Plane Strain Strength of Fine Sands

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요 지

실리카질 모래에 대한 많은 시험결과로부터 삼축압축시험과 평면변형시험간의 강도관계를 밀도와 파괴시 유효평균주응력의 함수로 표현하였다. 또한 파괴시 평균주응력과 축차응력간의 응력비가 내부마찰각의 함수로 잘 규정되었으며 그 비는 내부마찰각의 증가에 따라 감소하였다. 또한 중간주응력을 최대주응력과 최소주응력으로써 표현하였으며 이론적인 파괴면의 각도와 평면변형시험에서 관찰된 파괴면의 각도가 비교적 잘 일치함이 확인되었다.

Abstract

Based on many experimental results on fine silica sands, the strength relation between triaxial and plane strain tests is expressed as a function of both density and mean effective principal stress at failure. Stress ratio of mean normal stress to deviatoric stress at failure is a well defined function of shear angle of friction, This ratio decreases with increasing shear angle of friction. Intermediate principal stress is also expressed in terms of major and minor principal stresses and a relatively good agreement between theoretical and observed angles of failure plane in plane strain test is confirmed.

Keywords : Sand, Shear strength, Triaxial test, Plane strain test, Stress ratio

1. Introduction

The results from conventional triaxial tests are widely used for the design of structures. However, the majority of stability problems involving the shear strength approximately satisfy the plane strain condition, i.e., the strain in the direction of the intermediate principal stress is negligible. Methods for determining the strength of soil in plane strain involve simple prismatic samples, cubical samples, hollow cylindrical samples and direct and simple shear tests.

The strength from plane strain test is higher than that from a triaxial test. This is

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mainly due to the difference in the strain conditions. The differences in strength have been analyzed by many researchers. However, most of the expressions for the differences in shear strength are given simply in terms of shear angle of friction or densities without the consideration of the coupling effect from both quantities. Therefore, in this research emphasis is given to the establishment of the relationship between plane strain and triaxial tests that considers both the void ratio and the mean normal effective stress at failure.

2. Test Equipment and Materials

The test equipment used was developed originally at M.I.T. It can measure stress, strain and strength behavior of rectangular test specimens of soil with 88.9×88.9×35.6 mm(Photo 1).

With this apparatus the transducers for data acquisition were used. Mol sand is geologically referred to as Tertiary−Pliocene and is obtained from a deposit at Mol, located at north−east of Belgium. Mol sand is a uniform and fine quartzitic sand with a median grain size of 0.195mm and belongs to SP by unified soil classification system (Fig. 1 and Photo 2).

Because of the shape of the plane strain specimen, it is difficult to make loose sand specimen with the vacuum pressure technique. In this research a freezing method was adopted. The specimen was prepared with water content of 10 percent in the mold by the undercompaction method(Ladd, 1978). This method's advantage is that same density exists throughout the layers but simulation to the field condition is lacking. The specimen was frozen in the commercial freezer at a temperature of about −30℃ and tested later after thawing. The influence of freezing/thawing on the strength paramenter in sand is neglected. According to Aoyama and Fukuda(1990), the influence of freezing/thawing on the shear angle of friction is negligible in the case of clay.

