강구조 설계를 위한 가상하중 소성활절 해석

Notional-Load Plastic-Hinge Method for Steel Structure Design

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요 약

본 논문에서는 강구조물의 설계를 위한 가상하중 소성활절 해석기법이 연구되었다. 구조물의 기하학적인 불완전성을 가상하중기법으로 고려하였다. 본 해석기법을 통하여 구조물의 거동과 하중 지지능력을 직접적인 방법으로 예측할 수 있다. 즉 본 기법은 강구조 설계에서 전통적으로 사용되고 있는 유효길이 인자(K-factor)의 계산 및 각 부재의 강도계산을 필요로 하지 않으므로써 다음 세대의 설계기법이라고 할 수 있다. 본 기법에 의하여 예측된 강도와 변위는 정확한 해라고 알려진 Plastic-Zone 해석 결과와 비교 검증하였다. 본 기법의 해석 및 설계 세부지침과 순서를 제시하였으며, 본 해석기법 및 AISC-LRFD 방법에 의하여 결정된 부재크기를 비교하였다. 본 해석기법은 실제 설계에 효율적으로 사용될 수 있을 것으로 사료된다.

Abstract

This paper presents practical notional-load plastic-hinge method for a two-dimensional steel structure design. The proposed method incorporates the refined plastic-hinge concept for spread of plasticity together with a practical notional-load approach. The proposed method can assess realistically both strength and behavior of a structural system and its individual members in a direct manner. As a result, the method can be used for design without tedious separate member capacity checks, including the calculation of K-factor. The strengths predicted by the proposed method are then compared with those predicted by the exact plastic-zone analysis as well as by the conventional LRFD procedure. A good agreement is· generally observed. The displacement predictions are compared with the plastic-zone solutions. Analysis and design guidelines in using the proporsed method are given in detail. Analysis and design procedures are recommended. Member sizes determined by the proposed method are compared with those determined by the LRFD method. It is concluded that the procedures are suitable for adoption in practice.

Keywords : steel design, advanced analysis, plastic hinge, steel frame, material nonlinearity, geometric nonlinearity, notional load, LRFD

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1. Introduction

The steel design methods used are the Allowable Stress Design(ASD), the Plastic Design(PD), and the Load and Resistance Factor Design(LRED). In the ASD, the stress computation is based on a first-order elastic analysis, and the geometric nonlinear effects are implicitly accounted for in the member design equations. In the PD, a first-order plastic-hinge analysis is used for the structural analysis. The plastic design allows inelastic force redistribution in the structural system. Since geometric nonlinearity and gradual yielding effects are not accounted for in the analysis of plastic design, they are approximately accounted for in the member design equations. In the LRFD, a first-order elastic analysis with amplification factors or a direct second-order elastic analysis is used to account for geometric nonlinearity, and the ultimate strength of beam-column members is implicitly reflected in the design interaction equations. All three design methods require separate member capacity checks including the calculation of K-factor.

In order to account for the influence of a structural system on the strength of individual structural members, the effective length factor is employed. The effective length method generally provides a good solution in the design of a separate member. However, theoretical and practical difficulties are associated with the use of the effective length method in design of structural system. As the theoretical difficulty, the effective lengh approach cannot accurately account for the interactive behavior beween structural system and it members. This is because the interaction in a large structural system is too complex to be represented by the simple effective length factor K. As the practical difficulty, the effective length method requires a time consuming process of separate member capacity checks involving the calculation of K-factor since the effective length method is not user-friendly for a computer-based design.

An alternative method without the use of the effective length factor is necessary in order to overcome the limitations of the effective length method. One way to account for both system stability and force redistribution rationally is through a second-order inelastic frame analysis called "Advanced Analysis" Anvanced indicates any method that can sufficiently capture the limit strength and stability of a structural system and its individual members so that separate member capacity checks are not required. Since the power of personal computers and engineering workstations is rapidly increasing, it is becoming feasible to empoly advanced analysis techniques that have been considered impractical for disign office use in the past. Herein a notional-load plastic-hinge method as a practical advance analysis/design method will be presented for planar frames without the use of K-factor.

