강성측정법을 이용한 경제적인 비선형해석

ECONOMICAL NONLINEAR RESPONSE ANALYSIS USING STIFFNESS MEASURE APPROACH

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요 약

힌지가 발생하는 철근콘크리트 골조구조물의 비선형해석시에 부재강성값을 사용하는 새로운 방법에 대한 연구이다. 본 연구에서는 부재의 비선형상태에서 힌지영역의 접선강성을 평가하고 효율적으로 이용하는 방법을 제시하였다. 비선형응답을 얻기위해 고유벡터를 이용하는 해석법은 비선형범위에서 시각증분에 따라 강성이 변하고 따라서 고유벡터군도 그 변하는 수만큼 재산정 하여야 하기 때문에 일반적인 해석방법이 아니다. 그러나 부재의 비선형상태를 나타내는 강성값, 즉 고유벡터의 산정횟수를 줄이며 산정된 기존값을 적절하게 재사용하여 해석의 효율성을 입증하였다. 지진하중을 받는 철근콘크리트 골조구조물의 비선형 해석의 경제성은 고유벡터의 산정횟수에 의존되기 때문에 고유벡터의 산정횟수를 감소시키며 신뢰성있는 응답을 구하여 본 해석법의 효율성을 입증하였다.

Abstract

A method used for measuring the stiffness of hinging reinforced concrete frame structures is developed. The so called Stiffness Measure Method is used to evaluate the tangent stiffness of hinge regions while the structure is responding in nonlinear ranges. Eigenvector methods for nonlinear response have not been especially popular because of the need for regenerating eigenvectors as the time history proceeds. In the present work the eigenvectors sets and corresponding nonlinear state variables, i. e. , the tangent stiffnesses of the hinge regions, are stored. There is an expectation that previously generated eigenvectors can be reused as the analysis proceeds. The stiffness measure is used to compare the current tangent stiffnesses of hinge regions with those of previously stored eigenvectors sets. Since eigenvector calculations are diminished the method is effective in reducing computational effort for reinforced concrete frame structures subjected to strong ground motions.

Keywords : nonlinear analysis, stiffness measure, eigenvectors, reinforced concrete frame

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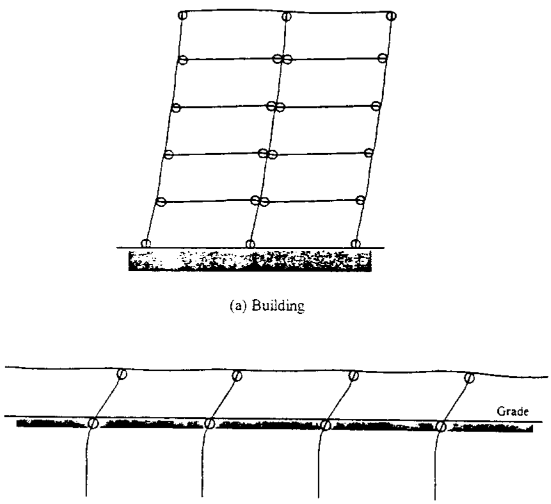
1. INTRODUCTION

The planned location and strength of flexural hinges in reinforced concrete structures provides strict control of nonlinear seismic behavior. Usually the hinges are selected to maximize inelastic energy dissipation while satisfying necessary constraints. In a building the so-called strong-column/weak-beam design philosophy [1] results in hinging at beam ends of all floors (Fig. 1-(a)). The columns of these buildings do not hinge except at the footings and roof. Bridge structures contrast with buildings because the prestressed box girder superstructure is not ductile. The hinges are designed to occur in the piers of bridges (Fig. 1-(b)). In both types of structures, the number of hinges is limited to a relatively small finite number. Therefore, the nonlinear behavior in buildings and bridges responding to earthquakes is confined to a relatively small number of regions. These structures are expected to have significant inelastic global displacements during a typical earthquake response, e. g., displacement ductility factors [2] equal to 8 are expected. Curvature ductility factors in the hinge regions, i. e., the exclusive locations of inelastic behavior, can reach 20.