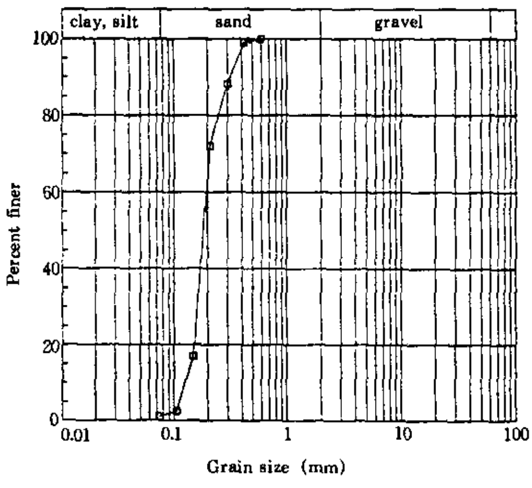
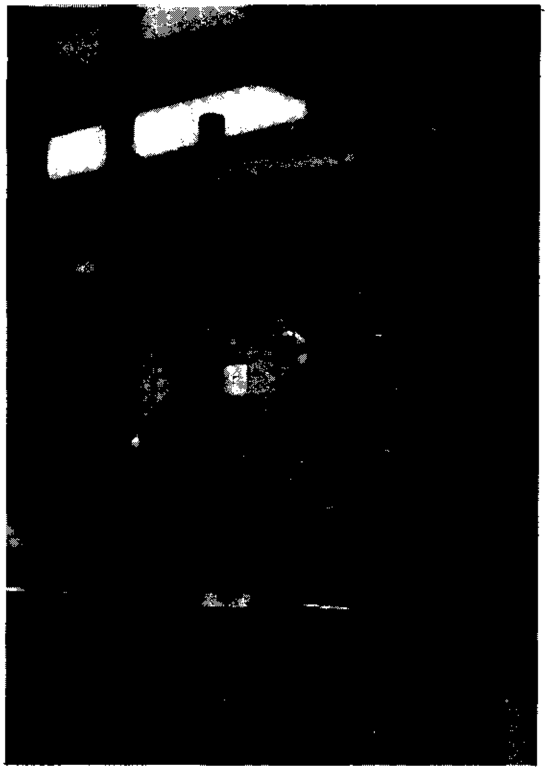
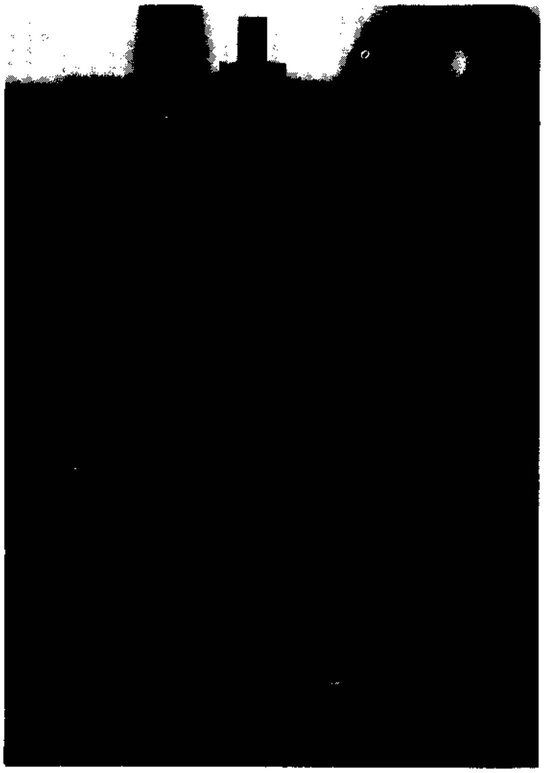


Fig. 1



(a)



(b)

Photo 1

Photo 2

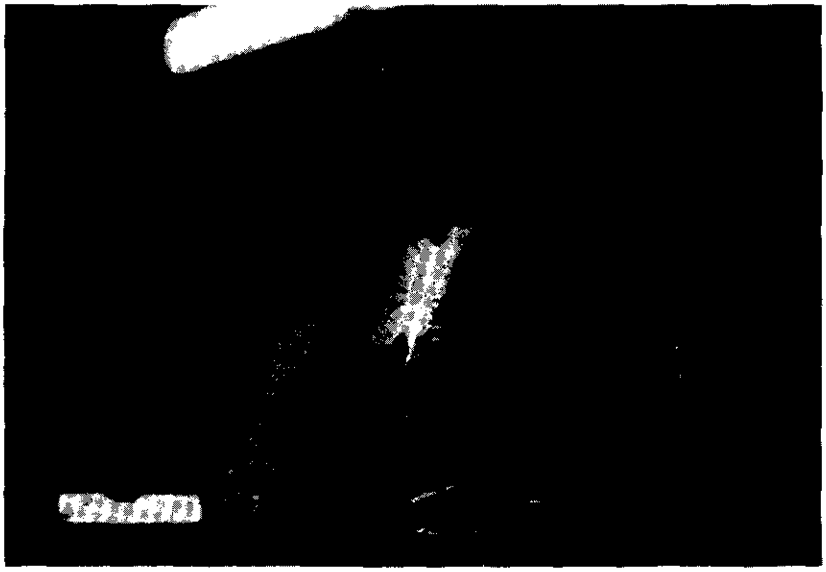


Photo 3

3. Relation of Triaxial and Plane Strain Strengths

The strength and deformation properties of sands are a funcition of many variables. Among them, the effects of the following factors complicate the comparison of the behavior of the different testing methods : (1) the parameter $b(=(\sigma_2'-\sigma_3)/(\sigma_1'-\sigma_3'))$, which indicates relative magnitude of the intermediate principal stress, (2) the initial anisotropy of the fabric, (3) the stress level and its changes during shearing, and (4) the continuous or discontinuous rotation of principal stress axes.

