Efficient Design of 2-D FIR Fan Filters Using New Formulas for McClellan Transform Parameters

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Abstract

New formulas for McClellan transform parameters for the design of 2-D zero-phase FIR fan filters are optimally derived under the integral squared error(ISE) criterion. By imposing the constraint that $F(0,0) = \cos w_c$, where $F(\cdot)$ is the McClellan transform and w_c is the cutoff frequency of the 1-D prototype filter, the ISE is directly minimized without modifying it and, as a consequence, closed-form formulas for the McClellan transform parameters are obtained. It is shown that these formulas lead to a very efficient design for 2-D zero-phase FIR fan filters.

I. Introduction

The McClellan transform [1] has been recognized as a powerful tool in designing two-dimensional (2-D) zero-phase FIR filters [2]-[6]. This transform, which maps the 1-D frequency points of a 1-D prototype filter onto 2-D frequency contours, is easy to perform and leads to a 2-D filter with an efficient structure. The parameters of this transform are so determined that a 1-D cutoff frequency is mapped onto a contour that best approximates the desired 2-D cutoff contour.

The 2-D zero-phase FIR fan filters were designed using the McClellan transform in [4], [5]. Simple formulas for the transform parameters were derived by approximating an optimality criterion [4], and in [5] the parameters were determined by applying the eigenfilter method. It has been shown that the eigenfilter approach results in a cutoff contour having smaller absolute deviation from the ideal as compared to the approximation approach. The former, however, requires some numerical search and is computationally more expensive than the latter.

In this letter, we present a new approach to the McClellan transform parameters for the design of 2-D zero-phase FIR fan filters. In particular, new closed-form formulas for the proposed method is as simple as the approximation method

transform parameters are derived. It will be shown that the in [4], and that its performance is comparable to that of the eigenfilter approach in [5].

II. The McClellan transform

Consider a 1-D zero-phase FIR filter with frequency response

$$H(w) = \sum_{n=0}^{N} a(n) T_n(\cos w) \tag{1}$$

where $T_n(\cdot)$ is the n-th Chebyshev polynomial. The 2-D frequency response $H(w_1, w_2)$ is obtained by substituting $F(w_1, w_2)$ for $\cos w$. Specifically,

$$H(w_1, w_2) = H(w) \mid_{\cos w = F(w_1, w_2)} = \sum_{n=0}^{N} a(n) T_n(F(w_1, w_2)). \tag{2}$$

Here

$$F(w_1, w_2) = t_{00} + t_{10} \cos w_1 + t_{01} \cos w_2 + t_{11} \cos w_1 \cos w_2 \tag{3}$$

in the original McClellan transform [1]. Since $|\cos w| \leq 1$, $F(w_1, w_2)$ should satisfy $|F(w_1, w_2)| \leq 1$. The transform $F(w_1, w_2)$ in (3), which is quadrantally symmetric, has been used for the design of 2-D zero-phase fan filters which also have quadrantal symmetry. If we map 1-D frequency points, $w = 0$ and $w = \pi$, respectively, onto 2-D points $(0, \pi)$ and $(\pi, 0)$ as in [4], [5], then the parameters of $F(w_1, w_2)$ are related as $t_{00} = t_{11}$ and $t_{10} = 1 + t_{01}$. Now $F(w_1, w_2)$ is

rewritten as

$$F(w_1, w_2) = t_{01}(\cos w_1 + \cos w_2) + t_{11}(1 + \cos w_1 \cos w_2) + \cos w_1. \quad (4)$$

It was shown in [4] that this $F(w_1, w_2)$ satisfies the inequality $|F(w_1, w_2)| \leq 1$ if and only if

$$|t_{11}| \leq min\ 1 + t_{01}, -t_{01}. \quad (5)$$

Note that this inequality holds only for $-1 \leq t_{01} \leq 0$. When (5) is satisfied, $|t_{11}| \leq 0.5$. In the following section, we shall derive simple formulas for fast calculation of t_{01} and t_{11}.

