The Resonance Frequency of Sound Channel in Shallow Water with a Thermocline

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Abstract

In shallow water with a thermocline, the characteristics of sound propagation strongly depend on the signal frequency. When only one of the source and the receiver is above the thermocline, it is known that the intensity of the received signal changes largely and almost periodically as the signal frequency varies. This is the so-called channel resonance. By using the ray-mode approach, the formula relating the resonance frequency and the sound speed profile is obtained, and the resonance phenomenon is analyzed. Also this analysis is verified by numerical calculation.

Ⅰ. Introduction

In summer, a thermocline often occurs in shallow water, and the water body can be divided into three layers. In the upper and the lower layers the sound speed changes little as the water depth increases, while it decreases sharply in the middle layer. In this kind of environment, the characteristics of sound propagation will not only vary with the positions of the source and the receiver, they will also strongly depend on the signal frequency. When only one of the source and the receiver is in the upper layer, the intensity of the received signal will change largely and almost periodically as the frequency varies. This is the so-called channel resonance.

The resonance phenomenon was first discussed by Jensen [1]. Using a normal mode code and assuming that the sea water consisted of two iso-speed layers, he calculated the incoherent sound transmission loss at range 30km and frequency up to 1000Hz. The result showed that the transmission loss changed up to 15dB, and the resonance frequency was about 85Hz.

In fact, this kind of phenomenon had been discovered in the field experiment even more earlier by Zhang et al [2]. Using the explosive detonating at depth 7m as the source, they found that the received signal was approximately comb-like structure, which meant that the signal was of periodic structure in the frequency domain. Zhu et al. [3] simulated the wide-band sound propagation in shallow water with an ideal thermocline by the ray-mode approach. They found that the resonance frequency of the transmission loss change was about 128Hz, and the period of the received pulse waveform was about 7.8ms. These coincided with the experimental results by Zhang et al [2].

Although the resonance phenomenon was discovered earlier, the reason has not yet been explained clearly, especially the relation between the resonance frequency and the sound speed profile. By using the ray-mode approach, these problems are dealt with in this paper. In section Ⅱ, the mode theory and the ray-mode approach are briefly expressed. In section Ⅲ, assuming the ocean channel consists of two homogeneous water layers overlying a fluid bottom with half-space, we analyze the sound propagation, explain why resonance occurs, and obtain a formula relating the resonance frequency and the sound speed profile. In section Ⅳ, we consider a more realistic ocean model, i.e., three water layers overlying a fluid bottom. The upper and the lower water layers are homogeneous, and in the middle layer, the square of the sound speed decreases linearly as the depth increases. The resonance phenomenon in this environment is discussed. In section Ⅴ, some numerical results are given to support our analysis. And the summary is given in section Ⅵ.

Ⅱ. Background

The Holmholtz equation describing sound propagation in a layered medium resulting from a harmonic point source of angular frequency ω and unit strength at range $r=0$ and depth $z=z_s$ is given by

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$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right)+\rho\frac{\partial}{\partial z}\left(\frac{1}{\rho}\frac{\partial p}{\partial z}\right)+k^2 p$$
$$=-\frac{\delta(r)\delta(z-z_s)}{2\pi r}, \qquad (1)$$

where δ is the Dirac delta function, p is the complex pressure (the time-related factor has been omitted), ρ is the medium density, and $k=\omega/c=2\pi/\lambda$ is the wavenumber, c is the sound speed, λ is the wavelength.

At long range, the solution of Eq. (1) can be written as

$$p(r,z)=\frac{i}{\rho(z_s)\sqrt{8\pi r}}\, e^{-i\pi/4}\sum_l \Psi_l(z_s)\Psi_l(z)\,\frac{e^{ik_l r}}{\sqrt{k_l}}, \qquad (2)$$

where $i=\sqrt{-1}$, k_l and Ψ_l are respectively the eigenvalues and eigenfunctions of the depth-separated equation

$$\rho(z)\frac{d}{dz}\left(\frac{1}{\rho(z)}\frac{d\Psi(z)}{dz}\right)+[k^2(z)-k_l^2]\Psi(z)=0,$$
$$l=1,2,\cdots \qquad (3)$$

with corresponding boundary conditions.