The nonlinear behavior of these structures is completely controlled by the behavior of the hinge regions. Then it seems to be a reasonable proposal that there is sufficient moment-curvature information in the hinge regions to completely predict the overall nonlinear structure behavior. If true, the proposal is helpful because it permits us to efficiently represent nonlinear behavior. In other words, the tangent stiffnesses of the hinge regions operating in nonlinear states can be linked directly to the overall stiffness of the structure. The overall stiffness can then be tied to the frequencies



(a) Building

(b) Bridge

○ Location of plastic hinge zones

Fig. 1 Plastic Hinge Forming Patterns

and mode shapes of the structure as it exists at any time during an earthquake simulation. As simulation time goes by, the nonlinear states of hinge stiffness undergo changes and therefore a different set of mode shapes and frequencies is needed each time the stiffness changes.

Nickell [3] showed that the reduced basis method can be successfully used to predict the nonlinear behavior of structures. The method [3] suggested that new mode shapes should be recalculated after each time step. Efficiency of the method was claimed through : 1) achieving completeness with a limited number of eigenvectors, and 2) using subspace iteration methods [4] for the eigenvector calculations. Nevertheless, the recomputation of eigenvectors after each time step is a burdensome process. The resulting eigenvalue or reduced basis method is not clearly superior to direct methods. There has been a continuing interest [5-6, 7] in using reduced bases for representing nonlinear behavior. These papers have focused on different formulations and applications of the reduced basis methods. There has been little or no work which links the overall structure stiffness to the appropriate set of eigenvectors that is needed as the nonlinear analysis proceeds.

Within the framework of the eigenvalue method, the concept of re-using the eigenvectors [8] is considered. This means that after an elapsed period of time, several different sets of eigenvectors have been calculated to represent the nonlinear behavior. Also these different sets of eigenvectors have been stored for reuse. The reuse of eigenvectors saves effort, but when necessary a new eigenvector calculation is performed during the step-forward integration. A new technique is needed to implement the decision making process, i. e., whether to reuse an old set of eigenvectors or to perform a new eigenvector calculation. The tangent stiffnesses, i. e., EI values of the hinge regions, will provide the information which is necessary for making the eigenvector decision. The main objective of this paper is to present the new technique which will be called the Stiffness Measure Approach (SMA).

2. STIFFNESS MEASURE WITH EIGENVECTORS

Each time that an eigenvector calculation is performed the resulting eigenvalues, eigenvectors, and hinge EI stiffnesses are stored for reuse. There is a unique connection between the current hinge EI values and the resulting eigenvalues and eigenvectors. The proposed SMM selects the eigenvector set for use or opts to calculate a new set of eigenvectors depending on the EI values.

During the time integration which is performed using the uncoupled normal coordinates, there is a need to know the curvatures within the hinge regions. The curvature, evaluated at the midpoint of the hinge zones (Fig. 2), can be found from differentiated forms of displacement interpolation functions, for subelement 1 the midpoint curvature is given by

$$k\left(\frac{l_1}{2}\right) = \frac{1}{l_1}\begin{Bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{Bmatrix}^T \begin{Bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{Bmatrix} \tag{1}$$

where $k(l_1/2)$ = curvature evaluated at the midpoint of the subelement 1 ; $v_1,\cdots,v_4$ = end displacement coordinates of subelement 1 ; and l_1 = length of subelement 1. The hinging is assumed to occur in zones 1 and 3 at the ends of the element (Fig. 2). Interior displacement coordinates are related to the exterior coordin-

