Performance of Track Formation of a Two-stage Cascaded Logic in a Cluttered Environment

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클러터가 존재하는 환경에서 2단계 접속 논리의 트랙생성에 대한 성능 분석

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요 약

2 단계 접속 논리(two-stage cascaded logic)는 관측 지역 내에 새로이 출현한 표적에 대한 트랙을 만드는 대표적인 방법 중의 하나이다. 2 단계 접속 논리의 트랙 생성(track formation)에 관한 성능 평가 방법 및 결과는 Bar-Schalom[2]에 의해 발표된 바가 있으나, 그 연구 결과는 트랙 생성 성능을 도출할 때 클러터로 인한 오경보율(false alarm probability)을 무시한다는 가정에 기초한 것이기 때문에, 오경보율이 높은 경우에는 적용할 수 없다는 단점을 지닌다. 이에 본 논문에서는 오경보율을 고려하여 2 단계 접속 논리의 트랙 생성 성능을 평가할 수 있는 개선된 방법을 제시하고자 한다. 그리고 2단계 접속 논리에서 사용하는 데이터 연관(data association) 기법으로 트랙 분리(track splitting) 기법과 최근접 데이터 선택 기법(nearest neighbor rule)을 사용하는 경우에 대하여 각각의 성능 평가 결과를 몇 가지 예시하고자 한다.

1. INTRODUCTION

Track formation is the process of associating several detections over time and deciding that these detections originated from the same target. The common approaches for track formation in a tracking system are an m out of n (m/n) logic[1] and a two-stage cascaded logic sup[2]. The m/n logic initiates a track when m correlative detections occur in n consecutive scans. The two-stage cascaded logic consists of two stages. In the first stage, a 2/2 logic is tested. If the 2/2 logic is satisfied, the second stage starts and waits a new detection to initiate the first stage otherwise. In the second stage, an m/n logic is tested using a validation gate determined by the Kalman filter based on the assumed target and measurement models.

Bar-Schalom *et al*[2]. presented a Markov-chain-based performance evaluation technique for the two-stage cascaded logic($2/2\times m/n$) for track formation in a cluttered environond stage when P_{FA} and P_D vary. Fig. 5 through Fig. 7 show that the limiting value of the cpmf P_{TT} for the case of using the nearest neighbor rule in the second stage is smaller than that for the case of using the track splitting method. This is due to the fact that the nearest neighbor rule is more susceptible to the false alarms than the track splitting method.

Table I shows the average true track formation time T_C of $2/2 \times 1/2$ cascaded logics using the track splitting method both in the first and the second stages. Table II shows the average true track formation time T_C of $2/2 \times 1/2$ cascaded logics using the track splitting method in the first and the nearest neighbor rule in the second stages. The results in Table 1 and 2

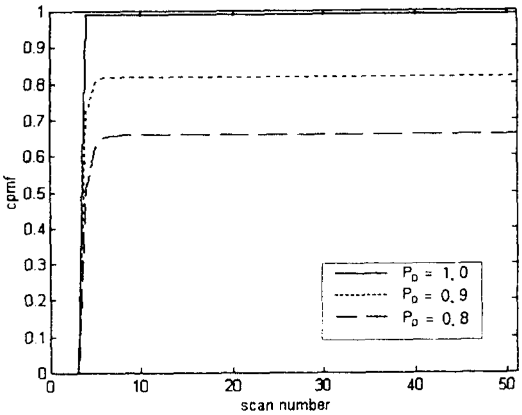

Fig. 4 Cumulative probability mass function of true track formation for the $2/2 \times 1/2$ logic using the track splitting method in the first and the second stages when $P_{FA}=10^{-2}$.

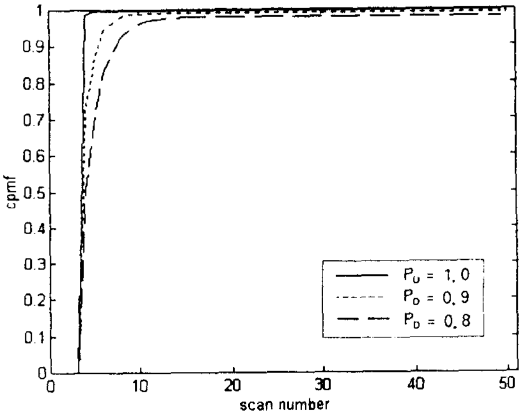

Fig. 6 Cumulative probability mass function of true track formation for the $2/2 \times 1/2$ logic using the track splitting method in the first and the neighbor rule in the second stages when $P_{FA}=10^{-4}$.