Following Brekhovskikh [4], we consider a homogeneous thin layer with wavenumber k and thickness h in the medium, and assume that there are two plane waves $e^{ik_l r \pm i\sqrt{k^2-k_l^2}z}$ in this layer (where r represents the horizontal coordinate), V_s is the reflection coefficient of the upper boundary for the wave upward incident, and V_b is the reflection coefficient of the lower boundary for the wave downward incident. Then, the eigenvalue k_l should satisfy

$$1-V_s(k_l)V_b(k_l)\,e^{i2\sqrt{k^2-k_l^2}h}=0. \qquad (4)$$

In the ray-mode approach, the eigenvalues are obtained by solving Eq. (4). In this sense, every mode has its corresponding ray (eigenray). Eq. (4) serves as a bridge between the mode and the ray theories, and forms the basis of the ray-mode approach.

It is often used the incoherent transmission loss defined by

$$TL(r,z)=-20\log\frac{1}{\rho(z_s)}\sqrt{\frac{2\pi}{r}}\sqrt{\sum_l\left|\Psi_l(z_s)\Psi_l(z)\frac{e^{ik_l r}}{\sqrt{k_l}}\right|^2}. \qquad (5)$$

This expresses the transmission loss of the incoherent sound intensity, i.e., the range-averaged sound intensity.

Ⅲ. Resonance in an idealized shallow water waveguide

For simplicity, we assume that the ocean channel consists of two homogeneous water layers overlying a fluid bottom with half-space, as shown in Fig. 1. For the upper layer, the thickness is h_0, the sound speed is c_0, and for the lower one, h_1 and c_1, respectively. In the bottom, sound speed is c_b, attenuation is α, and the density is ρ_b ($c_1<c_0<c_b$, $h_0\gg\lambda$, $h_1\gg\lambda$, and the density of the sea water is 1g/cm^3).

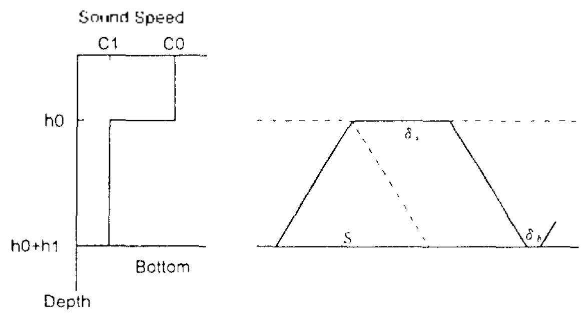


Figure 1. Shallow water waveguide consisting of two homogeneous water layers overlying a fluid bottom.

We note $k_0=\omega/c_0$, $k_1=\omega/c_1$, $k_b=k_{b0}(1+i\alpha)$, $k_{b0}=\omega/c_b$, $V_s(k_l)=e^{i\phi_s(k_l)}$, $V_b(k_l)=|V_b|\,e^{i\phi_b(k_l)}$, and $k_l=\mu_l+i\beta_l$, where, V_s and V_b are the reflection coefficients respectively at the interface of the two water layers and at the sea bottom, and μ_l is the horizontal wavenumber of the l'th mode, and β_l is its attenuation. Then Eq. (4) can be written as [5, 6]

$$2h_1\sqrt{k_1^2-\mu_l^2}+\phi_s(\mu_l)+\phi_b(\mu_l)=2l\pi, \qquad (6)$$

$$\beta_l=\frac{-\ln|V_b(\mu_l)|}{S(\mu_l)+\delta_s(\mu_l)+\delta_b(\mu_l)}. \qquad (7)$$

Here,

$$\phi_s(\mu)=\begin{cases}-\pi+2\arctan\left[\dfrac{\sqrt{k_1^2-\mu^2}}{\sqrt{\mu^2-k_0^2}}\tan(h_0\sqrt{\mu^2-k_0^2})\right], & \text{if } k_0\le\mu,\\[2ex] -\pi+2\arctan\left[\dfrac{\sqrt{k_1^2-\mu^2}}{\sqrt{k_0^2-\mu^2}}\tan(h_0\sqrt{k_0^2-\mu^2})\right], & \text{if } k_0\ge\mu.\end{cases} \qquad (8)$$

and

$$|V_b|\,e^{i\phi_b}=\frac{\rho_b\sqrt{k_1^2-\mu^2}-i\sqrt{\mu^2-k_{b0}^2(1-\alpha^2)-i2\alpha k_{b0}^2}}{\rho_b\sqrt{k_1^2-\mu^2}+i\sqrt{\mu^2-k_{b0}^2(1-\alpha^2)-i2\alpha k_{b0}^2}}. \qquad (9)$$