Based on the results from both tests, Lade and Lee(1974) proposed the following relationship :

$$\phi_p' = 1.5\phi_c' - 17^\circ (\phi_c' > 34^\circ) \quad (1)$$

$$\phi_p' = 1.5\phi_c' \qquad (\phi_c' \leq 34^\circ) \quad (2)$$

where subscripts p and c denote plane strain and triaxial cases respectively.

Ramamurthy and Tokhi(1981) derived theoretical expressions that predict the plane strain strength from the results of conventional triaxial compression tests. Their expressions are based on the experimental observation that failure points corresponding to both triaxial and plane strain types of tests trace a common locus in the stress space of deviatoric stress and effective mean stress. Ramamurthy and Tokhi's proposal is

$$\sin\phi_p' + 3\left(\frac{1}{\sin\phi_c'} - \frac{1}{\sin\phi_p'}\right) = 1 \quad (3)$$

$$3\sin\phi_p' - \sin\phi_c'(\sin\phi_p' + \cos\phi_p') = 2\sin\phi_c' \quad (4)$$

Equations (3) and (4) are based on the results from Bishop(1966) and Green(1972) respectively.

4. Experimental Results and Discussions

4.1 Angle of Failure Plane in Plane Strain Tests

For the tested sands, the angle of failure plane a_f(measured relative to the plane of major principal stress) was measured after the test (photo 3). The denser samples had a noticeably steeper slope than the looser samples. All measured slopes from the tests showed good agreement with Mohr's hypothesized angle ($45° + \phi_p'/2$) as shown in Fig 2. Bulging, however, was also observed in loose sands.

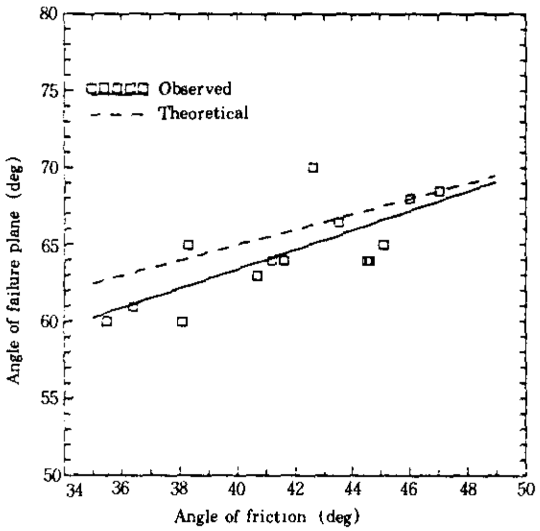


Fig. 2

4.2 The Interconnection among Principal Stresses

4.2.1 Intermediate Principal Stress as a Function of Major and Minor Principal Stress

Bishop(1966) and Green(1972) expressed the relationship among principal stresses in terms of shear angle of friction at failure in plane strain condition. For the sands in this research the approach given by Green seems well suited. The relationship for the tested sand is expressed as(Fig 3.)

$$(\sigma_2')_p = K_1\sqrt{(\sigma_1'\sigma_3')_p} \qquad (5)$$

with $K_1 = 0.90$ for Mol sand.

Using Eq.(5), the parameter b at failure, b_p, becomes

$$b_p = \frac{1}{2}\left(1 - \frac{1 - K_1\cos\phi_p'}{\sin\phi_p'}\right) \tag{6}$$

4.2.2 The Relationship between Mean Normal Stress and Deviatoric Stress at Failure

The failure points corresponding to the triaxial tests and the plane strain tests trace different loci in the principal stress space. The loci in the two types of tests might be expressed by using Mohr – Coulomb criteria :

(a) Triaxial tests : the ratio of mean normal stress at failure, $\sigma'_{m,c}$, to deviatoric stress at failure is expressed below(Yoon, 1991).

