An Adaptive Algorithm for Array System in the Presence of Faulty Element

Ki M. Kim, Il W. Cha, and Dae H. Youn

Abstract

Element failure occurs with high probability for every array used in the real world ; that is, it is a common phenomenon that there are one or more elements with no output. Element failure produces an elevated sidelobe level and fails to reject the interference signals in an adaptive beamformer. In this paper, we present the adaptive beamforming algorithm for array with element failure. The presented method minimizes the array output power subject to a set of linear constraints which maintain the frequency response in the look direction and force the weights of the inoperative elements to zero. Numerical results have been included.

I. Introduction

Adaptive beamformers have received much interest in the area of sonar, radar, communication, and seismic systems. The adaptive array system involves the weighting of received signals at a sensor array optimally so that the output closely approximates a desired signal from a look direction while minimizing the contributions from interference directions[1,2].

Element failure occurs with high probability for every array used in the real world, that is, that there are one or more elements with no output is a common phenomenon. Ramsdale and Howerton[3] have shown that element failure produces an elevated sidelobe level. Previously reported beamforming techniques for element failure have fallen into five groups the amplitude shading method[4], subaperture processing[5], cross-sensor beamforming[6], and a procedure for estimating the signal at inoperative elements by using those from neighboring elements. However, in these methods beamforming is done with fixed coefficients. For the case where interferences exist, a method for adaptively forming null patterns according to the interference signal incident angles is needed.

This paper presents an effective method to reject the interference signals in the presence of faulty elements. The formulation results in a linearly constrained optimization problem involving a set of linear constraints on the weights. Numerical results have been included.

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II. Formulation

Assume that the incident signals are narrowband in nature. The adaptive beamformer in the narrowband case is shown schematically in Fig. 1, consisting of K omnidirectional equispaced elements. The weight vector $\underline{w}(n)$ and input data vector $\underline{x}(n)$ at the n-th snapshot are defined as, respectively,

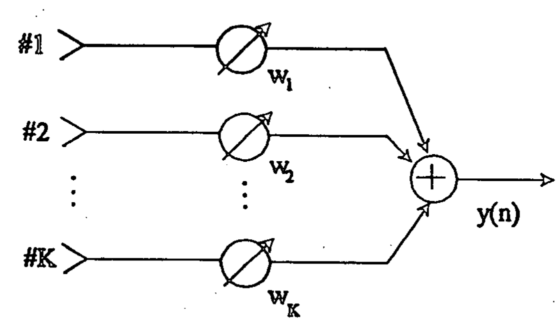

Fig.1. Schematic diagram of a narrowband adaptive beamformer.

$$\underline{w}(n) = [\ w_1(n)\ w_2(n) \cdots w_K(n)]^T \tag{1}$$

and

$$\underline{x}(n) = [\ x_1(n)\ x_2(n) \cdots x_K(n)]^T \tag{2}$$

Superscript T denotes the transpose. For generality both the data vector and the weight vector are assumed complex valued. We assume that the input signals can be modeled as zero mean random processes with unknown second order statistics. The output $y(n)$ can be expressed as the inner product of $\underline{x}(n)$ and $\underline{w}(n)$, that is

$$y(n) = \underline{w}^H(n)\,\underline{x}(n) \tag{3}$$

where H denotes Hermitian transpose. The covariance matrix of $\underline{x}(n)$ is designated as $\underline{R} = E[\,\underline{x}(n)\,\underline{x}^H(n)\,]$. $E[\,\cdot\,]$ denotes the expected value and the superscript H denotes the matrix conjugate transpose. The expected value of the array output power is given by

$$E[\,|y(n)|^2\,] = \underline{w}^H \underline{R}\,\underline{w} \tag{4}$$

We now assume that the array has M faulty elements whose positions are known. The optimum weights in the presence of element failure is the solution to the following constrained optimization problem :

$$Minimize \;\; \underline{w}^H \underline{R}\,\underline{w} \tag{5a}$$

$$\textbf{subject to} \;\; \underline{s}^H \underline{w} = 1 \tag{5b}$$

$$\underline{Q}^H \underline{w} = \underline{0}_M \tag{5c}$$

where $\underline{0}_M$ is an M-dimensional vector of all 0's, and $\underline{s}$ is the K-dimensional desired signal vector given by

$$\underline{s} = [\;1 \;\; \exp(j2\pi f_o \tau) \;\cdots\; \exp(j2\pi(K-1)f_o\tau)\,]^{\,T} \tag{6}$$

where f_o is the center frequency, τ is the propagation delay between the wavefront and the element, and $\underline{Q}$ is the K by M matrix given by

$$\underline{Q} = \begin{bmatrix} 0 & 0 & \cdots \\ 1 & 0 & \cdots \\ 0 & 1 & \cdots \\ \vdots & \vdots & \end{bmatrix} \tag{7}$$