In addition, as shown in Fig. 1,

$$S(\mu)=\frac{2h_1\mu}{\sqrt{k_1^2-\mu^2}} \qquad (10)$$

is the horizontal distance which the corresponding ray travels in the lower water layer between two consecutive reflection from the bottom,

$$\delta_s(\mu) = -\frac{\partial \phi_s(\mu)}{\partial \mu} \tag{11}$$

is the displacement for the ray reflecting from the interface between the two water layers, and

$$\delta_b(\mu) = -\frac{\partial \phi_b(\mu)}{\partial \mu} \tag{12}$$

is the displacement for the ray reflecting from the bottom.

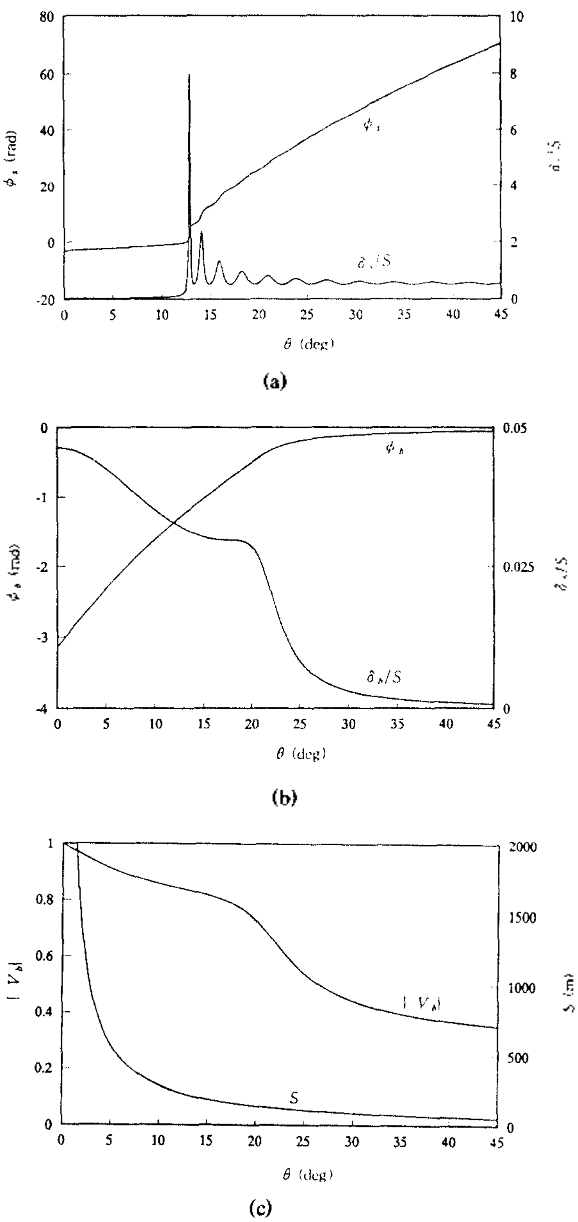


Figure 2. The curves of ϕ_s, ϕ_b, $|V_b|$, S, δ_s/S and δ_b/S, at sound frequency 1000Hz, as functions of the grazing angle θ. (a) ϕ_s and δ_s/S; (b) ϕ_b and δ_b/S; (c) S and $|V_b|$.