III. Formulas for the McClellan transform parameters

Fig. 1 shows the ideal specifications of a 2-D fan filter. Let $\theta \in (0, \pi/2)$ be the angle between the cutoff frequency line and the w_1-axis. The cutoff line is expressed as $w_2 = rw_1$ where $r = \tan \theta$. For a given θ, our goal is to find the values of t_{01} and t_{11} in (4) and the 1-D cutoff frequency w_c such that the resulting contour corresponding to w_c best approximates the cutoff frequency line. We define a deviation function as

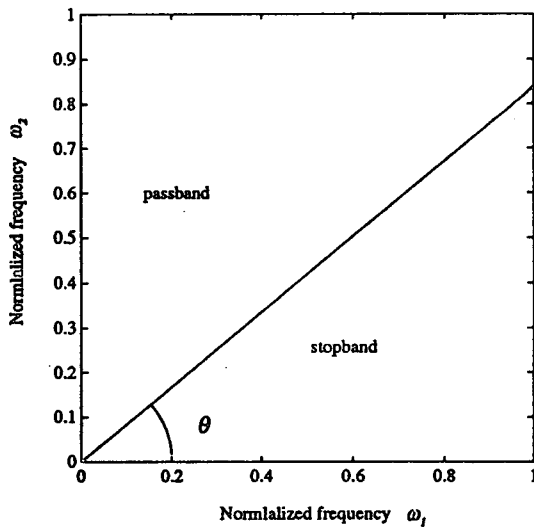$$D(w_1, rw_1, w_c) = F(w_1, rw_1) - \cos w_c. \quad (6)$$

Fig. 1. Specifications of an ideal fan filter within the first quadrant.

Since it is impossible to find $F(w_1, rw_1)$ and w_c that make $D(w_1, rw_1, w_c) = 0$ for all $w_1 \in [0, \pi]$, the following minimization problem is formulated: find t_{01}, t_{11} and w_c minimizing the integral squared error (ISE)

$$J = \int_0^u D^2(w_1, rw_1, w_c)\, dw_1 \quad (7)$$

where $u = \pi$ for $\theta \in (0, \pi/4)$ and $u = \pi/r$ for $\theta \in (\pi/4, \pi/2)$. As shown in [4], it is sufficient to consider only the case where $\theta \in (0, \pi/4)$. This is true because for $\theta \in (\pi/4, \pi/2)$, the parameters are given by $t_{11} = -\hat{t}_{11}$ and $t_{01} = -(1 + \hat{t}_{01})$ where $\hat{t}_{01}$ and $\hat{t}_{11}$ are those computed for $\hat{\theta} = \pi/2 - \theta$. Note that if $\hat{t}_{01}$ and $\hat{t}_{11}$ satisfy the condition in (5), so do t_{01} and t_{11}. The minimization problem in (7) was previously considered in [4], [5], but they modified the problem either by approximating the deviation function [4] or by employing a modified deviation function [5]. In our work, we minimize J in (7) without any modification except that a constraint is imposed on the minimization. In particular, we impose

$$F(0, 0) = \cos w_c. \quad (8)$$

This constraint is justified by the fact that any ideal fan filter cutoff frequency line passes through the origin. Under this constraint, $\cos w_c$ in (6) is replaced by $F(0, 0) = t_{00} + t_{10} + t_{01} + t_{11}$ and J is rewritten as

$$J = \int_0^\pi [1 - \cos w_1 - t_{01}(\cos w_1 + \cos rw_1) \quad (9)$$
$$- t_{11}(\cos w_1 \cos rw_1 - 1)]^2\, dw_1.$$