2. Key Factors Influencing Steel Structure Behavior

2.1 Geometric Nonlinearity

The bending moments in a beam-column consist of two types: primary bending moments; and secondary bending moments. Primary bending moments are caused by applied end moments and/or transverse loads on members. Secondary bending moments are from axial compressive force acting through the lateral displacements of a member. The secondary bending moments include the p-δ and p-Δ moments. Herein, stability functions are used for each member to capture these second-order effects in a direct manner.

Geometric imperfections result from unavoidable tolerance during fabrication or erection,and they may be classified as out-of-straightness and out-of-plumbness. These imperfections cause additional moments in column members that result in further degradation of members' bending stiffness. In this paper, geometric imperfections will be considered by equivalent notional load.[4,5]

2.2 Material Nonlinearity

Residual stresses result in a gradual axial stiffiness degradation. The fibers that have the highest compressive residual stress will yield first under compressive force, followed by the fibers with a lower value of compressive residual stress. Due to this spread of yielding or plasticity, the axial and bending stiffness of a colunn segment is degraded gradually along the length of a member. This stiffness degradation effect will be accounted for later by the tangent modulus concept.

When a wide flange section is subjected to pure bending, the moment-curvature relationship of a section has a smooth transition from elastic to fully plastic. This is because the section yields gradually from extreme fibers which have higher stresses than interior fibers. The gradual yielding effect leads to the concept of a hardening plastic hinge which may be represented simply by a parabolic stiffness reduction function of a plastic hinge. This will be described later.

3. Refined Plastic Hinge Analysis

The advanced analysis methods may be classified into three categories including: Plastic-zone method; Elastic-plastic hinge method; and Refined plastic-hingd method as shown in Fig. 1. Among these, the refined plastic hinge method wil be implemented here, since it was evaluated as a practical method without significant errors.[1,2]

The incremental force-displacement relationship of the refined plastic-hinge analysis may be expressed as:[1]

$$\begin{bmatrix} \dot{M}_A \\ \dot{M}_B \\ \dot{P} \end{bmatrix} = \frac{E_t I}{L} \begin{bmatrix} \eta_A[S_1 - \frac{S_2^2}{S_1}(1-\eta_B)] & \eta_A\eta_B S_2 & 0 \\ \eta_A\eta_B S_2 & \eta_B[S_1 - \frac{S_2^2}{S_1}(1-\eta_A)] & 0 \\ 0 & 0 & A/I \end{bmatrix} \begin{bmatrix} \dot{\theta}_A \\ \dot{\theta}_B \\ \dot{e} \end{bmatrix} \tag{1}$$

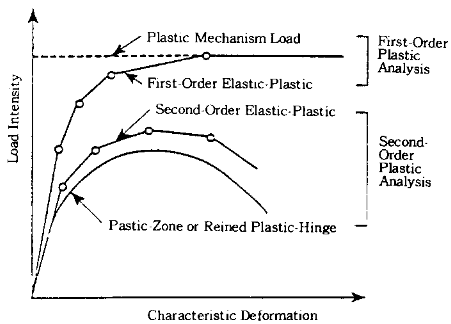
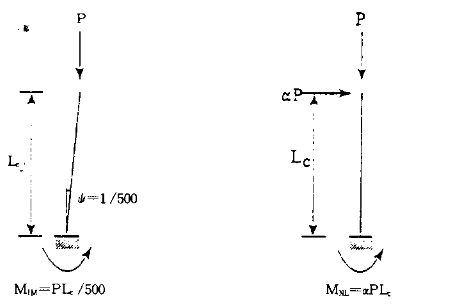


Fig.1 Deformation characteristics of plastic analysis methods

S_1 and S_2 are stability functions which account for stability effects. The benefit of using stability functions is that it enables only one element to predict accurately the second-order effect of each framed member. E_t stands for tangent modulus which accounts for gradual stiffness degradation due to residual stresses. η_A and η_B are scalar parameters for gradual inelastic stiffness reduction associated with flexure. $\dot{M}_A$, $\dot{M}_B$ $\dot{P}$ are incremental end moments and axial force, respectively. $\dot{\theta}_A$, $\dot{\theta}_B$ are incremental rotations at element ends A and B, and $\dot{e}$ is incremental axial deformation. I, L, and A are moment of inertia of cross section, length of element, and area of cross section, respectively.