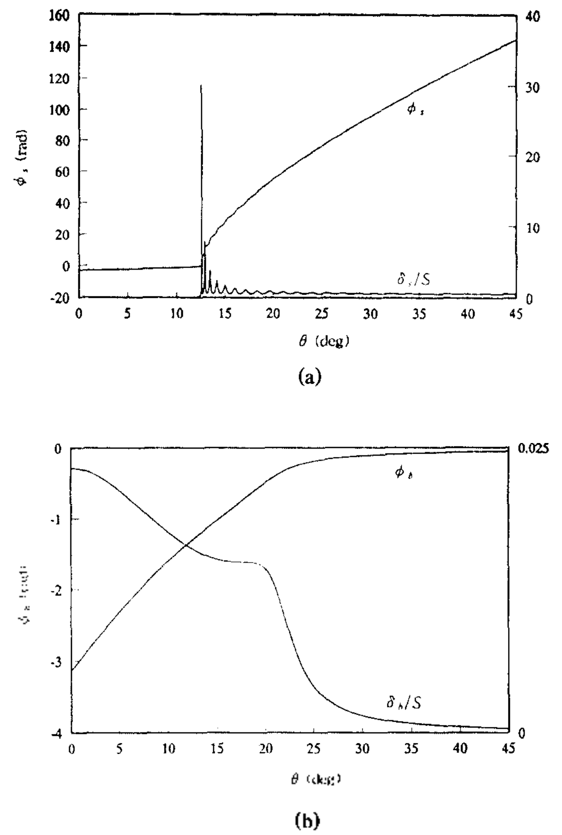


Figure 3. The curves of ϕ_s, ϕ_b, δ_s/S and δ_b/S, at sound frequency 2000Hz, as functions of the grazing angle θ. (a) ϕ_s and δ_s/S; (b) ϕ_b and δ_b/S.

For $c_0 = 1528$m/s, $c_1 = 1492$m/s, $h_0 = 13$m, $h_1 = 25$m, $c_b = 1600$m/s, $\alpha = 0.0183$dB/λ, and $\rho_b = 1.8$g/cm^3, Fig. 2 shows the curves of ϕ_s, ϕ_b, $|V_b|$, S, δ_s/S, and δ_b/S, at sound frequency 1000Hz, as functions of θ, where θ=arccos (μ/k_1) is the grazing angle of the corresponding ray in the lower water layer. Fig. 3 shows the curves of ϕ_s, ϕ_b, δ_s/S, and δ_b/S, at sound frequency 2000Hz. In fact, the values of ϕ_b, $|V_b|$, and S are frequency-independent.

When only one of the source and the receiver is in the upper water layer, the sound field will be dominated by the higher-order modes with $\mu_l < k_0$, and will not be affected by the lower-order modes with $\mu_l > k_0$ because the eigenfunction Ψ_l almost vanishes in the upper layer. Before discussing the contribution of the higher modes, we need to analyze the modal attenuation. From Eq. (7), β_l is related to $|V_b|$, S, δ_s, and δ_b. Of all these, δ_s varies largely as μ or ω changes, which can be seen from Fig. 2 and Fig. 3. So there will be obvious differences between modal attenuation values for different modes and frequencies. For some frequencies, there exists a mode, whose horizontal wavenumber μ_l is the maximum point of $\delta_s(\mu)$

and attenuation thus is much less than usual. At long range, the sound transmission losses for those frequencies will be much less than the others. Therefore, the resonance phenomenon occurs.

If both the source and the receiver are in the lower water layer, the sound field will be dominated by the lower-order modes. For these modes, δ_s and thus β_l don't change evidently as ω changes. So the resonance doesn't occur.

Below we will discuss the relation between the resonance frequency and the sound speed profile. We assume that the incoherent sound intensity reaches an apex at angular frequency ω_0, and the horizontal wavenumber $\mu_m(\omega_0)$ of the m'th mode corresponds to the maximum of $\delta_s(\mu, \omega_0)$. From Eqs. (8) and (11), solving

$$\frac{d\,\delta_s(\mu)}{d\mu}=0,$$

we get that

$$\mu_m \approx k_0-\frac{\pi}{2h_0k_0}, \tag{13}$$

and

$$\phi_s(\mu_m) \approx \pi. \tag{14}$$

Thus, Eq. (6) can be written as

$$2\omega_0 h_1\sqrt{\frac{1}{c_1^2}-\frac{1}{c_0^2}}+\pi+\phi_b(k_0)\approx 2m\pi, \tag{15}$$