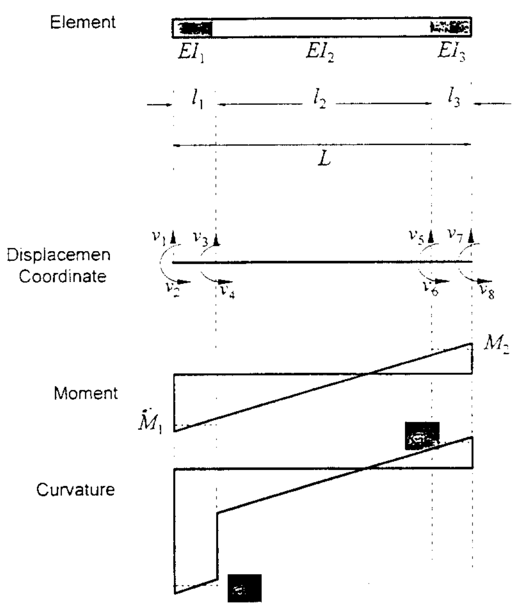


Fig. 2 Hinge Zone Element

ates through static condensation since lumped masses are only considered to act at the ends of elements.

As the integration of uncoupled normal equations proceeds it is necessary to evaluate the displacement coordinates of elements, but this can be done by identifying the appropriate elements of each set eigenvectors to be used for recovering the element displacements. Therefore, the subelement displacement coordinates v_1, v_2, v_3, and v_4 (Fig. 2) can be efficiently determined after each time step of the explicit formulation [2] and indirect method of integration [3] used herein. Then the curvatures are found from eqn. (1).

A time-history of midpoint curvature is known therefore for each hinge or subelement. Reinforced concrete building and bridge structures responding to earthquakes will have cyclic or reversing type curvature histories. The current EI values of the hinge zone can be determined from the slope of the moment-curvature relationship.

Moment-curvature relationships appropriate for reinforced concrete sections have been studied extensively [9]. Many different analytical formulations with varying levels of complexity have been proposed. A relationship which gives excellent prediction of reinforced concrete structure response [10] and yet is convenient to use is portrayed in Fig. 3. The hysteretic moment-curvature relation has 4 branches as follows : 1) initial elastic branch which should be based on cracked moment of inertia, 2) primary post yield branch, 3) unloading branch with amplitude dependent degrading stiffness, and 4) load reversal branch. The time varying EI, i. e., the tangent stiffness of the moment-curvature relationships, can be readily determined from the 4-branch relationship depicted in Fig. 3 because the curvature history of each hinge zone is known.

When the EI's are known for all hinge zones then the nonlinear stiffness of the structure is known. If we were to compute the eigenvectors and eigenvalues for the structure with these hinge stiffnesses, it would be prudent to save or store the EI's along with the eigenvectors and eigenvalues. Alternatively, it might not be necessary to compute the eigenvectors, i. e., if there is a stored set of them which closely resemble the needed set. The decision to compute new eigenvectors or to use a stored set can be based on the relation

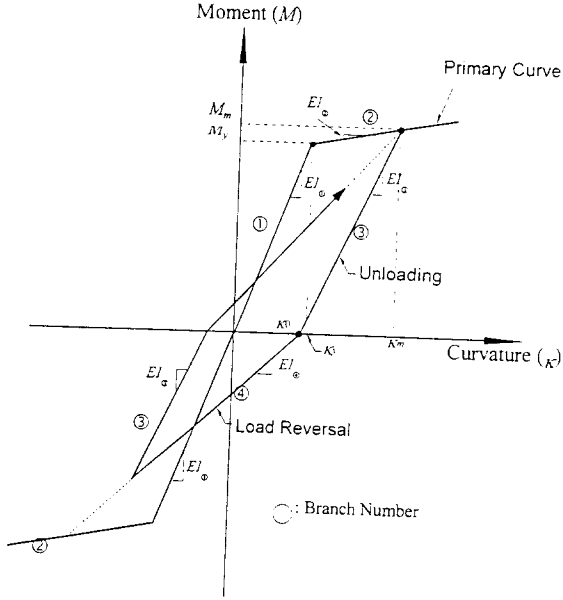


Fig. 3 Q-Hyst. Model

$$SM^k = \frac{1}{N}\sum_{i=1}^{N}\frac{MAX((EI)_i^t, (EI)_i^k)}{MIN((EI)_i^t, (EI)_i^k)} \tag{2}$$

where, SM^k is the stiffness measure for stored eigenvector (EV) set k ; $(EI)_i^t > 0$ is the current value of EI for hinge zone i at time t found using eqn. (1) and moment-curvature law for the hinge (Fig. 3) ; $(EI)_i^k > 0$ is the EI value for hinge zone i and stored EV set k ; N is total number of hinge zones.