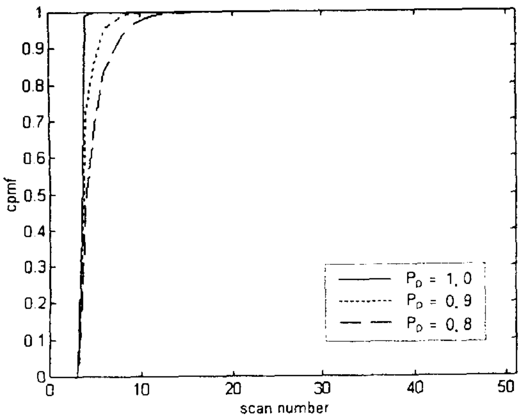

Fig. 5 Cumulative probability mass function of true track formation for the $2/2 \times 1/2$ logic using the track splitting method in the first and the neighbor rule in the second stages when $P_{FA}=0$.

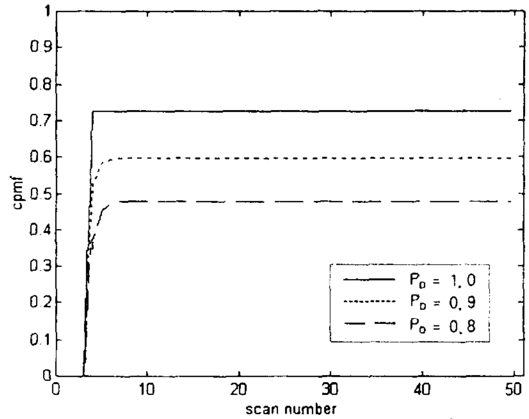

Fig. 7 Cumulative probability mass function of true track formation for the $2/2 \times 1/2$ logic using the track splitting method in the first and the neighbor rule in the second stages when $P_{FA}=10^{-2}$.

stage cascaded logic is in the state i at time k. The probability vector $\vec{\mu}(k) \equiv [\mu_1(k), \cdots, \mu_8(k)]$ propagates according to the following relation :

$$\vec{\mu}(k+1) = \Phi^T \vec{\mu}(k) \qquad (25)$$

where the initial value $\vec{\mu}(o)$ is assumed to be $[1, 0, \cdots, 0]$. Then, for the $2/2 \times 1/2$ cascaded logic, the cumulative probability mass function(cpmf) $P_{TT}(k)$ of true track formation at time k is

$$P_{TT}(k) = \mu_7(k) \qquad (26)$$

Another quantity of interest is the average true track formation time T_C

$$T_C = \frac{\sum_{k=1}^{\infty} k\{P_{TT}(k) - P_{TT}(k-1)\}}{\sum_{k=1}^{\infty} \{P_{TT}(k) - P_{TT}(k-1)\}} \qquad (27)$$

4. RESULTS AND DISCUSSION

Here, using the procedure described in Section 3, we present the performances of $2/2 \times 1/2$ in terms of the cpmf $P_{TT}(k)$ and the average true track formation time T_C.

For performance evaluation, we assume $V(2) = 400$(resolution cells) which follows from the assumption of a maximum velocity of 10 cells/scan in x and y directions. Also we assume that R_x and R_y which are respectively measurement error covariances in x and y coordinates are 1(resolution cell). We assume that the process noise is zero and the normalized gate size $\gamma = 9.21$ guaranteeing $P_G = 0.99$. The sampling interval T used in determining the volume of a gate is assumed to be 1(*sec*).

Fig. 2 through Fig. 4 show the cpmf $P_{TT}(k)$ of true track formation for $2/2 \times 1/2$ cascaded logics using the track splitting method both in the first and the second stages with P_{FA} and P_D varying. The figures show that the cpmf $P_{TT}(k)$ of true track formation increases more rapidly with time as the detection probability P_D increases. They also show that the limiting value of the cpmf $P_{TT}(k)$ becomes below unity when P_{FA} is high. This phenomenon occurs because a true track is unlikely to form at time k larger than three in a dense environment. Fig. 5 through Fig. 7 show the cpmf $P_{TT}(k)$ of true track formation for $2/2 \times 1/2$ cascaded logics using the track splitting method in the first stage and the nearest neighbor rule in the sec-