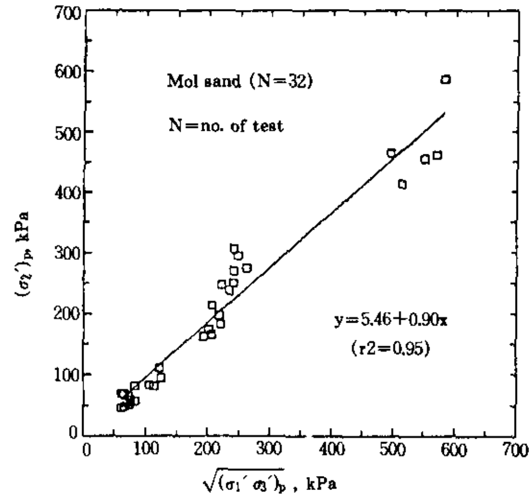


Fig. 3

Consider the ratio of mean normal stress to deviatoric stress at failure for axisymmetric case :

$$\frac{1}{3}\frac{(\sigma_1' + \sigma_2' + \sigma_3')_c}{(\sigma_1' - \sigma_3')_c} = \frac{1}{3}\left[\frac{(\sigma_1' + \sigma_3')_c}{(\sigma_1' - \sigma_3')_c} + \frac{\sigma_{3,c}'}{(\sigma_1' - \sigma_3')_c}\right] \tag{7}$$

As mentioned before, subscripts p and c denote plane strain and triaxial cases respectively. According to Mohr – Coulomb criteria,

$$\frac{(\sigma_1' - \sigma_3')_c}{(\sigma_1' + \sigma_3')_c} = \sin\phi_c' \tag{8}$$

The terms in the above equation may be rearranged to yield

$$\frac{\sigma_{3,c}'}{(\sigma_1' - \sigma_3'{}_c)} = \frac{1}{2}\left(\frac{1}{\sin\phi_c'} - 1\right) \tag{9}$$

Substituation of Eq.(9) into Eq.(7) gives

$$\frac{1}{3}\frac{(\sigma_1' + \sigma_2' + \sigma_3')_c}{(\sigma_1' - \sigma_3')_c} = \frac{1}{3}\left(\frac{3}{2}\frac{1}{\sin\phi'_c} - \frac{1}{2}\right) \tag{10}$$

From Eq.(8)

$$\sigma_{1,c}' = \sigma_{3,c}' \; (1+\sin\phi'_c)\,/\,(1-\sin\phi'_c) \tag{11}$$

Substituting Eq.(11) into Eq.(10) and arranging the terms yields

$$\frac{(\sigma_1' + \sigma_1' + \sigma_3')_c}{3} = \sigma_{m,c}' = \frac{\sigma_{3,c}'(3-\sin\phi_c')}{3(1-\sin\phi_c')} \tag{12}$$

And deviatoric stress at failure is

$$(\sigma_1'-\sigma_3')_c = \sigma_{3,c}'\frac{(2\sin\phi_c')}{(1-\sin\phi_c')} \tag{13}$$

Therefore, using Eqs.(12) and (13), the above equation can be expressed as

$$\frac{\sigma_{m,c}'}{(\sigma_1' - \sigma_3')_c} = \frac{1}{3}\,\frac{3-\sin\phi_c'}{2\sin\phi_c'} \tag{14}$$

Eq.(14) has a linear relationship in a log−log plot as shown in Fig. 4. Eq.(14) can be rewritten in a power form following Eq.(15) :

$$\frac{\sigma_{m,c}'}{(\sigma_1' - \sigma_3')_c} = \alpha\phi_c'^{\beta} \tag{15}$$

in which $\alpha = 30.298$ and $\beta = -1.058$.

Eq.(15) coincides with the measured data for Mol sand with very high accuracy(determination coeff., $r^2 \approx 1.0$) as shown in Fig. 4 and must be correct under the given definition.