4. Equivalent Notional-Load Approach Accounting for Geometric Imperfection

The AISC Code of Standard Practice (AISC 1994) limits the out-of-plumbness equal to $L_c/500$ in a story, and the out-of-straightness equal to $L_c/1000$ in a member in order to consider the erection tolerance. The geometric imperfections of a frame may be replaced by the equivalent notional lateral loads that are expressed as a fraction of the gravity loads acting on the story. A cantilever column shown in Fig. 2 is used here to discuss the basic idea of the equivalent notional load method. The moment M_{IM} caused by the geometric imperfection is equal to $PL_c/500$ at the base of the column shown in Fig. 2(a). The moment M_{NL} caused by equivalent notional load is equal to αPL_c in Fig. 2(b), where α is an equivalent notional load factor. M_{IM} and M_{NL} should be identical, and thus the notional load factor α results in the value of 0.002.



(a) Explicit imperfection model (b) Equivalent notinal load

Fig.2 Equivalent notional load concept for modeling geometric imperfection

Based on this concept, the geometric imperfections of frames at a story may be replaced by the equivalent notional lateral loads expressed as a fraction of the gravity loads acting on that story. The proposed equivalent notional load for practical use is $0.002\Sigma P_u$, where ΣP_u is total gravity load in story.[2] The notional load should be applied laterally at the top of each story.

For sway frames subjected to combined grav ity and lateral loads, the notional loads should be added to the lateral loads.

For braced frames, an equivalent notional load should be applied at mid-height of a column since the ends of the column are braced. An equivalent notional load factor equal to 0.004 is proposed here.[2] This value is equivalent to the geometric imperfection of $L_c/1000$. The notional load factor α is equal to 0.002 with respect to one half of the member length for the diagram.

5. Verification Study

A number of benchmark problems available in the open literature are used to evaluate the proposed method. The verification studies are carried out by comparing the results of the proposed method with those of the accurate plastic-zone analysis and the conventional LRFD method.

5.1 Axially Loaded Column

For verification of the proposed method, the AISC-LRFD column strength curve is used here since it properly accounts for the seconed-order effect, residual stresses, and geometric imperfections in a practical manner.[3] The notional loads equal to 0.004 times the gravity loads are applied at the mid-height of the column in modeling. The column strength of the proposed method is compared with the LRFD column strength in Fig. 3. The error is less than 5% for the columns within slenderness ratio L/r up to 140 which covers almost all columns used in engineering practice.

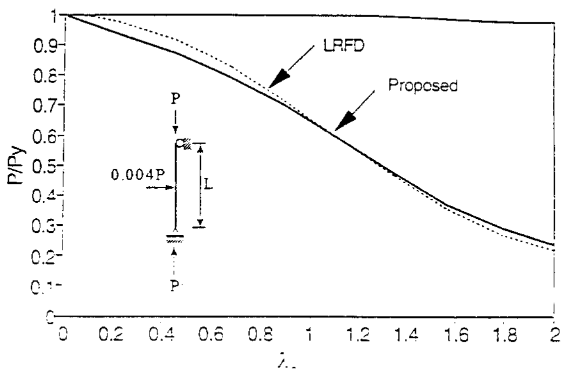


Fig.3 Comparison of strength curves for axially loaded pin-ended column

5.2 Beam-Columns

Galambos and Ketter developed interaction curves of pin-ended beam-columns subjected to axial thrust combined with bending moment using the plastic-zone method.[4] Galambos and Kettr's interaction curves account for residual stresses of $0.3F_y$ but not geometric imperfections. As a result, the curves are adjusted to account for geometric imperfections. In addition, the LRFD interaction curves have also been used for strength comparison without the resistance factors Φ_c and Φ_b.

