# A Finite Element Formulation for Vibration Analysis of Rotor Bearing Systems

*Myung-Jin Choi, and *Se-Gye Oh

Abstract

To get accurate vibration analysis of rotor bearing systems, finite element models of high speed rotating shaft, unbalance disk, and fluid film journal bearing are developed. The study includes the effects of rotary inertia, gyroscopic moment, damping, shear deformation, and axial torque in the same model. It does not include the axial force effect, but the extension is straightforward. The finite elements developed can be used in the analysis and design of any type of multiple rotor bearing systems. To show the accuracy of the models, numerical examples are demonstrated.

Keywords; Rotor bearing system, High speed shaft element, Unbalance disk element, Fluid film journal bearing element, Strong form, Weak form

## I. Introduction

Many analytical methods have been used to determine the free and forced response characteristics of rotor bearing systems. Most of the methods have been based on the transfer matrix concept or on the direct stiffness approach such as the finite element method. Using the finite element method, it is posible to formulate increasingly complicated problems and the use of powerful computers makes it possible to solve large ordered system equations. Large ordered eguations are not desirable, because they require more storage space, more computational time, and have more computational errors. So, to reduce the computational efforts, the banded property of the system matrices can be utilized.

Finite element models of rotor bearing systems have been reported by several researchers in the area of rotordynamics since 1970. Ruhl and Booker[1] reported the first examples of the studies. In their studies, the effects of rotary inertia, gyroscopic moment, shear deformation, axial load, and internal damping have been neglected. Since that time several investigators[3, 4, 5, 6] have studied similar problems including different effects. Nelson used Timoshenko beam theory to establish the shape functions of a rotating shaft element[3]. In his model the effects of rotary inertia, gyroscopic moment, axial load, and shear deformations are considered. Ozguben and Ozkan[7] developed the most generalized finite element model. They considered the effect of internal damping but neglected the effect of axial torque, using the shape functions developed by Nelson. Although the various effects on dynamics of rotor bearing systems have been studied using the finite element method by the researchers, the combined effects of flexural and torsional deformations are not considered. Their models have 4 or less degrees of freedom per each node.

In this study, to get a more accurate analysis of high speed rotor bearing systems, the previous studied are generalized, and finite element models are developed. The study includes the effects of rotary inertia, gyroscopic moment, dampings, shear deformation, and axial torque in the same model, which has 5 degres of freedom per node. It does not include axial force but the extension is straightforward, using the same scheme. The finite elements developed in this study can be used in the analysis and design of any type of multiple rotor bearing system. To show the accuracy of the models, numerical examples are demonstrated.

## II. Modelling

A typical rotor bearing system is composed of shafts, disks, and bearings. In this study, finite elements of a high speed rotating shaft, an unbalanced disk, and a fluid film journal bearing are developed. The common way to formulate finite element equations is through variational methods such as Rayleigh-Ritz method, weighted residual methods such as Galerkin's method, and the least square methods. In this study, Galerkin's finite element method

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is used.

### 2.1 High speed shaft Element

Figure 1 shows a shaft element of length of L with the coordinate used to describe the end point displacements and rotations. The shaft element is considered to be initially straight and the cross section is circular and modelled as a 2-node element. Each node has 5 degree of freedom, 2 translations and 3 rotations. Figure 2 shows the free body diagram, we can write the partial differential equations of motion and the finite element equations of motion can be derived based on Galerkin's finite element method. Equilibrium equations in x-y plane become

$$\sum F_y = \rho A \Delta x \ddot{v} = Q_y(x + \Delta x, t) - Q_y(x, t) \quad (1)$$

$$\rho A \ddot{v} = \frac{1}{\Delta x}\left(Q_y(x + \Delta x, t) - Q_y(x, t)\right) \quad (2)$$

$$\sum M_z = \rho I \Delta x \ddot{\theta} - \rho I_p \Delta x \Omega \dot{\phi} = M_z(x + \Delta x, t) - M_z(x, t)$$
$$+ Q_y(x + \Delta x, t)\frac{\Delta x}{2} + Q_y(x, t)\frac{\Delta x}{2} \quad (3)$$