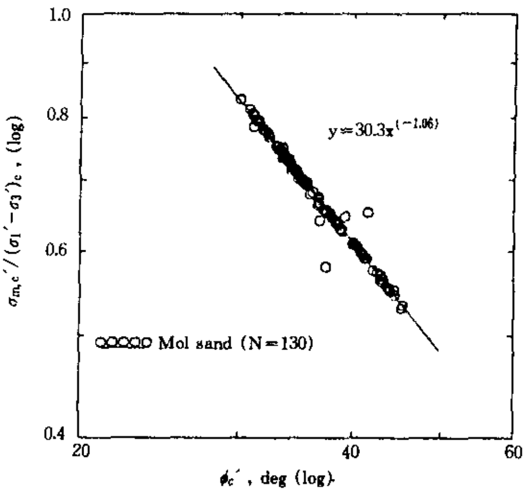


Fig. 4

Note that all the shear angles of friction are expressed in secant angle for each Mohr circle ($\phi'=\sin^{-1}[(\sigma_1'-\sigma_3')/(\sigma_1'+\sigma_3')]$). As far as the shear angle of friction is concerned in the tested range of mean normal stress at failure from 50 kPa to 650 kPa, the following assumptions are valid with only small margin of $\pm 0.1°$:

$$\phi \approx \sin^{-1}[(\sigma_{1f}'-\sigma_{3f}')/(\sigma_{1f}'+\sigma_{3f}')] \approx \tan^{-1}(\tau_{ff}'/\sigma_{ff}') \approx \tan^{-1}(\tau_m'/\sigma_m') \tag{16}$$

(b) Plane strain test : the ratio of mean normal stress to deviatoric stress at failure can be determined by considering the relationship between principal stresses. Therefore, considering the ratio of mean effective normal stress to deviatoric stress at failure for the plane strain tests and deviatoric stress ratio, b_p, the following(see Ref. 8 for further details) can be stated :

$$\frac{\sigma_{m,p}'}{(\sigma_1'-\sigma_3')_p} = [\frac{1}{2}\frac{1}{\sin\phi_p'} - \frac{1}{6} + \frac{1}{3}b_p] \tag{17}$$

B_p in Eq.(17) is expressed in Eq.(6). The stress ratio is given in Fig. 5 and the slopes in the figure depend on b_p values. Eq.(17) can be expressed similar to Eq.(15),

$$\frac{\sigma_{m,p}'}{(\sigma_1'-\sigma_3')_p} = 26.148\phi_p'^{-0.982} \tag{18}$$

ϕ' at failure clearly depends on the ratio of the mean effective stress on the failure plane to the corresponding deviatoric stress.

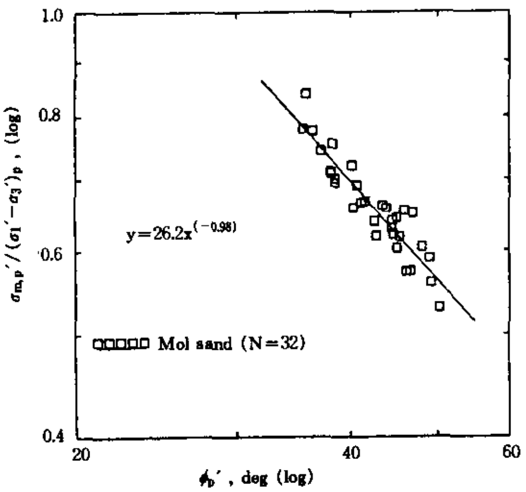


Fig. 5

4.3 Variation of Shear Strength with Density and Mean Effective Stress at Failure

The influence of the mean effective stress on the shear strength of sand can easily be seen by the equations derived in the previous sections and has been reported repeatedly by many researchers(De Beer; 1965, Ramarmurthy; 1981). For the sands the shear angle could be expressed as a function of both density of sand and mean effective normal stress at failure for the triaxial test results. Fig 6. shows the plane strain test results for the Mol sand. Similar to the triaxial test results, the shear angle can be expressed as a function of $\phi' = f(\sigma_m', e)$. Triaxial test results for Mol sand are expressed as follows (Yoon, 1991) :

$$\phi_c' = 25.70 - 38.03 \cdot \ln e - [(7.48 - 7.46e) \cdot \ln(\sigma_{m,c}' / \sigma_{atm})] \tag{19}$$

where e = void ratio,

$\sigma_{m,c}'$ = mean effective normal stress at failure plane,

σ_{atm} = atmospheric pressure(assumed 1 kg/cm^2 = 98.1 kPa).