When the equivalent notional-loads equal to 0.004 times gravity loads are applied at mid-height of beam-column, the strength curves are compared well with the LRFD interaction curvesn and the adjusted plastic-zone curves. The errors are no more than 5% as shown in Fig.4.

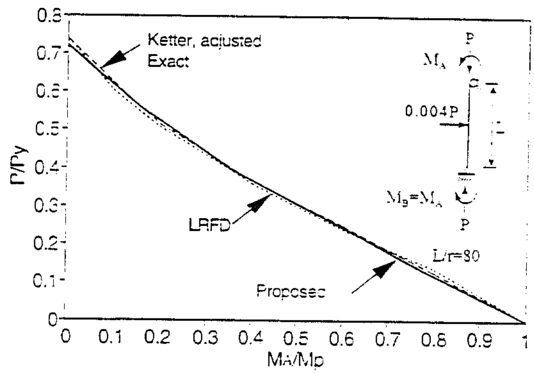


Fig.4 Comparison of strength curves for beam-column

5.3 Sway Structures

Kanchanalai developed exact interaction curves based on the plastic-zone analysis for sway structures.[5] In his studies, the members were assumed to have maximum compressive residual stresses of $0.3F_y$ but not geometric imperfections. As a result, the curves are adjusted to account for geometric imperfections. The AISC-LRFD interaction curves are obtained based on LeMessurier K-factor approach.[6]

When the equivalent notional loads equal to 0.004 times gravity loads are applied at top of the column, most strength curves are found within the area bounded by the plastic-zone curves and the LRFD curves (Figs. 5-6). The conservative errors are less than 7% which is better than 17% error of the LRFD, and maximum unconservative error is not more than 1%.

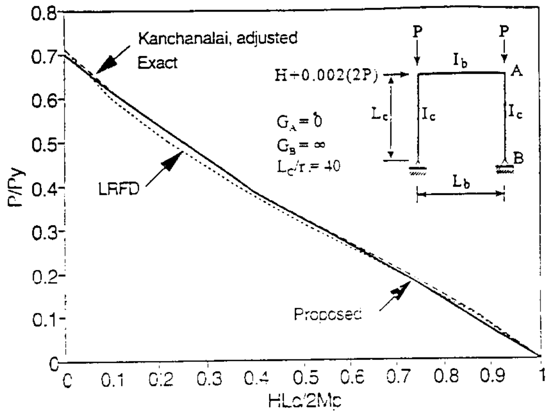


Fig.5 Comparison of strength curves for portal frame

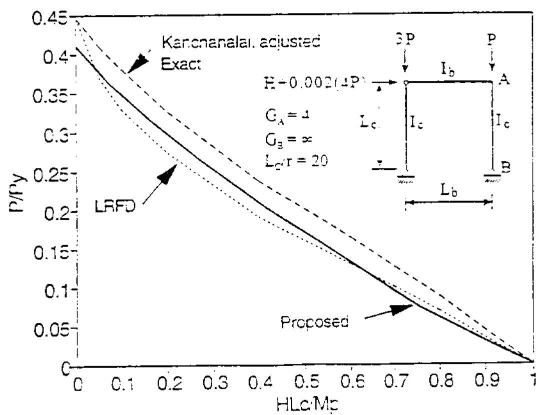


Fig.6 Comparison of strength curves for leaned-column frame

5.4 A 6-Story and 2-Bay Frame

Vogel provided the load-displacement relationships of a 6-story frame using plastic-zone analysis.[7] The load-displacement curves of the proposed method and Vogel's plastic-zone analysis are compared in Fig.7. The errors of the proposed method in strength predictions are less than 1%. The proposed method predicts well the lateral displacements. The method underpredicts the lateral displacement by only 3% at service load levels when compared with the Vogel's solutions.

