A Variable Step Size LMS Algorithm Using Normalized Absolute Estimation Error

D. W. Kim, S. H. Han, H. K. Hong, H. B. Kang, and J. S. Choi

Abstract

Variable step size LMS(VS-LMS) algorithms improve performance of LMS algorithm by means of varying the step size. This paper presents a new VS-LMS algorithm using normalized absolute estimation error. Normalizing the estimation error to the expected value of the desired signal, we determined the step size using the relative size of estimation error. Because parameters and computational load are less, our algorithm is easy to implement in hardware. The performance of the proposed algorithm is analyzed theoretically and estimated through simulations. Based on the theoretical analysis and computer simulations, the proposed algorithm is shown to be effective compared to conventional VS-LMS algorithms.

I. Introduction

A transmitted signal is usually corrupted and transformed through communication channels. The corruption of the signal may be gaussian thermal noise, impulse noise, and additive and/or multiplicative noise owing to fading. The effect of transformation by imperfect channels appears as frequency translation, nonlinear or harmonic distortion, and time dispersion. Even though these distortions cause significant waveform distortion of the transmitted signal in analog communication, their effects are much more serious in digital communication. Especially, the multipath formed by time delay and phase transformation of the transmitted signal gives rise to large ISI, which is one of the major causes for bit detection error.

Thus, to detect the original signal in the receiver, we need channel equalization[1], which decreases the bit error ratio by compensating for transformation of the signal in receiver. Equalization can be adaptive to compensate for unknown or slowly time-varying channel because the characteristic of a channel is often time-varying depending on a location of a transmitter-receiver, distance, configuration of the ground, temperature and moisture distribution, etc.

In adaptive signal processing, one of the most popular algorithms to cope with the time-varying characteristic of a channel is the least mean square(LMS) algorithm by Widrow and Hoff[1] due to its simplicity for implementation. The LMS algorithm, while relatively simple to implement, does not always converge in an acceptable manner and particularly gets worse when the input eigenvalue spread is large. This is because, unless the input signal is white(when all the eigenvalues are equal), the gradient vectors generally do not point to the bottom of the error surface.

To achieve faster convergence rate with a reasonable computational load for real-time applications, many variations of the LMS algorithm have been proposed with different choices of the scalar step size. The step size in the LMS algorithm controls the convergence rate of the filter and determines the final excess mean square error. A large step size for fast convergence in applications is often selected. This selection, however, results in increased misadjustment or error residual.

The idea of variable step size LMS (VS-LMS) is to somehow sense how far away the adaptive filter coefficients are from the optimal filter coefficients, and uses small step sizes when adaptive filter coefficients are close to the optimal values and large step sizes otherwise. In previous studies, many algorithms have been proposed to adjust the step size. These algorithms are based on the polarity of the successive samples of the estimation error[2], the magnitude of the squared estimation error[3], the negative of the gradient of the squared estimation error[4], the time constant concept [5], and so on.

The LMS algorithm should be considered in the following issues. First, the algorithm must have a simple structure to implement hardware systems. Second, the parameters of the algorithm must be controlled easily and be robust to noise.

Manuscript received April 20, 1996; accepted June 8, 1996.

The authors are with Department of Electronic Engineering, Chung-Ang University, Seoul, Korea.

The previous methods have some drawbacks that the above issues are not considered simultaneously.

This paper is organized as follows. In Section II, we derive the variable step size LMS algorithm using normalized absolute estimation error. In Section III, we present the mean squared behavior and steady state result. Simulation results obtained using the algorithm are described in Section IV. Finally, the conclusions are made in Section V.