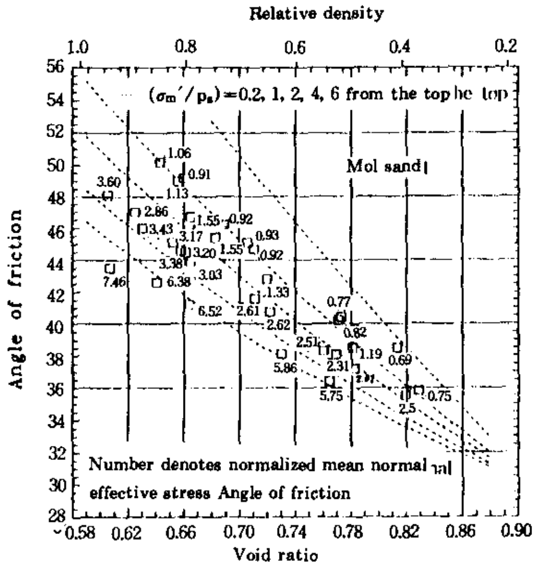


Fig. 6

Based on the data taken from the curves in Fig. 6 for each mean effective stress level and void ratio, the angle of shear resistance can be expressed as a function of mean effective stress in logarithmic type of equation :

$$\phi_p' = C + D\ln(\sigma_{m,p}' / \sigma_{atm}) \tag{20}$$

The costant C in Eq.(20) can be expressed as a function of log-void ratio while the constant D seems to be best correlated linearly with the void ratio for the tested sands. Therefore,

$$C = E_1 + E_2 \cdot \ln e \tag{21}$$

$$D = E_3 + E_4 \cdot (e) \tag{22}$$

The final proposal is the following :

$$\phi_p' = E_1 + E_2 \cdot \ln e + [(E_3 + E_4 \cdot e) \cdot \ln(\sigma_{m,p}' / \sigma_{atm})] \tag{23}$$

The values for the constants C, D, E_1, E_2, E_3, and E_4 in Eqs.(21), (22) and (23) are shown in Table 1.

Table 1. The constants C, D, E_1, E_2, E_3 and E_4 in Eqs. (21, 22 and 23)

e	0.62	0.66	0.70	0.74	0.78
C	52.3	49.8	45.5	42.2	38.8
D	−4.5	−3.9	−3.3	−2.7	−2.1

E_1	E_2	E_3	E_4
30.89	−61.72	−33.56	34.75

4.4 Comparison of Strength between Plane Strain and Triaxial Tests

As mentioned in Section 3, the relationships in Eq.(3) and Eq.(4) by Ramamurthy and Tokhi(1981) are based on the observation that failure points corresponding to these two types of tests trace a common locus in the stress space of deviatoric stress and mean effective stress.

For Mol sand, the comparison between plane strain and triaxial shear angles of friction using Ramamurthy and Tokhi's theory is shown in Fig 7.(Eq. 5 with K_1=0.9), together with the results by Bishop(1966), Green(1972), and by Lade and Lee(1976). Lade and Lee's relationship gives lower plane strain strength in the range of loose to medium dense compaction.

The results from the present research which is composed of a large number of tests (triaxial tests N=130, plane strain tests N=32) show that the ratio of σ_m' to $(\sigma_1'-\sigma_3')_f$ is a function of shear angle (see Eqs. 15 and 18). For example, in Fig. 8, the local slopes do not converge to the origin although a general regression line can pass through the origin. And each local slope converges to different points depending on its shear angle of friction and it changes with stress history. Therefore, Ramamurthy and Tokhi's observation (they assumed the slope in Fig. 8 (bold line) converges to the origin) and assumption (shear angel is unique irrespective of stress history) are based on oversimplification of the problem.

The strength relationship between two tests with a simple linear line has difficulty in considering the variation of shear angle with different stress levels. Ramamurthy and Tokhi' expressions, including the mean normal stress term, (Eqs. 3 and 4) should probably be modified. The shear angle of friction can be expressed by Eq.(19) for triaxial test and by Eq.(23) for plane strain test for Mol sand. Eqs. (15) and (18) imply that the variation of shear angle can be expressed best with a power function although some researchers pointed out its inconvenience for practical use.