This gives the frequency corresponding to an apex of the incoherent sound intensity, that is, the resonance frequency. Now we increase the sound frequency to ω_1, so the incoherent sound intensity reaches another apex. Then the horizontal wavenumber $\mu_{m+1}(\omega_1)$ of the (m+1)'th mode corresponds to the maximum of $\delta_s(\mu, \omega_1)$. Following the same procedures as above, we obtain

$$2\omega_1 h_1\sqrt{\frac{1}{c_1^2}-\frac{1}{c_0^2}}+\pi+\phi_b(k_0)\approx 2(m+1)\pi, \tag{16}$$

By subtracting Eq. (15) from Eq. (16), it yields

$$\omega_1-\omega_0\approx\frac{\pi}{h_1\sqrt{\frac{1}{c_1^2}-\frac{1}{c_0^2}}}.$$

Thus, the resonance frequency

$$\Delta f\approx\frac{1}{2h_1\sqrt{\frac{1}{c_1^2}-\frac{1}{c_0^2}}}. \tag{17}$$

From Eq. (17), it can be seen that the resonance frequency is mainly related to the sound speed values of the two water layers and the thickness of the lower water layer. From Eqs. (8) and (11), however, the thickness of the upper layer partially determines the maximum value of δ_s, thus is related to the resonance magnitude. The incoherent transmission loss changes furthermore as the thickness of the upper layer increases.

Ⅳ. Resonance in shallow water with a thermocline

Now we assume the ocean channel consists of three water layers overlying a fluid bottom, as shown in Fig. 4. The upper and lower water layers are homogeneous. The sound speed and the thickness are c_0 and h_0 for the upper layer, and c_1 and h_1 for the lower layer. In the middle layer with thickness d, the square of the sound speed decreases linearly as the depth increases.

For this ocean model, the eigenvalues are still determined by Eqs. (6) and (7). Here ϕ_s is the phase of $V_s(z=h_0+d)$, the reflection coefficient at the interface of the middle and the lower water layer. Although the closed form expression for $V_s(z=h_0+d)$ can be obtained, it is too complicated and prevents us from deriving useful results. Here we calculate $V_s(z=h_0+d)$ by using the global matrix approach [7].

Taking $d=5$m and other parameters be the same as in Section Ⅲ, Fig. 5 and Fig. 6 show curves of ϕ_s and δ_s/S versus the grazing angle, at sound frequency 1000Hz and 2000Hz, respectively. The values of ϕ_b, $|V_b|$, S, and δ_b/S are the same as in Fig. 2 and Fig. 3.

From Fig. 5 and Fig. 6, we see that δ_s will also change evidently versus μ and ω. For some frequencies, there will

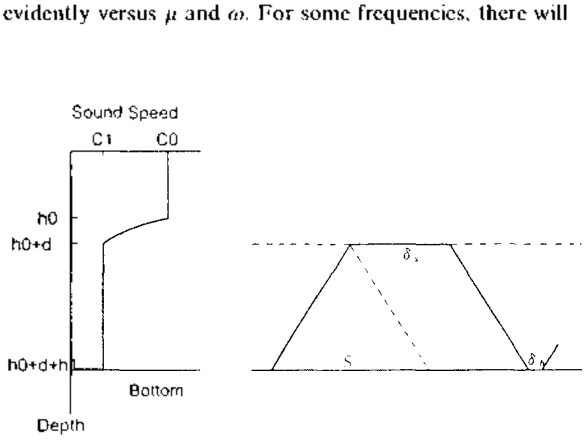


Figure 4. Shallow water waveguide consisting of three water layers overlying a fluid bottom.

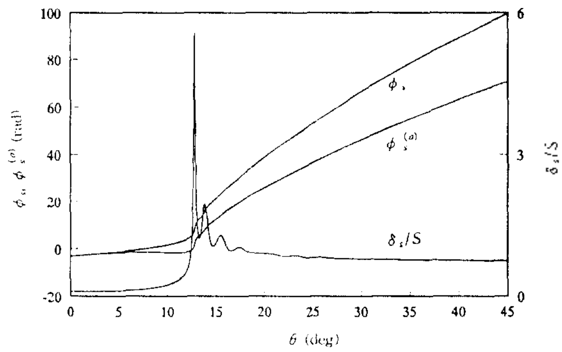


Figure 5. The curves of ϕ_s, $\phi_s^{(a)}$ and δ_s/S, at sound frequency 1000Hz, as functions of the grazing angle θ.