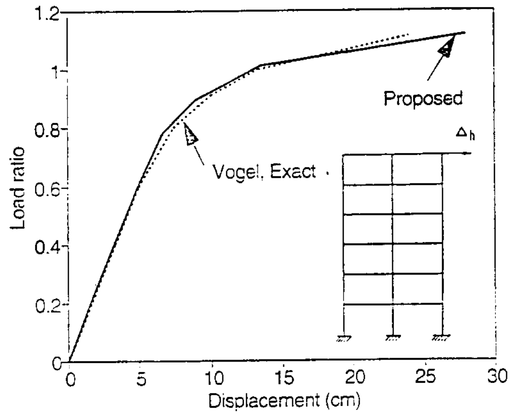


Fig.7 Comparison of load-displacements of Vogel's 6-story frame

6. Analysis and Design Guidelines

Analysis and design guidelines in using the proposed method are summarized as follows:

6.1 Design Format

The proposed method is based on the limit state approach to strength design. The limit state format may be written as:[3)]

$$\sum \gamma_i Q_i \leq \phi R_n \qquad (2)$$

where γ_i = load factors

Q_i = nominal design loads

ϕ = resistant factors

R_n = nominal resistances.

6.2 Load Combinations

The load combinations in the proposed method are based on the LRFD load combinations.[3)] The member sizes of structures are determined from an appropriate combination of factored loads.

6.3 Live Load Reduction

The live load reduction is based on the ASCE 7-88 as:

$$L = (0.25 + \frac{15}{\sqrt{A_f}})L_o \geq \alpha L_o \qquad (3)$$

where L= reduced design live load

A_I = mem ber influence area in square feet (A[illegible]36 m^2 (400fr^2))

L_o = unreduced design live load

α = 0.5 for members supporting on floor,

α = 0.4, otherwise.

It is important to carry out properly the application of the live load reduction in analyzing a structural system. This is because the influence area for each beam and column is generally different, and different influence area results in a different reduction factor.

In the present study, the live load reduction procedures follow the work of Ziemian and McGuire.[8] The method is based on the use of "compensation forces" calculated by:

(1) applying live load reduction factor to beams.

(2) applying live load reduction factor to columns.

(3) determining compensating forces due to different reduction factors between columns and beams at the beam-to-column intersections.

The compensating forces are generally directed upward since columns typically have a larger influence area and a larger reduction factor than beams.

6.4 Resistance Factors

In the present study, the resistance factors as used in the LRFD cross-section strength equations are:[3]

$$\frac{P}{\phi_c P_y} + \frac{8}{9}\frac{M}{\phi_b M_p} = 1.0 \qquad \text{for } \frac{P}{\phi_c P_y} \geq 0.2 \qquad (4a)$$

$$\frac{P}{2\phi_c P_y} + \frac{M}{\phi_b M_p} = 1.0 \qquad \text{for } \frac{P}{\phi_c P_y} < 0.2 \qquad (4b)$$

where P, M = second-order axial force and bending moment

P_y = squash load

M_p = plastic moment capacity

ϕ_c, ϕ_b = resistance factors for axial strength and flexural strength.

The resistance factors are selected to be 0.85 for axial strength and 0.9 for flexural strength as in LRFD.

6.5 Serviceability Limit

According to the ASCE A_n Hoc Committee report,[9] the normally accepted range of overall drift limits for buildings is 1/750 to 1/250 times the building height H with a typical value of H/400. The general limits on the interstory drift are 1/500 to 1/200 times the story height. Based on the studies by the An Hoc Committee, and by Ellingwood,[10] the deflection lim its for girder and story are selected as:

(1) Floor girder live load deflection : L/360

(2) Roff girder deflection : L/240

(3) Lateral drift : H/400 for wind load

(4) Interstory drift : H/300 for wind load

At service load levels, no plastic hinges are allowed to occur in order to avoid permanent deformation under service loads.