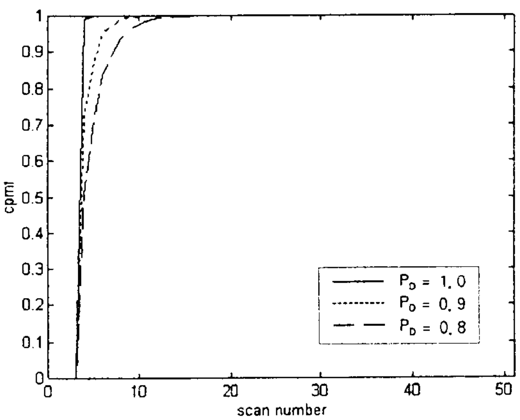

Fig. 2 Cumulative probability mass function of true track formation for the $2/2 \times 1/2$ logic using the track splitting method in the first and the second stages when $P_{FA} = 0$.

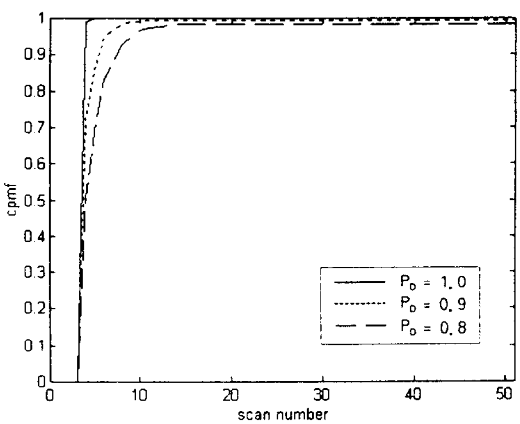

Fig. 3 Cumulative probability mass function of true track formation for the $2/2 \times 1/2$ logic using the track splitting method in the first and the second stages when $P_{FA} = 10^{-4}$.

stage and the nearest neighbor rule in the second stage, we define several quantities related to the nearest neighbor rule. Let $P_{CC|D}$ denote the probability of a correct correlation, given that a target detection occurs. Let P_{NE} be the probability of no correlation, given that a target is not detected. Let P_E be the probability that an incorrect measurement is detected closer to the predicted position than the correct one, conditioned upon that the correct one is within a validation gate. Then the probabilities of the three events(CC, FC, and NC), which are associated with the states 3, 4, 5, and 6 in the second stage, can be expressed as

$$P[CC] = P_D P_{CC|D} \tag{15}$$

$$P[FC] = P_D P_G P_E + (1 - P_D P_G)(1 - P_{NE}) \tag{16}$$

$$P[NC] = (1 - P_D P_G) P_{NE} \tag{17}$$

These probabilities depend on the detection sequence indicator $\delta(k)$.

The innovation $\tilde{y}(k)$ of the Kalman filter at time k is assumed to have an M - dimensional Gaussian distribution $f(\tilde{y}(k))$ with a covariance $S(k)$, i.e.,

$$f(\tilde{y}(k)) = \frac{1}{(2\pi)^{M/2}\sqrt{|S(k)|}} \times \exp\left\{-\frac{1}{2}\tilde{y}(k)^T S^{-1}(k)\tilde{y}(k)\right\} \tag{18}$$

where M denotes the dimension of a measurement and the innovation $\tilde{y}(k)$ is a $M \times 1$ vector. Defining $\tilde{z}(k) \equiv \tilde{y}(k)^T S^{-1}(k)\tilde{y}(k)$, $\tilde{z}(k)$ has an M - dimensional chi - square distribution. That is

$$f(\tilde{z}(k)) = \frac{\tilde{z}(k)^{M/2-1}\exp\left\{-\frac{1}{2}\tilde{z}(k)\right\}}{2^{M/2}\Gamma(M/2)} \tag{19}$$

where $\Gamma(\cdot)$ denotes the gamma function.

Since the number of incorrect measurements was assumed to have a Poisson distribution with density P_{FA} and the nearest neighbor rule is used for data association, $P_{CC|D}$ can be expressed as[3]

$$P_{CC|D} = \int_0^G f(\tilde{z}(k))\exp\{-P_{FA}V(k)\}d\tilde{z}(k) \tag{20}$$

where G is a normalized gate size and $V(k)$ is the volume of an ellipsoidal gate of a normalized gate size $\tilde{z}(k)$ at time k. In this paper, since we consider a target moving in two - dimensional space($M=2$), $P_{CC|D}$ becomes

$$\begin{aligned} P_{CC|D} &= \int_0^G \frac{\exp\left\{-\frac{1}{2}\tilde{z}(k)\right\}}{2\Gamma(1)} \times \exp\{-P_{FA}C_2\sqrt{|S(k)|}\tilde{z}(k)\}d\tilde{z}(k) \\ &= \frac{1-\exp\left\{-\left(\frac{1}{2}+P_{FA}C_2\sqrt{|S(k)|}\right)G\right\}}{1+2P_{FA}C_2\sqrt{|S(k)|}} \end{aligned} \tag{21}$$