II. A New Variable Step Size LMS Algorithm

In this section, we present a new VS-LMS algorithm using normalized absolute estimation error. At each iteration, the filter weight(W) updating formula is given by

$$W_{k+1} = W_k + \mu_k e_k X_k \tag{1}$$

where X_k represents an input signal vector, e_k signifies the error signal defined as a difference between the desired signal and the filter output, and μ_k represents the step size. For adjusting the step size μ_k, the formula is given by

$$\mu_{k+1} = \frac{|e_k|}{|d_k|} \cdot \mu_{\max}, \tag{2}$$

and

$$\mu_{k+1} = \begin{cases} \alpha\mu_k, & \text{if } \mu_{k+1} < \alpha\mu_k \\ \frac{1}{\alpha}\mu_k, & \text{if } \mu_{k+1} > \frac{1}{\alpha}\mu_k \\ \mu_k, & \textit{otherwise}. \end{cases} \tag{3}$$

where d_k is a desired signal. The initial step size μ_0 is usually taken to be $\mu_{\max}$ and the step size is limited by $\mu_{\max}$. The constant $\mu_{\max}$ is chosen to ensure that the Mean Square Error (MSE) of the algorithm remains bounded. A sufficient condition for $\mu_{\max}$ to guarantee bounded MSE is [3] [6],

$$0 < \mu_{\max} < \frac{2}{3tr(R)} .$$

As can be seen from (2) and (3), the step size μ_k is always positive and is controlled by the parameter α and the relative size of the estimation error. If the input power of the equalizer equals the desired signal and initial value of each weight is zero, estimation error in initial iteration is similar to expected value of desired signal. Normalizing the estimation error to the expected value of the desired signal, we determine the step size using the relative size of estimation error. Since the convergence characteristic curves of LMS algorithms have an exponential function, α must be chosen in the range (0,1) to provide an exponential forgetting. By using the chosen α, the proposed algorithm is robust to noise in addition. That is, α prevents the step size from fluctuating in the optimum step size to some extent. Until estimation error decreases to expected value of desired value, the step size μ_k is taken to be close to $\mu_{\max}$. If the magnitude of the estimation error decreases, the step size will be decreased exponentially to reduce the misadjustment. In our study, α is taken to be 0.996.

III. Performance Analysis

In this Section, we analyze the performance of proposed algorithm. Let the input vector to the system be denoted by X_k and the desired scalar signal d_k, and W_{opt} the optimal weight vector that makes the system output track a desired signal. We write the time varying weight vector W_{opt} by

$$W_{opt,n} = W_{opt,n-1} + C_{n-1} \tag{4}$$

where C_{n-1} is a disturbance process with mean zero and variance $\sigma_c^2 I$. We use following assumptions and approximations for performance analysis.

i. X_k and d_k are jointly Gaussian processes that have zero mean and are independent of X_n and d_n for $n \neq k$

ii. $d_k = X_k^T W_{opt,k} + \zeta_k$ (5)

where ζ_k is an error for the optimal weight. In (5) we assume that X_k, C_k, and ζ_k are statistically independent processes.

iii. μ_k is statistically independent of X_k and e_k.

1. Mean Behavior of Weight Vector

Denoting the weight deviation V_k by

$$V_k = W_k - W_{opt,k} \tag{6}$$

we can write the following expression for the estimation error.

$$e_k = \zeta_k - V_k^T X_k . \tag{7}$$

Substituting (5), (6) and (7) into (1), we obtain

$$V_{k+1} = (I - \mu_k X_k X_k^T) V_k + \mu_k X_k \zeta_k - C_k . \tag{8}$$

If μ_k is independent with X_k and e_k, the expression (8) becomes

$$E[V_{k+1}] = (I - E[\mu_k] R) E[V_k] \tag{9}$$

where R is the autocorrelation of input signal vector X_k.

2. Mean Squared Behavior of Weight Vector

The second moment matrix of V_k vector is defined as

$$K_k = E[V_k V_k^T] . \tag{10}$$

(8) is rewritten as (11),

$$V_{k+1} V_{k+1}^T = (I - \mu_k X_k X_k^T) V_k V_k^T (I - \mu_k X_k X_k^T) + \mu_k^2 \zeta_k^2 X_k X_k^T + C_k C_k^T + g(\mu_k, X_k, V_k, \zeta_k, C_k). \quad (11)$$