In each fractional term of eqn. (2) the larger quantity is in the numerator while the smaller quantity is in the denominator. The value of each fraction will be equal to or greater than unity. When the fraction for hinge i is unity it means that the current EI value of hinge i is equal to the stored EI value of hinge i for eigenvector set k. After summation the result is normalized with N. Accordingly, the SM^k value will be equal to or greater than unity. If SM^k is unity it means that for all hinges there is complete equality between the current EI values and the EI values for eigenvector set k.

Eqn. (2) is a measure of the quality of the representation of each stored eigenvector set k. In the use of eqn. (2) there are two important considerations which are :

1) Among the available eigenvector sets, k, the one with the smallest SM^k value will provide the best normal coordinate representation of the response.

2) Even the best set k may not provide a sufficiently accurate representation of the response. For example, if at time, t, there is a sudden shift in the participation factors [2] then the distribution of current hinge EI values will be radically different from the stored EI values available for all sets. Therefore none of the stored sets is sufficiently accurate and a new set of eigenvectors must be generated.

It is a straightforward procedure to select the best available normal coordinate representation, i. e., consideration 1). Let

$$SM_{MIN} = MIN(SM^1, SM^2, \cdots, SM^M) \tag{3}$$

where, SM_{MIN} is the smallest SM^k value and M is a total number of stored EV sets. The k value corresponding to SM_{MIN}, i. e.,

$$SM^k = SM_{MIN} \tag{4}$$

and k is the integer identifying the best available EV set.

Consideration 2) pertains to a significant change in the EI values from those which had previously occurred during the simulation. This means that SM_{MIN} is too large and the stored EV sets are not sufficiently accurate to represent the structural response. Chang [9] verified that previously generated EV sets may not accurately represent response of frames in earthquakes. This is where the option to generate a new set of eigenvectors enters. The decision to generate a new set of eigenvectors is based on

$$SM_{MIN} > 1 + \varepsilon \tag{5}$$

where $\varepsilon > 0$ is the tolerance in the stiffness measure of the best available EV set. If eqn. (5) is satisfied then a new EV set must be generated. The tolerance quantity, ε, can be used in convergence studies. In general, when smaller values of ε are used then new sets of eigenvectors are generated more frequently.

3. DEVELOPMENTS OF SOLUTION PROCEDURE

The equations of motion in generalized coordinates for iteration with current eigenvalues and eigenvectors are.

$$[I]\{\ddot{y}(t)\}^i + [c^*]\{\dot{y}(t)\}^i + \left[(w(t)^{i-1})^2\right]\{\delta y\}^i = \left[\Phi(t)^{i-1}\right]^T\left(\{F(t)\} - \{R(t)\}^{i-1}\right) \tag{6}$$

where, superscript i denotes an iteration number and greater than 1 ; $w(t)$i-1 and $\Phi(t)$ i-1 are natural frequency and mode shape representing the stiffness state at the end of the

$(i\text{-}1)$th iteration ; and $\{\delta y\}^i=\{y(t)\}^i-\{y(t)\}^{i-1}$. To solve the Eq. (6), define the i-th incremental velocity, $\{\delta \dot{y}\}^i$ and incremental acceleration, $\{\delta \ddot{y}\}^i$, which correspond to the i-th incremental displacements, $\{\delta y\}^i$.

$$\begin{aligned}
&\{\delta \dot{y}\}^i=\{\dot{y}(t)\}^i-\{\dot{y}(t)\}^{i-1} \quad , \text{or}\\
&\{\dot{y}(t)\}^i=\{\dot{y}(t-\Delta t)\}+\sum_{n=1}^{i}\{\delta \dot{y}\}^n\\
&\{\delta \ddot{y}\}^i=\{\ddot{y}(t)\}^i-\{\ddot{y}(t)\}^{i-1} \quad , \text{or}\\
&\{\ddot{y}(t)\}^i=\{\ddot{y}(t-\triangle t)\}+\sum_{n=1}^{i}\{\delta \ddot{y}\}^n
\end{aligned} \tag{7}$$