According to the definition of P_{NE} and (12), P_{NE} for an ellipsoidal gate of a normalized size G becomes

$$P_{NE} = \exp\{-P_{FA}C_2\sqrt{|S(k)|}G\} \tag{22}$$

Since

$$P_{CC|D} = P_G(1-P_E) \tag{23}$$

P_E can be expressed in terms of $P_{CC|D}$ and P_G as follows

$$\begin{aligned} P_E &= 1 - \frac{P_{CC|D}}{P_G} \\ &= 1 - \frac{1-\exp\left\{-\left(\frac{1}{2}+P_{FA}C_2\sqrt{|S(k)|}\right)G\right\}}{P_G\{1+2P_{FA}C_2\sqrt{|S(k)|}\}} \end{aligned} \tag{24}$$

Until now, we have illustrated the procedure for modeling the operation of the 2/2 × 1/2 cascaded logic by a Markov chain. In case of $P_{FA}=0$, the derived Markov - chain model is reduced to that in the reference 2), which confirms the correctness of the technique presented in this paper.

Let $\mu_i(k)$ be the probability that the two -

transition probabilities must be defined. If we ignore the effect of clutter, the state transition probabilities can be expressed with only the detection probability P_D and the gating probability P_G[2]. However, if we consider the effect of clutter, the state transition probabilities are dependent upon the size of a validation gate, the false alarm probability P_{FA}, a correlation method in addition to P_D and P_G.

Let Φ be the state transition matrix associated with the Markov chain. Let the (i, j) component $\phi_{i,j}$ of Φ be the state transition probability from state i to state j. Determination of the state transition matrix Φ depends on the data association technique. We first consider the case of employing the track splitting method[4] in the first and the second stages.

There are two events(TD and NTD) associated with the state 1. Since the probabilities of the two events are related to only the detection probability P_D

$$\phi_{1,1} = 1 - P_D, \quad \phi_{1,2} = P_D \tag{8}$$

There are three events(CC, FC, and NC) associated with the states 2, 3, 4, 5, and 6. Since the number of incorrect measurements was assumed to have a Poisson distribution with density P_{FA} and the track splitting method is employed for data association, the probabilities of the three events(CC, FC, and NC) can be expressed as

$$P[CC] = P_D P_G \tag{9}$$

$$P[FC] = (1 - P_D P_G)(1 - \exp(-P_{FA} V(k))) \tag{10}$$

$$P[NC] = (1 - P_D P_G)\exp(-P_{FA} V(k)) \tag{11}$$

where $V(k)$ denotes the volume of a validation gate at time k. Since $V(2)$ is usually taken to be large enough to contain almost a target detection, P_G is almost equal to unity. The second stage is assumed to use an ellipsoidal gate for data association. In general, the volume $V(k)$ for an ellipsoidal gate of a normalized gate size G at time k is related to the innovation covariance $S(k)$ of the Kalman filter as follows : [3]

$$V(k) = C_M \sqrt{|S(k)|}\, G^{M/2} \tag{12}$$

where C_M is

$$C_M = \frac{\pi^{M/2}}{\Gamma(M/2+1)} = \begin{cases} \dfrac{\pi^{M/2}}{(M/2)!} & \text{for an even M} \\ \dfrac{2^{M+1}\{(M+1)/2\}!\,\pi^{(M-1)/2}}{(M+1)!} & \text{for an odd M.} \end{cases} \tag{13}$$

The states 7 and 8 are terminating states. Accordingly

$$\phi_{7,7} = \phi_{8,8} = 1. \tag{14}$$

The state transition probabilities without explicit assignment of specific values are assumed to be zero.

In case of using the track splitting method in the first and the second stages, the number of initiated tracks can be considerable in a heavily cluttered environment, which increases the computational load required for multiple target tracking. As a solution to this problem, we may consider the case of using the tracking splitting method in the first stage and the nearest neighbor rule in the second stage. Since this solution uses the nearest neighbor rule for data association in the second stage, the number of initiated tracks is small compared with the case of using the track splitting method. However, this solution may eliminate potential true tracks.