where $g(\mu_k, X_k, V_k, \zeta_k, C_k)$ consists of six items and its mean is zero. Assuming that μ_k and μ_k^2 are independent with input signal, we can derive (11) as

$$K_{k+1} = K_k - 2E[\mu_k]\sigma_x^2 K_k + E[\mu_k^2](2\sigma_x^4 K_k + \sigma_x^2 \sigma_{e,k}^2 I) + \sigma_c^2 I. \quad (12)$$

where

$$\sigma_{e,k}^2 = \xi_{\min} + \sigma_x^2 tr(K_k) \quad (13)$$

and

$$\xi_{\min} = E[\zeta_k^2]. \quad (14)$$

So, mean and mean-squared behavior of step size be written as

$$E[\mu_k] = \alpha E[\mu_{k-1}], \quad (15)$$

and

$$E[\mu_k^2] = \alpha^2 E[\mu_{k-1}^2]. \quad (16)$$

where μ_k is assumed to be convergent. Mean-squared behavior characteristics of V_k vector are represented in (12)-(16). It is difficult to derive parameter satisfying the convergence condition of the above equation. But, if μ_k is bounded value to guarantee convergence, the convergence of K_k is valid.

3. Steady State Misadjustment

Let the steady state values of $E[\mu_k]$, $E[\mu_k^2]$, $\sigma_e^2(k)$, and K_k be $\overline{\mu_\infty}$, $\overline{\mu_\infty^2}$, $\sigma_e^2(\infty)$, and K_∞. Then, $\overline{\mu_\infty}$, $\overline{\mu_\infty^2}$, σ_e^2 and K_∞ are represented by

$$\overline{\mu_\infty} = \sqrt{\frac{\xi_{\min} + \sigma_x^2 tr(K_k)}{\sigma_d^2}} \cdot \mu_{\max} = \sqrt{\frac{\xi_{\min} + \sigma_x^2 tr(K_k)}{\sigma_d^2}} \cdot \frac{2}{3N\sigma_x^2}, \quad (17)$$

$$\overline{\mu_\infty^2} \approx \frac{\xi_{\min} + \sigma_x^2 tr(K_k)}{\sigma_d^2} \cdot \frac{4}{9N^2\sigma_x^4}, \quad (18)$$

$$\sigma_e^2 = \xi_{\min} + \sigma_x^2 tr(K_\infty). \quad (19)$$

and K_∞ is

$$K_\infty = K_\infty - 2\overline{\mu_\infty}\sigma_x^2 K_\infty + \overline{\mu_\infty^2}(2\sigma_x^4 K_\infty + \sigma_x^2 \sigma_e^2(\infty) I) + \sigma_c^2 I. \quad (20)$$

As can be seen from (12) and (19), K_∞ is diagonal matrix.

$$K_\infty = k_\infty I. \quad (21)$$

Then, using (19)-(20) we obtain the following equation for K_∞

$$k_\infty = \frac{\overline{\mu_\infty^2}\xi_{\min} + \frac{\sigma_c^2}{\sigma_x^2}}{2\overline{\mu_\infty} - \overline{\mu_\infty^2}(N+2)\sigma_x^2} \quad (22)$$

In stationary environment, excess MSE is less than Minimum Mean Square Error(MMSE), that is,

$$\xi_{\min} \gg N\sigma_x^2 k_\infty. \quad (23)$$

So $\overline{\mu_\infty}$ and $\overline{\mu_\infty^2}$ are simplified as

$$\overline{\mu_\infty} \approx \sqrt{\frac{\xi_{\min}}{\sigma_d^2}} \cdot \frac{2}{3N\sigma_x^2}, \quad (24)$$

$$\overline{\mu_\infty^2} \approx \frac{\xi_{\min}}{\sigma_d^2} \cdot \frac{4}{9N^2\sigma_x^4}. \quad (25)$$

Under the same condition, it follows generally that

$$\overline{\mu_\infty} \gg \overline{\mu_\infty^2}(N+2)\sigma_x^2 \quad (26)$$

Using (24)-(26) and if σ_c^2 is zero in the stationary process, (22) is approximated by

$$k_\infty \approx \frac{1}{3N\sigma_x^2} \cdot \sqrt{\frac{\xi_{\min}^3}{\sigma_d^2}} \quad (27)$$