Rewrite $\{\delta \ddot{y}\}^i$, $\{\dot{y}(t)\}^i$ and $\{\delta y\}^i$ with the known responses from the $(i\text{-}1)$th iteration in conjunction with the Newmark's constant average acceleration method and transferring all known terms to the right hand side gives the following uncoupled equations of motion.

$$\begin{aligned}
&\left(1+\frac{\Delta t}{2}c^*{}_m\frac{\Delta t^2}{4}\left(w_m(t)^{i-1}\right)^2\right)\delta \ddot{y}^i_m\\
&=\left\{\Phi(t)^{i-1}\right\}^T_m\left(\{F(t)\}-\{R(t)\}^{i-1}\right)\\
&-\ddot{y}_m(t)^{i-1}-c^*{}_m\dot{y}_m(t)^{i-1}
\end{aligned} \tag{8}$$

Eqn.(8) can be solved for the incremental acceleration of the i-th iteration, $\{\delta \ddot{y}\}^i$. The i-th incremental displacement $\{\delta \ddot{y}\}^i$ is calculated based on $\{\delta \ddot{y}\}^i$.

$$\{\delta \ddot{y}\}^i=\frac{\Delta t^2}{4}\{\delta \ddot{y}\}^i \tag{9}$$

In order to evaluate the restoring force, the displacements in generalized coordinates need to be transformed to real coordinates by

$$\{\delta v\}^i = [\Phi(t)^{i-1}]\{\delta y\}^i \tag{10}$$

$$\{v(t)\}^i = \{v(t)\}^{i-1} + \{\delta v\}^i \tag{11}$$

Then we can calculate the restoring force, $R(t)\}^i$, corresponding to the real displacements of the i-th iteration, $\{v(t)\}^i$. According to the moment-curvature relation the stiffnesses of hinge zones can be traced for displacement, $\{v(t)\}^i$.

As the stiffness changes, the SM value (Eq. 2) between the current stiffness state and the stiffness state of the previous iteration is calculated. If Eq. (5) is satisfied, the iteration is continued with eigenvectors and eigenvalues used in the previous iteration. Otherwise, basis vectors need to be changed. Then SM values (Eq. 2) between the current stiffness state and stiffness states of the stored HZS sets are evaluated. If Eq. (5) is satisfied for the value of SM_{MIN}, Eq. (3), the eigenvectors and eigenvalues of the HZS set with SM_{MIN} will be chosen and be used for the next iteration. Otherwise, the new eigenvalues and eigenvectors are calculated and saved as a new HZS set for later use.

The iteration process should be continued until the convergence tolerance meets the allowable value within a time step. An appropriate convergence measure need be employed. The convergence tolerance used here is the ratio of absolute values of incremental displacements and total displacements,

$$\frac{\|\{\delta v\}^i\|}{\|\{v(t)\}^i\|} \leq CONV \tag{12}$$

in which $CONV$ is the maximum allowable value for the convergence. When a change of basis vectors takes place, acceleration and velocity should be updated for changed basis vectors by the Eq. (12) is satisfied, continue the response calculation to the next time step. The process may be continued step-by-step until any desired time.

4. NUMERICAL STUDIES

4.1 5-story Panar Moment Resisting Frame

A 5-story 2-bay planar moment resisting frame (Fig. 4) is used for evaluating the SMM. The frame which behaves as a strong column/weak beam structure was developed in Ref. [1]. The Q-hyst model (Fig. 3) is used for the moment-curvature relations of the hinge zones. The post-yield slope of the moment-curvature relations, i. e. , branch ②, is assumed to be 0.5% of the initial value.