6.6 Ductility Requirement

Adequate inelastic rotation capacity is required for members in order to develop their full plastic moment capacity. The required rotation capacity may be achieved when members are adequately braced and their cross sections are compact.

Compact sections are capable of developing the full plastic moment capacity M_p and sustaining large hinge rotation before the onset of local buckling. The compact section in the LRFD Specification is defined as:[3]

(1) Flange

$$\frac{\mu_f}{2t_f} \leq \frac{65}{\sqrt{F_y}} \tag{5}$$

where b_f = width of flange; t_f = thickness of flange; and F_y = yield stress in ksi.

(2) Web

$$\frac{h}{t_w} \leq \frac{640}{\sqrt{F_y}}\left(1-\frac{2.75P_u}{\phi_b P_y}\right)$$

$$\text{for } \frac{P}{\phi_b P_y} \leq 0.125 \tag{6a}$$

$$\frac{h}{t_w} \leq \frac{191}{\sqrt{F_y}}\left(2.33-\frac{P_u}{\phi_b P_y}\right) \geq \frac{253}{F_y}$$

$$\text{for } \frac{P}{\phi_b P_y} > 0.125 \tag{6b}$$

where h = clear distance between flanges
t_w = thickness of web.

In addition to the compactness of section, the lateral unbraced length of member is also a limiting factor for the development of the full plastic moment capacity of members. The LRFD seismic provisions provide the limit on spacing of braces for beam-columns as:[3]

$$\frac{L}{r_y} \leq \frac{17200}{F_y} \tag{7}$$

where L = unbraced length
r_y = radius of gyration about y-axis.

When the yield stress is equal to 250 MPa (36 ksi) and the radius of gyration is assumed to be approximately 50 mm (2 in), the permissible unbraced length L results in 3.5 m (11.6 ft). Since the unbraced length of 3.5 m (11.6 ft) is similar to the typical story height of 3.0-3.7 m (10-12ft), the lateral torsional buckling may not be a governing factor for the ductility of beam-columns in typical building frames.

6.7 Geometric Imperfection

The factors of equivalent notional-loads are determined as α=2/1000 for unbraced frame and α=4/1000 for braced frame.

7. Recommended Analysis/ Design Procedures

A possible design procedure in using the proposed method is recommended as follows:

Step 1: Preliminary analysis/design assuming rigid frame. The preliminary member sizing is intrinsically dependent on engineer experiences, the rule of thumb, or some simplified analysis. For example, beam members are usually selected assuming that beams are simply supported and subjected by gravity loads only. For the preliminary sizing of column members, the overall drift requirements should be a good guideline to determine preliminary member sizes rather than the tedious strength checks of the individual column.

Step 2: Analysis of structural system. Once the preliminary members are determined in Steps 1, the analysis using the proposed method may be performed for the structural model with notional-loads accounting for the effect of geometric imperfections.

Step 3: Check for strength, serviceability, and ductility. The adequacy of system and its component member strength can be directly evaluated by comparing the predicted ultimate loads with the applied factored loads. The serviceability of a structural system should be also checked to ensure the adequacy of the system and member stiffness at service loads. Adequate ductility is required for members in order to develop their full plastic moment capacity. The required ductility may be achieved when members are adequately braced and their cross sections are compact.

Step 4: Local strength checks of members. Since the proposed analysis account for only the global behavior effects, the independent local strength checks of members are required based on the LRFD Specification.

Step 5: Adjustment of member and connection sizes. If the conditions of steps 3-4 are not satisfied, appropriate adjustments of member sizes should be made. For illustration, if an excessive lateral drift occurs in a structural system, the drift may be reduced by increasing column sizes. Once the member sizes are adjusted, iteration of Steps 2-5 leads to an optimum design.

8. Case Study

8.1 Frame Configuration and Load Condition

Figure 8 shows a leaned-column frame used by LeMessurier.[6] The exterior columns are leaned to the central column which supplies lateral rigidity of the frame. Lateral braces are assumed to be fully provided for the beams and columns. The frame is subjected to a distributed gravity load and a concentrated horizontal load.