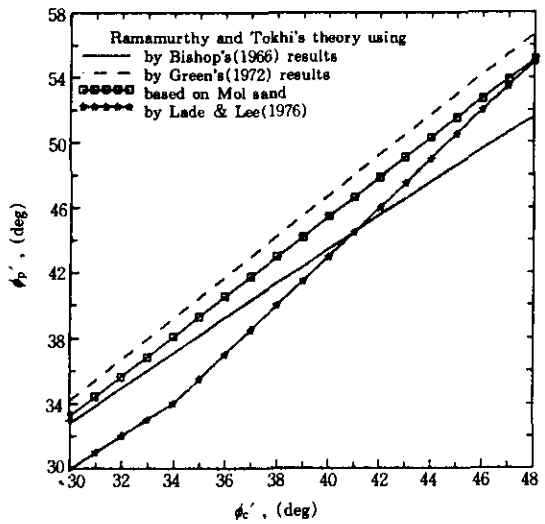


Fig. 7

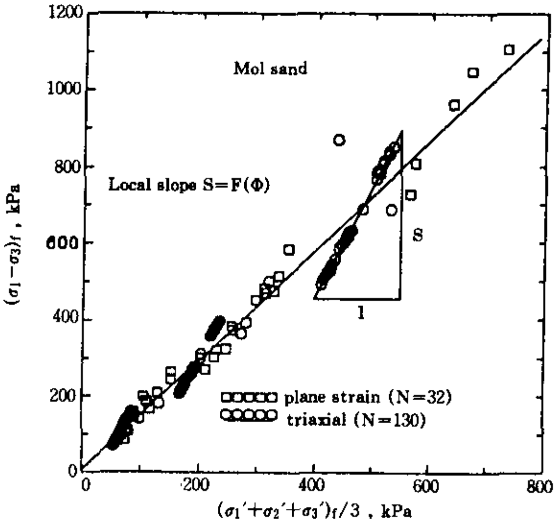


Fig. 8

For Mol sand under the same mean effective stress, the relationship between two tests can be expressed by a combination of Eq.(19) for triaxial tests and Eq.(23) for plane strain tests :

$$\phi_p' - \phi_c' = (6.4 + 7.6 \cdot e) \cdot \ln(\frac{\sigma_m'}{\sigma_{atm}}) - 20.2 \cdot \ln e - 1.2 \tag{24}$$

The differences are shown in Fig 9. As shown in the figure, for the same density the differences change with the level of mean effective normal stress at failure and the differences increase as the densities increase. From the comparison at the same level of mean effective stress at failure, it was observed that plane strain tests give higher shear angle of friction. It can also be seen that there is no difference in strength for very loose compaction.

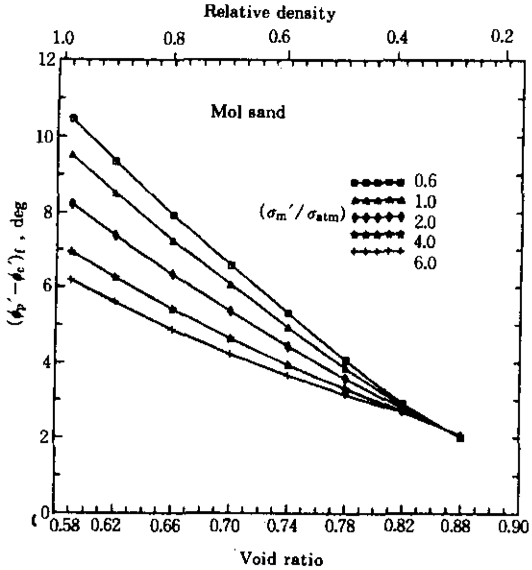


Fig. 9

5. Conclusions

Based on the results of plane strain and triaxial tests on Mol sand, the following conclusions are drawn.

(1) Good agreement between shear angle of friction and failure angle of plane with Mohr's hypothesized angle of $\alpha_f = 45^\circ + \phi_p'/2$ is confirmed as shown in Fig. 2. (2) The differences in the shear angle of friction between plane strain condition and triaxial condition are expressed in terms of both density and stress level. The differences increase with increasing density and decrease with increasing effective mean normal stress at failure. The difference in shear angle of friction between two types of test is given in Eq.(24) with logarithm of void ratio and mean effective normal stress at failure. (3) The ratio of mean normal stress to deviatoric stress at failure is a well expressed function of shear angle of friction in both triaxial and plane strain tests. The ratio decreases with increasing shear angle of friction.

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