Using the matrix-vector notation,

$$J = T^t\, C\, T - 2\, T^t\, D + s \quad (10)$$

where t means transposition, $T = [t_{01}, t_{11}]^t$, C is a 2×2 symmetric matrix whose elements are denoted by c_{ij} with

$$c_{11} = 5\pi + [1/(1-r) - 1(1+r) - 4/r]\sin r\pi$$
$$+ (1/(4\,r))\sin 2r\pi$$

$$c_{22} = 5\pi/4 + [1/(1+r) - 1/(1-r)]\sin r\pi$$
$$+ [1/(16(1+r)) - 1/(16(1-r)) + 1/(8r)]\sin 2r\pi$$

$$c_{12} = c_{21}$$
$$= 2\pi + [1/(4(2+r)) - 1/(4(2-r)) + 1/(1+r) \quad (11)$$
$$-1/(1-r) -1/(2r)]\sin r\pi + [1/(4(1-2r))$$
$$-1/(4(1+2r))]\sin 2r\pi,$$

$D = [d_1, d_2]^t$ with

$$d_1 = -5\pi/2 + [1/(2(1+r)) - 1/(2(1-r))$$
$$+ 1r]\sin r\pi$$

$$d_2 = -\pi + [1/(4(2-r)) - 1/(4(2+r)) + 1/(2(1-r)) \quad (12)$$
$$-1/(2(1+r)) -1/(2r)]\sin r\pi$$

and $s = 3\pi/2$. Since J is a quadratic function of T, its minimum occurs at $T = C^{-1}D$. This result leads to the following closed-form formulas for the parameters:

$$t_{01} = \frac{c_{22}d_1 - c_{12}d_2}{c_{11}c_{22} - c_{12}^2}$$

$$t_{11} = \frac{-c_{12}d_1 + c_{11}d_2}{c_{11}c_{22} - c_{12}^2}. \quad (13)$$

From (8),

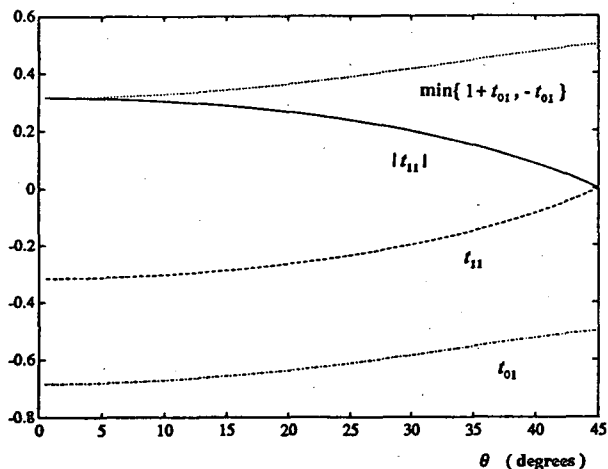$$w_c = \cos^{-1}(t_{00} + t_{10} + t_{01} + t_{11}). \tag{14}$$

Using these formulas, we evaluate t_{01} and t_{11} for $\theta = k/2$ degree, $k = 1,2,\cdots,90$. The results are shown in Fig. 2. This figure also shows $|t_{11}|$ and $\min\{1 + t_{01}, -t_{01}\}$. Note that $\min\{|t_{11}| \le (1 + t_{01}), -t_{01}\}$ for all θ, as desired. Table I lists t_{01}, t_{11} for $\theta = 5, 10, \cdots, 45$ degrees, and the corresponding normalized integral squared error (NISE) J/π. For comparison, NISE values corresponding to the parameter values obtained in [4] and [5] are also shown. It is seen that the derived formulas result in smaller NISE than the others. However, the difference between our NISE values and those obtained from [5] is almost negligible. In fact, t_{01} and t_{11} values in Table I are almost identical to those in [5]. In summary, the proposed method outperforms the approximation method in [4]; it performs like the eigenfilter approach but requires much less computation.