Lumped masses are placed at the beam-column joint coordinates. Viscous damping is assumed to be proportional to the mass. The proportionality coefficient, a_o, was chosen so that a decay equivalent to 5% of critical damping of the first elastic mode was achieved, i. e., let

$$a_o = 2\xi_1 w_1 \tag{13}$$

and

$$C = a_o M \tag{14}$$

where C, M = damping and mass matrices respectively ; ξ_1 = damping coefficient, 0.05 ; w_1 = first natural frequency of linear system.

The 15 beam-column joints (Fig. 4) were represented with 45 degrees of freedom. During the earthquake simulation eigenvector sets consisting of 5 mode shapes were generated as required by eqns. (2) through (5). The 5 mode shape sets are sufficiently complete for the purpose of representing the lateral response. Five mode shapes for a hinge yield pattern occurring during the simulation are presented in Fig. 5. The rapid angle changes, i. e., kinks, which occur in the hinge zones are apparent in the various mode shapes.

The frame has an elastic fundamental period

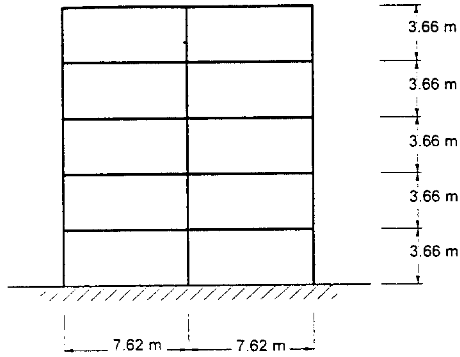


Fig. 4 Configuration of Ductile 5-Story 2-Bay Strong Column/Weak Beam Moment Resisting Frame

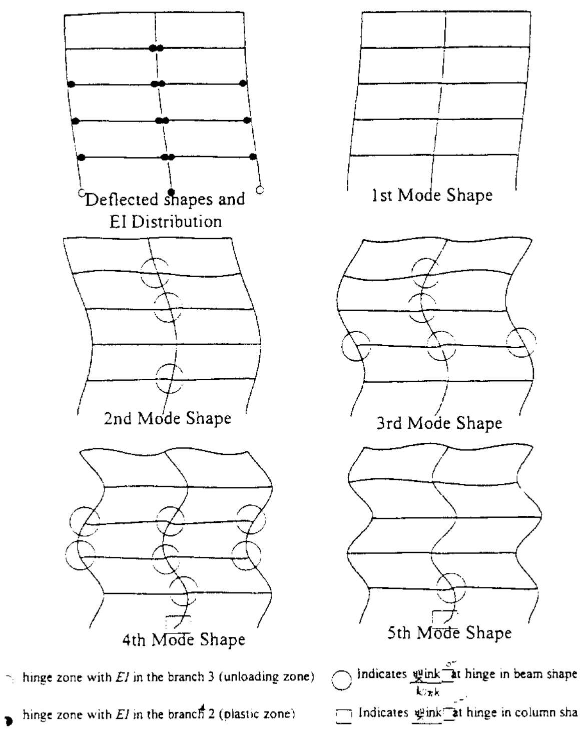


Fig. 5 Nonlinear Mode Shapes (When frame has 15 Plastic hinges)

of 0.84 sec. The seismic coefficient, i. e., the base story yield shear divided by the building weight, was 0.352. This coefficient is in the normal range. The frame was subjected to the

S16E component of the 1971 Pacoima Dam record. The record is considered to be severe because of the high values of peak velocity and acceleration. We chose it for the present study because strong nonlinear behavior was guaranteed to occur.

Results

The roof displacement and base story shear results are given in Fig. 6. Roof displacement ductility of 3 is reached during the simulation. The purpose of the analysis is to show how the results are affected by convergence studies with ε given in eqn. (5). A comparison of the results with the direct integration is also made. Convergence studies with Δt, the time step interval, have also been performed but are not indicated in Fig. 6. There is excellent agreement in roof displacement for $\varepsilon \leq 0.1$. The base story shear requires smaller e for convergence, i. e., $\varepsilon \leq 0.075$.

