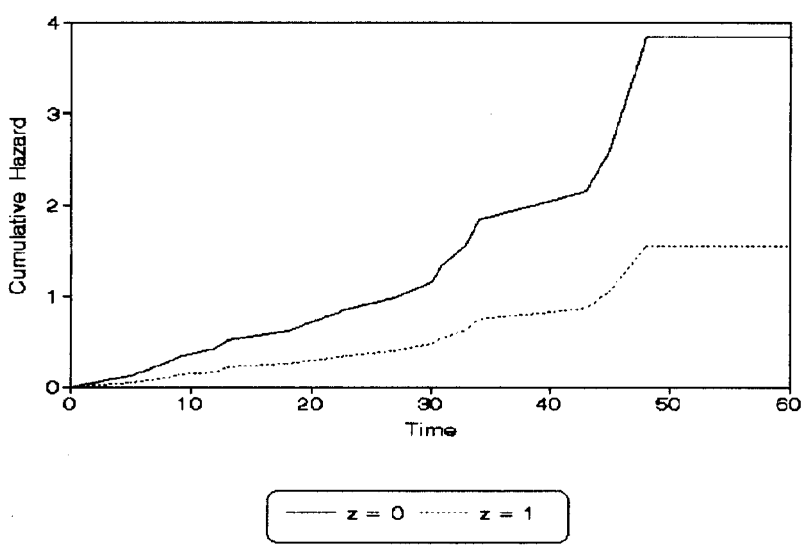

Figure 2. Estimator for $\widehat{\Lambda}(t;0)$, $\widehat{\Lambda}(t;1)$ in PHM

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