So, excess MSE in the steady state is represented as

$$e_{ex} = N\sigma_x^2 k_\infty \approx \frac{1}{3}\sqrt{\frac{\xi_{\min}^3}{\sigma_d^2}} \quad (28)$$

Assuming that σ_c^2 is small in the nonstationary process, we obtain

$$k_\infty = \frac{1}{3N\sigma_x^2}\sqrt{\frac{\xi_{\min}}{\sigma_d^2}} + \frac{3}{4}N\sigma_c^2\sqrt{\frac{\sigma_d^2}{\xi_{\min}}} \quad (29)$$

and

$$e_{ex} = N\sigma_x^2 k_\infty \approx \frac{1}{3}\sqrt{\frac{\xi_{\min}^3}{\sigma_d^2}} + \frac{3}{4}N^2\sigma_c^2\sigma_x^2\sqrt{\frac{\sigma_d^2}{\xi_{\min}}}. \quad (30)$$

(27)-(30) show the characteristics of the proposed algorithm in the steady state.

IV. Simulation results

In Fig. 1-4, we show simulation results of several LMS algorithms including the proposed VS-LMS algorithm. The ISI channel model[7] used for our simulation is given by

$$h_n = \begin{cases} \frac{1}{2}\left[1+\cos\frac{2\pi(n-1)}{K}\right], & n = 1,2,3 \\ 0, & otherwise \end{cases}$$

where K represents a parameter to adjust the degree of ISI. The received signal, x_k, is then given by

$$x_k = d_k * h_n + n_k$$

where n_k is additive white gaussian noise with variance σ_n^2 and d_k is the desired stationary random signal with the value of ± 1. For all equalizers, 11 taps were used. The parameters used in each of the algorithms are as shown in Table 1.

Fig. 1-4 show the MSE curves on the channels in which the parameter K is set to 2.7 or 3.3 and σ_n^2 is taken to be 0.001 or 0.005. The simulation curves were generated by averaging over 2000 runs. While it would be better to be compared with all conventional algorithms, the proposed algorithm is compared with one of them because it is difficult to distinguish between one and another.

Fig. 1 compares the proposed algorithm with Harris's algorithm and the Fixed Step Sized(FSS) algorithm when K =2.7 and σ_n^2=0.001. The parameters used in Harris's algorithm in this channel are α=2, m_0=3, and m_1=3. The parameter for FSS algorithm is μ=0.01. Table 1 shows the parameters which used for each algorithm in the other channels.

Table 1. Parameters of VS-LMS.

Algorithm	μ_{max}	μ_{min}	parameters
LMS	0.01		
Harris et al.	0.0625	0.000976	α = 2, m_0 = m_1 = 3
Kwong et al.	0.08333	0.0	α = 0.985, γ = 4.8×10^{-4}
Mathews et al.	0.08333	0.0	ρ = 0.003, 0.002
Y.K.Won et al.	0.08333	0.01	
Proposed	0.08333	0.0	α = 0.996

As may be seen, the proposed algorithm has the same MSE curve as that of Harris's algorithm to 200 iterations but it has better performance than Harris's algorithm after 200 iterations. Table 2 shows that the proposed algorithm has better result than the other four algorithms by 5dB in this channel. Because Harris's algorithm updates a step size at each tap independently, it needs additive M memory in updating weight.

In Fig. 2, we compared the simulation result of the proposed algorithm with that of Kwong's and FSS algorithms in the channel with K=2.7 and σ_n^2=0.005. Fig. 2 shows that the excess MSE and fluctuation of the proposed algorithm are smaller than Kwong's algorithm.

Fig. 3 shows the convergence curve of the proposed and Won's and FSS algorithms in the channel with K=3.3 and σ_n^2=0.001. The proposed algorithm has faster convergence speed and smaller excess MSE than any other algorithms for this channel. When the channel changes after convergence Won's algorithm cannot track itself, because it decreases the step size every constant time.