It is also of interest to consider the number of eigenvector set calculations versus ε (Fig. 7). The number of eigenvector set calculations required depends on the strength of nonlinearity and the value of ε. For the 1971 Pacoima record, the required number of EV set calculations is plotted against ε for values less than 2. The number hovers around 50 for $0.08 \leq \varepsilon \leq 0.2$. When $\varepsilon = 0$ then 899 eigenvector set calculations are required out of 4000 time steps.

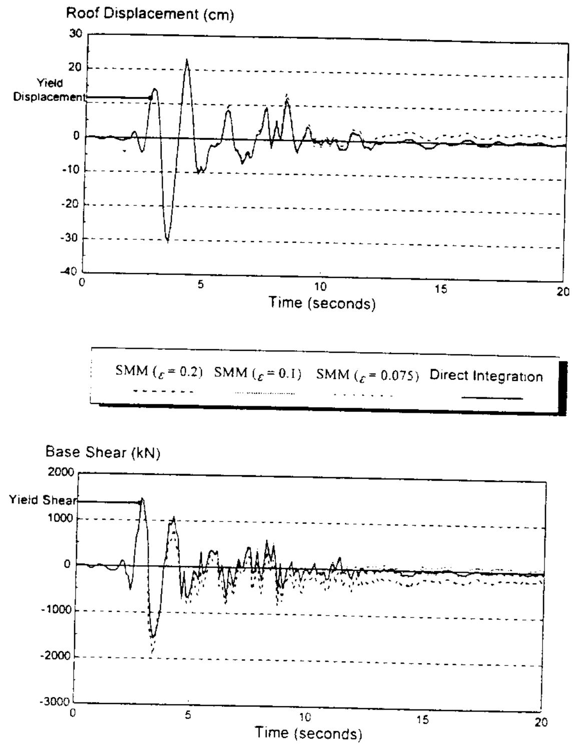
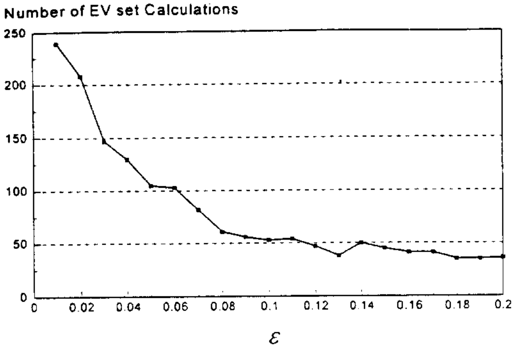


Fig. 6 Response Time Histories for Various Values of ε

4.2 Bridge Example

A 5 span frame bridge similar to Fig. 1 was studied using the SMM. The equal spans were 40.23m (130ft) and shaft heights were 15.24m (50ft) above grade and 24.38m (80ft) below grade. Ten beam elements were used to represent each span. Each shaft had 5 elements below grade and 3 elements above grade, and nodes touching the soil had springs representing the lateral subgrade modulus. Hysteretic-like soil yielding and gapping effects were represented in the subgrade modulus p-y relations. The hinging in the structure was represented to occur at the tops of the shafts and slightly below the grade line (Fig. 1). The representations of hinge zones, masses, and damping follow the procedures used in the building example.

83 nodes were represented with 249 degrees of freedom. During the earthquake simulation eigenvector sets consisting of 17 mode shapes were generated as required to satisfy completeness criteria. All but the fundamental mode were representing the vertical motion of the deck, and these did not significantly affect the shaft hinging or soil yielding. The fundamental mode occurring in the different eig-



Note : (1) when ε = 0, number of EV set calculations = 899
(2) total number of time steps = 4000

Fig. 7 Influence of Stiffness Measure Tolerance, e, on EV set Calculations for Frame Structure.