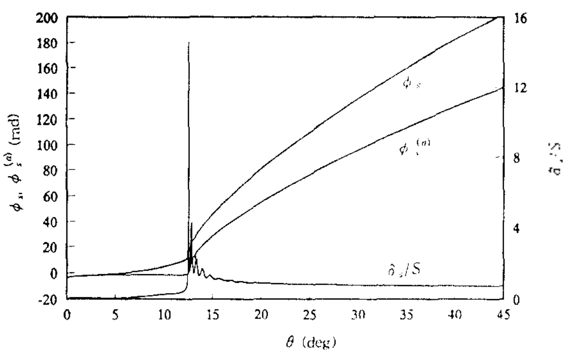


Figure 6. The curves of ϕ_s, $\phi_s^{(a)}$, and δ_s/S, at sound frequency 2000Hz, as functions of the grazing angle θ.

again exist a mode with $\mu_l < k_0$ and attenuation much less than usual, as in the case of the idealized ocean model in Section Ⅲ. Thus the resonance phenomenon occurs.

Below we calculate the resonance frequency. We write

$$\phi_s(\mu) = \phi_s^{(r)}(\mu) + \phi_s^{(a)}(\mu) \tag{18}$$

$$\text{where, } \phi_s^{(a)}(\mu) = \begin{cases} \dfrac{4d}{3}\dfrac{(k_1^2-\mu^2)^{3/2}}{k_1^2-k_0^2}, & \mu > k_0 \\ \dfrac{4d}{3}\dfrac{(k_1^2-\mu^2)^{3/2}-(k_0^2-\mu^2)^{3/2}}{k_1^2-k_0^2}, & \mu < k_0 \end{cases}$$

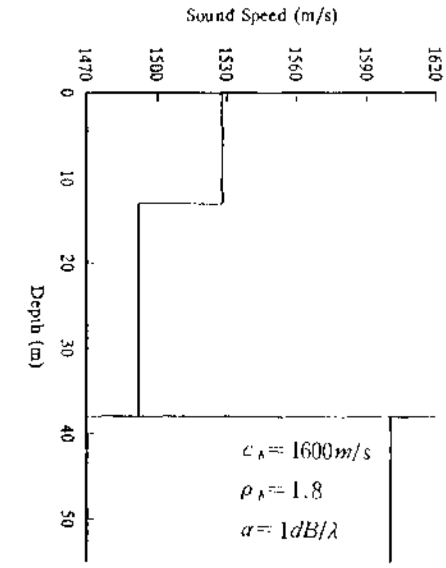
is the phase shift for the corresponding ray traveling up and down in the middle water layer, as given by ray theory [3]. Thus Eq. (6) becomes

$$2h_1\sqrt{k_1^2-\mu_l^2} + \phi_s^{(r)}(\mu_l) + \phi_s^{(a)}(\mu_l) + \phi_b(\mu_l) = 2l\pi. \tag{19}$$

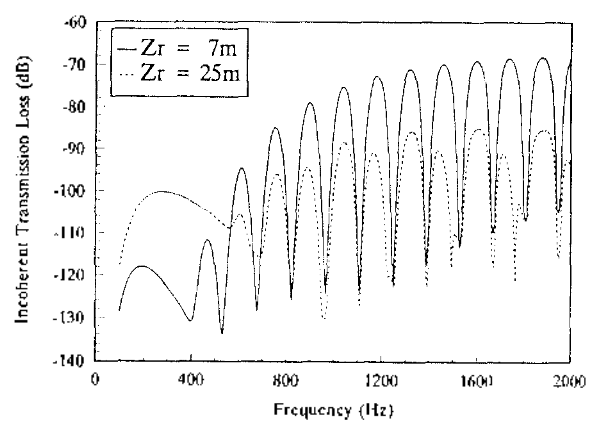
Noticing that, when the sound frequency changes, the value of $\phi_s^{(a)}(\mu)$ corresponding to the maximum point of $\delta_s(\mu)$ changes little (as shown in Fig. 5 and Fig. 6), we get, following the same procedures as in Section Ⅲ, the resonance frequency

$$\Delta f \approx \frac{1}{2\left(h_1 + \dfrac{2}{3}d\right)\sqrt{\dfrac{1}{c_1^2} - \dfrac{1}{c_0^2}}}. \tag{20}$$

In this case, the resonance frequency is determined mainly by the sound speed values of the upper and the lower water layers, and the thickness values of the middle and the lower water layers.