8.2 Design by Proposed Method

Preliminary member sizes are assumed as W27x94, W14x43, and W6x9 for the beams, central column, and exterior leaned-columns, respectively. Each column is modeled as one element, and each beam is modeled as four elements. The notional load of 1.71 kN (0.384 kips) is calculated by 0.2% times the total vertical load of 854 kN (192 kips), and is added to the lateral load. Herein, the uniform gravity load of 23.7 kN/m (1.6 kips/f_t) in Fig. 8 is converted into four concentrated loads of 107 kN (24 kips) on the beam. The vertical incremental loads are determined as 5.338 kN (1.2 kips) by dividing the concentrate load by the scaling number 20.

The horizontal incremental load is calculated as 0.8896 kN (0.2 kips). The proposed method predicts the ultimate load carrying capacity of 111kN (25.0 kips) with respect to vertical loads which satisfies the applied factored load of 107kN (24.0 kips). As a result, the preliminary member sizes are adequate.

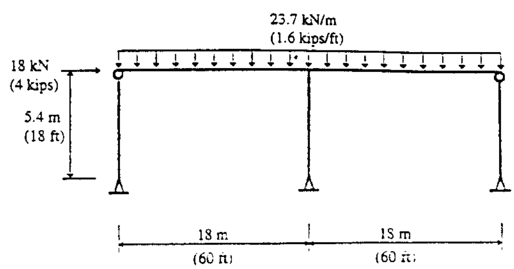


Fig.8 Configuration and load condition of leaned-column frame for case study

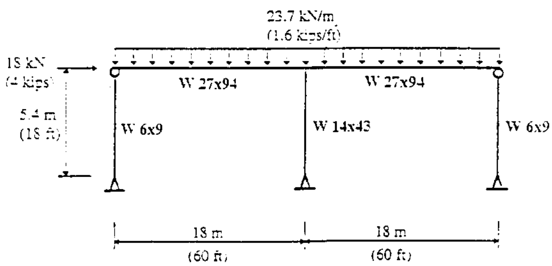
8.3 Design by the AISC-LRFD Method

Hereim, the LRFD procedures for the leaned-column frame design are briefly described. In the LRFD method, two first-order analyses are performed; one for the sway case and the other for the nonsway case. The member capacity checks were carried out based on the LRFD equations. Member sizes were adjusted according to the results of member capacity checks.

Through these tedious procedures, member sizes were determined as W33x118, W14x43, and W6x9 for the beams, central column, and exterior leaned-columns, respectively.

8.4 Comparison of Results

The proposed method predicts the identical member sizes with those determined by the LRFD method shown in Fig.9. This is because the present leaned-column frame is not a highly redundant structure and does not possess much benefit of inelastic moment redistribution using the advanced method.



* Same member sizes are predicted by the proposed and LRFD method

Fig.9 Comparison of member sizes of leaned-column frame for case study

9. Summary and Conclusions

The proposed method is adequate in assessing the strengths when compared with the exact plastic-zone solutions. The maximum unconservative errors of the method are no more than 5% for a wide range of frames. The proposed method predicts well the lateral displacements of sway frames when compared with that of the plastic-zone solution. The appropriate factor of equivalent notional loads associated with the method is selected as $\alpha=2/1000$ for unbraced frames and $\alpha=4/1000$ for braced frames. Specific analysis and design guidelines are suggested for the application of the proposed method, and analysis/design procedures are also recommended. Since member sizes determined by the propsed method are close to the determined by the LRFD method, the proposed method may be used as an alternative for the current LRFD design method. The proposed method does not require separate member capacity checks, including the calculation of K-factor. Since the proposed method takes into account the inelastic moment redistributions, they allow some reduction of steel weight, especially for highly indeterminate steel frames. Since the proposed method strikes a balance between the requirement for realistic representation of actual behavior and failure mode of a structural system and the requirement for simplicity in use, it is therefore recommended for general use.

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