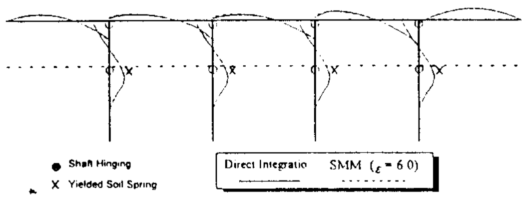
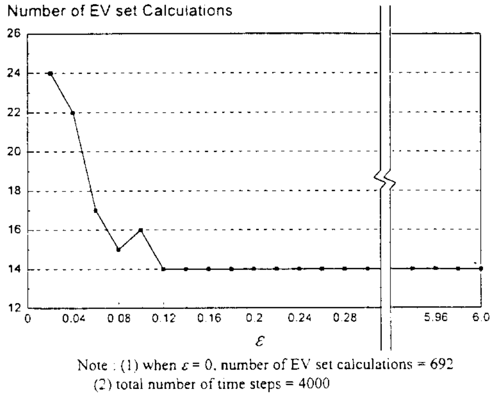


Fig 8. Comparison of Moment Diagrams when the Moments in the Shafts were the Maximum. (at 4.495 sec.)



Note : (1) when ε = 0, number of EV set calculations = 692
(2) total number of time steps = 4000

Fig. 9 Influence of Stiffness Measure Tolerance, ε, on EV set Calculations for Bridge Structure.

envector sets represented lateral response. The different fundamental modes in the sets depicted the various states of hinging and soil yielding. The bridge model had an elastic fundamental period of 1.85 sec. and lateral seismic coefficient of 0.3.

A vertical downward gravity load and vertical upward force caused by the draped prestress cable were applied while the bridge were subjected to the horizontal component of the 1994 Pacoima Dam accelerogram. The 1971 and 1994 records are amazingly similar with near equal peak accelerations and velocities.

Results

The diagram of maximum moments occurring at 4.459 sec. is presented in Fig. 8. At that time, the shafts have hinges at two locations, and the surface soil springs representing the subgrade modulus are in yield conditions. The SMM results which are compared with direct integration are obtained for a large value of ε, i. e., ε=6. 0.

Relatively few sets of eigenvectors are needed in this case because it is primarily the fundamental mode that is contributing to the nonlinearity. In Fig. 9 it is shown that for ε ≥ 0. 012, there are only 14 required eigenvector set calculations. There is tremendous efficiency in this case because only 14 sets of eigenvectors need be generated out of a total of 4000 time steps. The computational effort is comparable with a linear eigenvector type of solution.

Each of the 14 sets has a different fundamental mode. Please note the fundamental period T_1 = 1.85sec. for the elastic or stiffest state and T_1 = 2.08sec. for set used at the time of maximum moment (Fig. 8). The variation in T_1 is an indication of the strength of the nonlinearity. The requirement for the low number of sets (14) is consistent with a lateral stiffness history which was : 1) constant and highest during the initial elastic condition, 2) wildly swinging between highest, lowest, and intermediate values during the single cycle high impulse response, and 3) operating in a narrow intermediate range for the lower amplitude cyclic response occurring thereafter. The response to 1971 and 1994 Pacoima records is characterized by a large single cycle followed by lower amplitude motions with varying frequencies, e. g., Figs. 5 and 6.

5. CONCLUSION

The representation of reinforced concrete operating dynamically in the nonlinear range can be greatly simplified because the nonlinear behavior occurs in a small number of regions called hinges. Further the response can be represented with sets of eigenvectors, each set corresponding to various nonlinear states occurring in the hinge regions.

The so called Stiffness Measure Approach is used to match the appropriate set of eigenvectors with the nonlinear state of the hinge regions. Eigenvector sets and tangent stiffnesses of the hinges are stored for reuse after they are generated. As the step-forward time integration proceeds the Stiffness Measure Approach is used to decide if the stored eigenvectors can be used or if a new eigenvector set must be generated. The decision to generate a new set is based on a quantity called the stiffness measure tolerance.

The reduction of numerical effort achieved by the Stiffness Measure Approach is implied in the building and bridge examples presented herein. The reduction occurs because the various sets of eigenvectors representing different nonlinear hinged states of the structure are reused thus saving the effort of re-computing the eigenvectors.

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이에 귀 센타에 깊이 감사드립니다.

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