If the second stage employs the nearest neighbor rule for data association instead of the track splitting method, the associated state transition probabilities changes. To express the state transition probabilities $\phi_{i,j}$ for the case of using the track splitting method in the first where $P(k \mid k-1)$ is the prediction error covariance and $K(k)$ is the Kalman gain at time k. The Kalman gain $K(k)$ at time k is given by

$$K(k)=\delta(k)P(k \mid k-1)H^T S(k)^{-1} \qquad (7)$$

where the detection indicator $\delta(k)$ is 1 if there is a detection in the validation gate at time k and 0 otherwise. Therefore the innovation covariance $S(k)$ used to determine the size of the validation gate depends on the time sequence of $\delta(k)$ as well as the target and the measurement models.

3. TRACK FORMATION IN A CLUTTERED ENVIRONMENT

A Markov - chain technique has been used for performance evaluation of logic - based track formation approaches[1,2]. In this paper, we employ the Markov - chain technique as a tool for performance evaluation of the two - stage cascaded logic.

Here we illustrate the procedure for performance evaluation of a general two - stage cascaded logic for true track formation in a cluttered environment by presenting the procedure for evaluating the performance of a $2/2 \times 1/2$ cascaded logic which is a simple example of the two - stage cascaded logic.

The operation of a $2/2 \times 1/2$ cascaded logic can be represented by a Markov chain with 8 states as shown in Fig. 1 and its states are defined as

(1) Initial state ; *TD* → 2, *NTD* → 1
(2) $\delta(2)=[1]$; *CC* → 3, *FC* → 4, *NC* → 1
(3) $\delta(3)=[1\ 1]$; *CC* → 7a, *FC* → 8a, *NC* → 5
(4) $\delta(4)=[1\ \bar{1}]$; *CC* → 8b, *FC* → 8c, *NC* → 6
(5) $\delta(5)=[1\ 1\ 0]$; *CC* → 7b, *FC* → 8d, *NC* → 1
(6) $\delta(6)=[1\ \bar{1}\ 0]$; *CC* → 8e, *FC* → 8f, *NC* → 1
(7a) $\delta(7a)=[1\ 1\ 1]$; terminate
(7b) $\delta(7b)=[1\ 1\ 0\ 1]$; terminate
(8a) $\delta(8a)=[1\ 1\ \bar{1}]$; terminate
(8b) $\delta(8b)=[1\ \bar{1}\ 1]$; terminate
(8c) $\delta(8c)=[1\ \bar{1}\ \bar{1}]$; terminate
(8d) $\delta(8d)=[1\ 1\ 0\ \bar{1}]$; terminate
(8e) $\delta(8e)=[1\ \bar{1}\ 0\ 1]$; terminate
(8f) $\delta(8f)=[1\ \bar{1}\ 0\ \bar{1}]$; terminate

The two states 7a - b represent the status of true track initiation and the six states 8a - f the status of false track initiation. Each state is characterized by the corresponding $\delta(k)$ which is a sequence of the detection indicator $\delta(k)$ from initial to time k. 1 and $\bar{1}$, which are possible values of $\delta(k)$, represent detection of a correct measurement and an incorrect one, respectively. The notation *event* → *number* means that if an *event* occurs, go to the *number* - th state. *TD* denotes the event of a target detection and *NTD* the event of no target detection. *CC* indicates an event that a correct measurement is selected, *FC* an event that an incorrect measurement is selected, and *NC* an event that no measurement occurs in a validation gate.

For the Markov chain to be complete, state

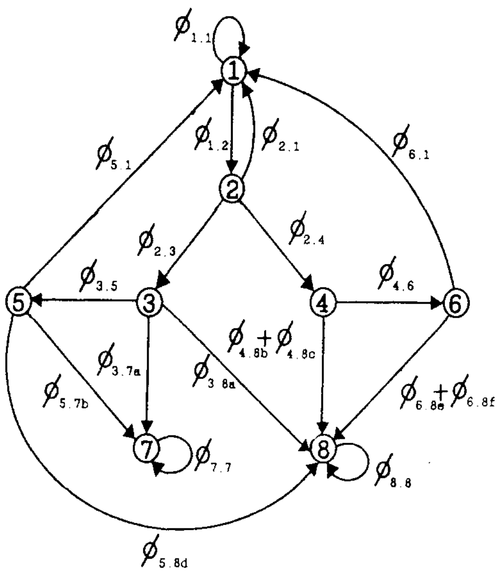

Fig. 1 Markov chain model for the $2/2 \times 1/2$ cascaded logic

ment. However, the technique did not consider the effect of clutter in evaluating the performance of the two - stage cascaded logic for true track formation in clutter.