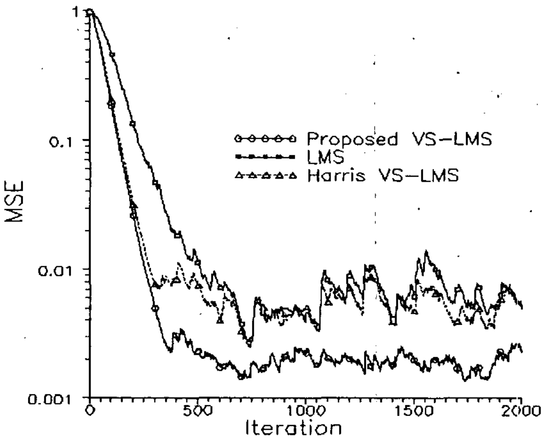

Fig. 1. MSE curve(K = 2.7, σ_n^2=0.001).

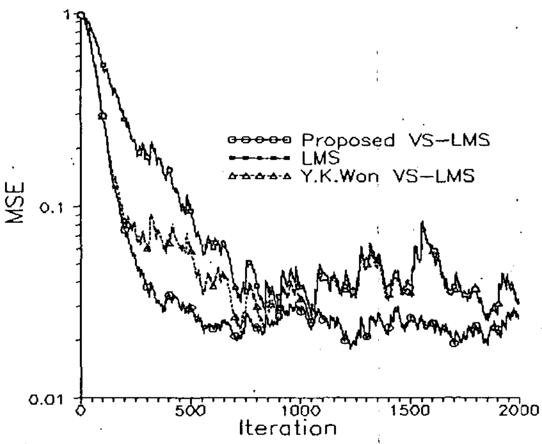

Fig. 2. MSE curve(K = 2.7, σ_n^2=0.005).

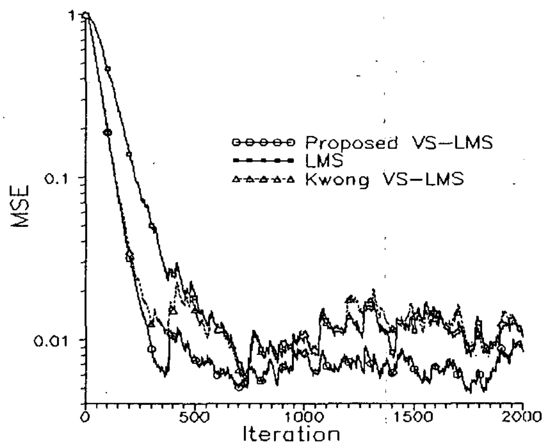

Fig. 3. MSE curve(K = 3.3, σ_n^2=0.001).

The proposed algorithm is compared with Mathew's and FSS algorithms in the channel with a large correlation and much gaussian noise in Fig. 4. The proposed VS-LMS algorithm is more stable in the steady state than the other VS-LMS algorithms.

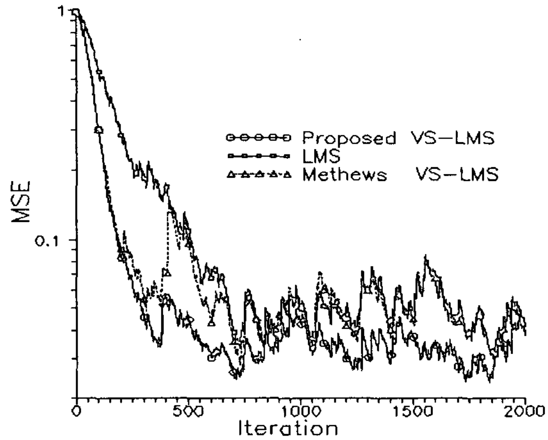

Fig. 4. MSE curve(K = 3.3, σ_n^2=0.001).