The weights are chosen to minimize the total mean output power as a way of rejecting interferences incident on the array, subject to a set of linear constraints which maintain the frequency response in the look-direction and force the weights of the inoperative elements to zero. The constraints of equation (5b) and (5c) may be as the following expression:

$$\underline{C}^H \underline{w} = \underline{d} \tag{8a}$$

$$\textbf{where} \;\; \underline{C} = \begin{bmatrix} \underline{s} & \underline{Q} \end{bmatrix} \tag{8b}$$

$$\underline{d} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \end{bmatrix} \tag{8c}$$

Here, matrix $\underline{C}$ has the signal vector $\underline{s}$ as the first column, with the remaining columns being the $\underline{Q}$ matrix. Also, $\underline{d}$ is an $(M+1)$ dimensional vector with the first element 1 and the rest zeros. Finding the optimal weight vector $\underline{w}_{opt}$ to satisfy (5a) and (8a) can be accomplished by the method of Lagrange multiplier.

$$\underline{w}_{opt} = \underline{R}^{-1}\,\underline{C}(\,\underline{C}^T\,\underline{R}^{-1}\,\underline{C})^{-1}\,\underline{d} \tag{9}$$

The equation with updated weights can be expressed as

$$\underline{w}(n+1) = \underline{P}[\;\underline{w}(n) - \mu y^*(n)\,\underline{x}(n)] + \underline{w}(0) \tag{10a}$$

$$\textbf{where} \;\; \underline{P} = I_K - \underline{C}(\,\underline{C}^T\,\underline{C})^{-1}\,\underline{C}^T \tag{10b}$$

$$\underline{w}(0) = \underline{C}(\,\underline{C}^T\,\underline{C})^{-1}\,\underline{d} \tag{10c}$$

Here * is the complex conjugate, I_K is the K-dimensional identity matrix, $\underline{w}(0)$ is the initial weight vector, and μ is the step size.

III. Results

The computer simulations are carried out to validate and investigate the performance of the proposed method. We used linearly periodic arrays with 16 elements and half-wavelength interelement spacing. The received signal in each sensor consists of a desired signal and two interferences buried in a white Gaussian noise. The desired signal is assumed to be broadside along the array. The interference signal has an identical center frequency as the desired signal and is assumed to be incident at angles of $-40°$ and $10°$ having input INR (Interference-to-Noise Ratio) of 40 dB and 30 dB, respectively. The input SNR (Signal-to-Noise Ratio) is -27 dB. Also, it is assumed that there is no correlation between the desired and interference signals.

Fig.2 shows beampatterns for a gain-only constrained beamformer and for the proposed method when the 2nd and 5th elements are inoperative. It can be seen that the gain-only constrained beamformer is unable to form a null in the incident direction of the interference signals. On the other hand, the presented method forms null of -65 dB and -55 dB, respectively, for each of the two interference directions. Fig.3 compares the output SINR (Signal-to-Interference plus Noise Ratio) of the proposed method with the case where all the elements are operative. To have a specific problem to study, we shall suppose that a desired signal and one interference signal are incident on the array. The input signals have an input SNR and INR of -30 dB and 30 dB, respectively. The interference incident angle is $30°$. It can be seen that with two inoperative elements, as the number of total elements increases, the output SINR of the proposed method converges to that with all elements operating normally.

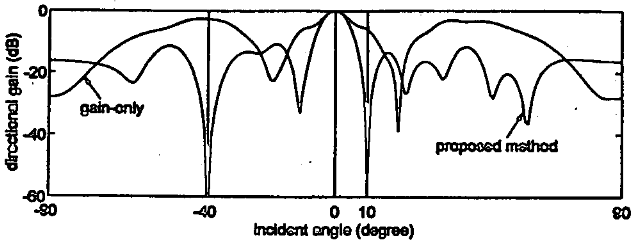

Fig. 2. Beampatterns of the gain-only constrained beamformer and the proposed method.

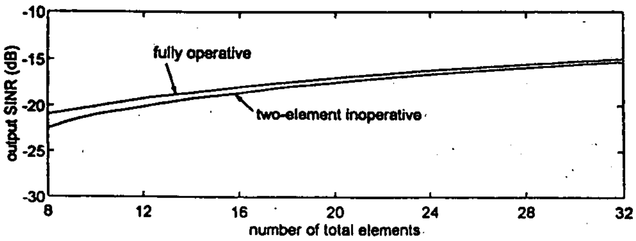

Fig. 3. Output SINR of beamformers in the presence of the 2nd and 5th elements failure.

VI. Conclusions

In this paper, we presented a method for an adaptive array in the presence of faulty elements. The weights are chosen to minimize the total mean output power as a way of rejecting interferences incident on the array, subject to a set of linear constraints which maintain the frequency response in the look-direction and force the weights of the inoperative elements to zero.

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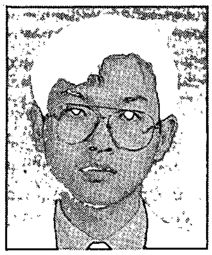

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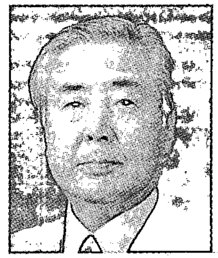

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