In this paper, we present a technique for evaluating the performance of the two - stage cascaded logic for true track formation in a cluttered environment taking the effects of clutter into account. Data association technique is necessary for track formation in a cluttered environment, since observations more than one may be within the validation gate. Two cases of employing the track splitting method[4] and the nearest neighbor rule are considered in evaluating the performance of the two - stage cascaded logic for true track formation in a cluttered environment.

Following this section, we describe the assumed target and measurement models in Section 2. In Section 3, we illustrate the proposed procedure for evaluating the performance of the two - stage cascaded logic for true track formation in clutter. Finally, in Section 4, we conclude with presenting the performances of $2/2\times1/2$ as an example of performance evaluation.

2. TARGET AND MEASUREMENT MODELS

In the two - stage cascaded logic, a validation gate is determined in a different way according to the associated stage. In the first stage, the size of the validation gate is determined by the maximum target maneuverability. In the second stage, it is determined by the Kalman filter based on a target and a measurement models. Therefore it is necessary to assume the target and the measurement models for performance evaluation of the two - stage cascaded logic.

We assume that a target moves in the two - dimensional space and the model for the two - dimensional motion can be decomposed into two independent and identical one - dimensional motions, i.e., motions in x and y coordinates. For each coordinate, the motion of a target is assumed to be modeled by

$$x(k+1)=Fx(k)+Gw(k) \tag{1}$$

where $x(k)$ is a 2×1 state vector consisting of position and velocity ; $w(k)$ is a 1×1 white Gaussian noise vector with zero mean and covariance Q ; F and

$$F=\begin{bmatrix}1 & T\\ 1 & 1\end{bmatrix},\quad G=\begin{bmatrix}T^2/2\\ T\end{bmatrix} \tag{2}$$

A correct measurement which is defined as one from a target of interest is assumed to be modeled by

$$z(k+1)=Hx(k+1)+v(k+1) \tag{3}$$

where $z(k)$ is a 1×1 measurement vector at time k ; $v(k)$ is a 1×1 white Gaussian vector with zero mean and covariance R ; H is

$$H=[1\ \ 0] \tag{4}$$

An incorrect measurement which is defined as one from clutter is assumed to be uniformly distributed, and the number of incorrect measurements is assumed to have a Poisson distribution with density P_{FA}. Then the probability P_n that there are n incorrect measurements within a volume V per unit time is

$$P_n=\frac{(P_{FA}V)^n}{n!}\exp(-P_{FA}V). \tag{5}$$

For the state - space model (1) and (3), the innovation covariance $S(k+1)$ of the Kalman filter at time $k+1$ is

$$S(k+1)=HF(I-K(k)H)P(k\,|\,k-1)F^TH^T +HQH^T+R \tag{6}$$

Table 1. Average true track formation time for 2/2 × 1/2 logic using the track splitting method in the first and the second stages

P_D	P_{FA}		
	0	10^{-1}	10^{-2}
1.0	3.0103	3.0101	3.0018
0.9	3.4962	3.4814	3.1340
0.8	4.2023	4.1586	3.2942

Table 2. Average true track formation time for 2/2 × 1/2 logic using the track splitting method in the first stage and the nearest neighbor rule in the second stage

P_D	P_{FA}		
	0	10^{-1}	10^{-2}
1.0	3.0103	3.0101	3.0013
0.9	3.4962	3.4810	3.1290
0.8	4.2023	4.1580	3.2850

indicate that T_C gets smaller as P_D or P_{FA} becomes higher. As P_D becomes higher, T_C gets smaller since a true track is expected to be formed more earlier. Similarly, for high P_{FA}, the probability that a true track is formed beyond three scans is very small, which explains the result that T_C gets smaller as P_{FA} becomes higher.

5. CONCLUSIONS

The previous work[2] on performance evaluation of the two - stage cascaded logic did not take the effects of clutter into account. In this paper, we presented a technique for evaluating the performance of the two - stage cascaded logic for true track formation in a cluttered environment taking the effects of clutter into account. Two cases of employing the track splitting method and the nearest neighbor rule for data association were considered in evaluating the performance of the two - stage cascaded logic for true track formation in a cluttered environment.

The presented technique is computationally efficient since it can predict the performance of a two - stage cascaded logic without the Monte Carlo simulation and can be utilized in selecting a two - stage cascaded logic satisfying the requirement of a given radar tracking system.

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