(a)



(b)

Figure 7. Incoherent transmission losses versus frequency in an idealized shallow water waveguide. (a) Sound speed profile. (b) Incoherent transmission losses for a range of 20km, a source of 7m depth, a receiver of 7m depth (solid line) and a receiver of 25m depth (dotted line).

V. Numerical results

In order to verify the analysis above, we calculate the incoherent transmission losses over frequency range 100~2000Hz, at the sample space of 1Hz, by using MOATL code [8]. The source is taken to be at 7m depth, and the two receivers at 20km range, 7m and 25m depth respectively.

Fig. 7 shows the curves of the incoherent transmission loss versus frequency in an idealized waveguide of two water layers. It can be seen that there is an evident resonance phenomenon, and the resonance frequency is about 141Hz, agreeing favorably with the result from Eq. (17), which is 138Hz. In addition, for the receiver of 25m depth, the incoherent transmission loss values are different at two consecutive resonance frequencies. This is due to the change of eigenfunction value $\Psi_l(z=25m)$ of the least-attenuation mode as the frequency changes.

Fig. 8 shows the results in a waveguide of three water layers. The resonance frequency gotten from the curves is about 133Hz. Eq. (20) gives the frequency value of 131 Hz. Again they agree well.

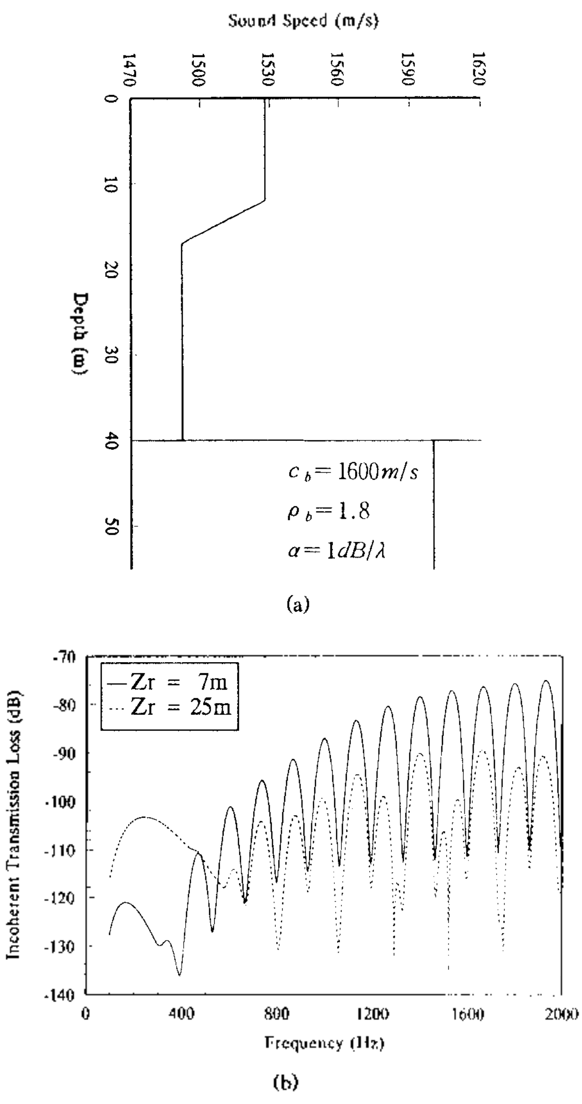


Figure 8. Incoherent transmission losses versus frequency in a shallow water waveguide with a thermocline. (a) Sound speed profile. (b) Incoherent transmission losses for a range of 20km, a source of 7m depth, a receiver of 7m depth (solid line) and a receiver of 25m depth (dotted line).

VI. Summary

By using the ray-mode approach, the resonance phenomenon of sound propagation in shallow water with a thermocline has been explained, its relation to the sound speed profile has been analyzed, and the resonance frequency has been obtained. Although the ocean models we use are relatively simple, the results obtained in this paper provide some insight into the sound propagation problems in the real shallow water with a thermocline.

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