Table 1. McClellan transform parameters and the correspoding NISE for $\theta = 5, 10, \cdots, 45$ degrees.
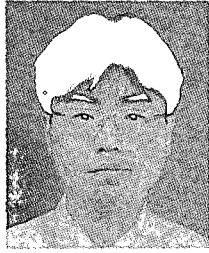
θ	Proposed		NISE (J / π)		
	t_{01}	t_{11}	Proposed	[5]	[4]
5	-0.6807778	-0.3131697	2.0263×10^{-7}	2.1032×10^{-7}	6.5695×10^{-7}
10	-0.6719625	-0.3038498	3.1494×10^{-6}	3.1499×10^{-6}	1.0081×10^{-5}
15	-0.6574929	-0.2881636	1.5122×10^{-5}	1.5125×10^{-5}	4.7360×10^{-5}
20	-0.6377381	-0.2658235	4.3885×10^{-5}	4.3896×10^{-5}	1.3313×10^{-4}
25	-0.6132962	-0.2362771	9.3627×10^{-5}	9.3649×10^{-5}	2.7190×10^{-4}
30	-0.5851066	-0.1984593	1.5573×10^{-4}	1.5575×10^{-4}	4.2653×10^{-4}
35	-0.5546773	-0.1502503	1.9405×10^{-4}	1.9409×10^{-4}	4.9107×10^{-4}
40	-0.524608	-0.0872481	1.3621×10^{-4}	1.3623×10^{-4}	3.0876×10^{-4}
45	-0.5	0.5	0	0	0

Fig. 2. The values of t_{01} and t_{11} obtained using derived formulas.

References

[1] J. H. McClellan, "The design of two-dimensional digital filters by transformation," in *Proc. 7th Annu. Princeton Conf. on Information and Sciences and Systems*, pp. 247-251, 1973.

[2] R. M. Mersereau, W. F. G. Mecklenbrauker, and T. F. Quatieri. Jr., "McClellan transformations for two-dimensional digital filtering: Part I - Design," *IEEE Trans. Circuits and Systems*, vol. CAS-23, pp. 405-422, July, 1976.

[3] J. H. McClellan and D. S. K. Chan, "A 2-D FIR filter structure derived from the Chebyshev recursion," *IEEE Trans. Circuits and Systems*, vol. CAS-24, pp. 372-378, July, 1977.

[4] E. Z. Pasarakis, V. G. Mertzios, and G. Ph. Alexiou, "Design of two-dimensional zero phase FIR fan filters via the McClellan transform," *IEEE Trans. Circuits and Systems*, vol. CAS-37, pp. 10-16, Jan. 1990.

[5] S.-C. Pei and J.-J. Shyu, "Design of 2-D FIR digital filters by McClellan transformation and least squares eigencontour mapping," *IEEE Trans. Circuits and Systems*, vol. CAS-40, pp. 546-555, Sept. 1993.

[6] D. T. Nguyen and M. N. S. Swamy, "Formulas for parameters scaling in the McClellan transform," *IEEE Trans. Circuits and Systems*, vol. CAS-33, pp. 108-109, Jan. 1986.

Young-Seog Song received the B.S. degree in electronical engineering from Seoul National University in 1989 and the M.S. degree in electronical engineering from Korea Advanced Institute of Science and Technology(KLIST) in 1992. He is currently working toward the Ph.D. degree at KAIST. His research interest are in digital signal processing and its applications to digital communications. His current research topic is FIR digital filter design.

Yong Hoon Lee was born in Seoul, Korea, on July 12, 1995. He received the B.S. and M.S. degrees in electrical engineering from Seoul National University, Seoul, Korea, in 1978 and 1980, respectively, and the Ph.D. degree in systems engineering from the University of Pennsylvania, Philadelphia, in 1984. From 1984 to 1988, he was an Assitant Professor at the Department of Electrical and Comupter Engineering, State University of New York at Buffalo. Since 1989, he has been with the Department of Electrical Engineering at the Korea Advanced Institute of Science and Technology, where he is currently a professor. His research activities are in the areas of 1-D and 2-D signal processing.