All simulation results are shown in Table 2. They are the average of Echo Return Loss Enhancement(ERLE) from 2000 to 5000 iterations in each channel. ERLE is represented as follow.

$$\text{ERLE} = 10 \log_{10}\left(\frac{E[d^2(n)]}{E[e^2(n)]}\right)$$

Table 2. The results of simulation under the channel conditions.(Unit : dB).

Algorithm \ Conditions	K = 2.7 σ_n^2=0.001	K = 3.3 σ_n^2=0.001	K = 2.7 σ_n^2=0.005	K = 3.3 σ_n^2=0.005
LMS	22.2548	14.5087	19.7716	13.6503
Harris et al.	22.6520	14.4430	19.5900	13.4256
Kwong et al.	21.8776	14.0450	19.7469	13.4056
Methews et al.	22.2403	14.4225	19.8088	13.7551
Y.K.Won et al.	22.2548	14.5100	19.7716	13.6522
Proposed	27.2584	16.3507	21.7328	14.8986

As may be seen in the simulation results, the proposed algorithm has a better performance in less correlated and less gaussian noisy channel by 5dB and in much correlated and much gaussian noisy channel by 1dB. It cannot conceal the error owing to white noise because the proposed algorithm use the estimation error. But it has a better result than the other conventional algorithms in this channel. In overall, we can see that the proposed algorithm is better than any other existing algorithms, especially for the case where correlation is large and gaussian noise is small.

V. Conclusions

The VS-LMS algorithms have better performances than LMS algorithm. We proposed a new VS-LMS using normalized estimation error. Compared with conventional VS-LMS algorithms, the proposed algorithm has better performances as shown in computer simulations. Because of parameter and computational load is less, our algorithm is easy to implement in hardware.

The stationary condition is assumed in this paper, but it is not always true in practical environment. So the advanced VS-LMS algorithm that is not affected by the characteristics of input signal will be proposed in the future.

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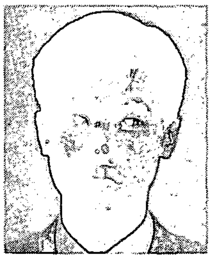
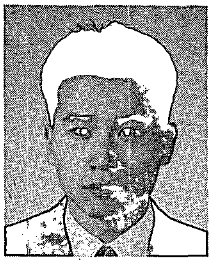
Dong-Wook Kim received the B.S. degree from Sung Kyun Kwan University, Suwon, Korea, the M.S. degree from Chung-Ang University, Seoul, Korea, in 1987 and 1992, respectively. He is currently working toward a Ph.D. degree in Electronic Engineering, Chung-Ang University. His research interests include signal and image processing, data compression, computer vision.



Hyun-Ki Hong received the B.S. and the M.S. degrees in Electronic Engineering from Chung-Ang University, Seoul, Korea, in 1993 and 1995, respectively. He is currently working toward a Ph.D. degree in Electronic Engineering, Chung-Ang University. His research interests include computer vision, electro-optical systems, image processing.

Jong-Soo Choi received the the B.S. degree from Inha University, Inchon, Korea, the M.S. degree from Seoul National University, Seoul, Korea, and the Ph.D. degree from Keio University, Yokohama, Japan, all in Electrical engineering, in 1975, 1977, and 1981, respectively. He joined the the faculty at Chung-Ang University in 1981, where he is now a Professor in the Department of Electronic Engineering. His current research interests are in computer vision and image coding.



Sung-Hyun Han received the B.S. and the M.S. degrees in Electronic Engineering from Chung-Ang University, Seoul, Korea, in 1990 and 1992, respectively. He is currently working toward a Ph.D. degree in Electronic Engineering, Chung-Ang University. His research interests include signal and image processing, data compression, electro-optical systems.

Han-Bit Kang received the B.S. and the M.S. degrees in Electronic Engineering from Chung-Ang University, Seoul, Korea, in 1993 and 1995, respectively. Since 1995, he has been with Daewoo Electronics Co., LTD., where he has been involved in the development of channel equalization algorithms and systems. His research interests include digital signal processing and channel equalization systems.