Let $\Delta$x approach to zero. Then

$$\rho A \ddot{v} = Q_{y,x} \quad (4)$$
$$\rho I \ddot{\theta} - \rho I_p \Omega \dot{\phi} = M_{z,x} + Q_y \quad (5)$$

In the same way, equilibrium equations in x-z plane and torsion become

$$\rho A \ddot{w} = Q_{z,x} \quad (6)$$
$$\rho I \ddot{\theta} - \rho I_p \Omega \dot{\theta} = M_{y,x} + Q_z \quad (7)$$
$$\rho I \ddot{\psi} - G I_p \psi_{,xx} \quad (8)$$

where F and Q represent forces, M means moment, and subscript x, y, and z are directions, $\rho$, A, I, $I_p$, and $\Omega$ represent density, cross sectional area, moment of inertia, polar moment of inertia, and rotating speed, respectively, $v$, $w$, $\theta$, and $\psi$ represent deformations and $_{,x}$ and $_{,xx}$ mean $d/dx$ and $d^2/dx^2$, and $G$ is shear modulus.

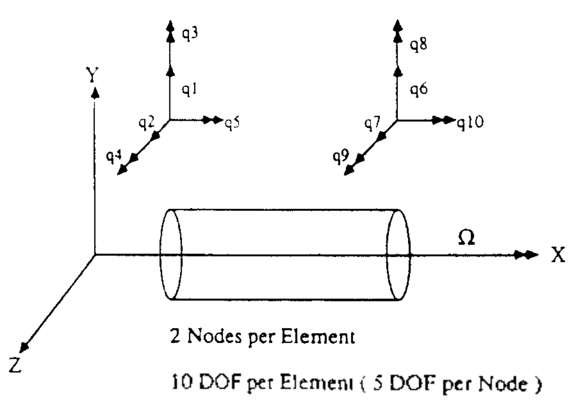


Figure 1 High speed shaft element

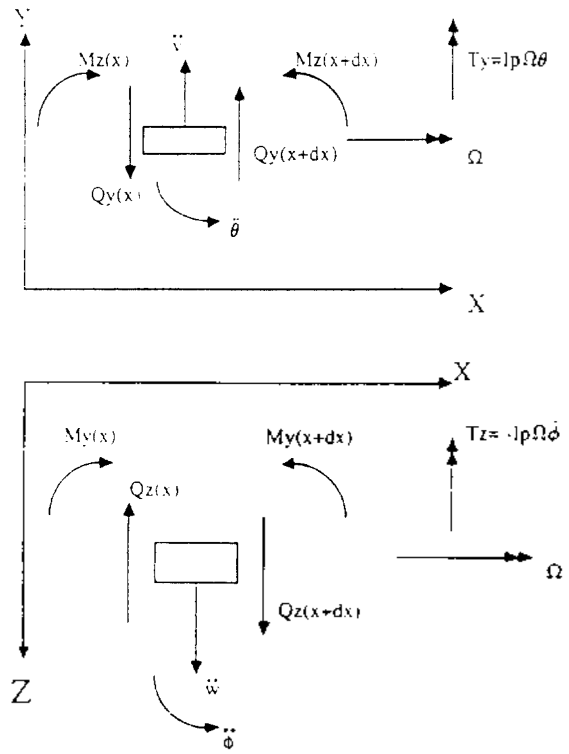


Figure 2 Free body diagram for the shaft element

From Timochenko beam theory, shear and bending strains in x-y plane are defined by $u_{,x} - \theta$ and $\theta_{,x}$ and in x-z plane are defined by $w_{,x} + \varphi$ and $\varphi_{,x}$. Then from the stress strain relationship we can write

$$Q_y(x, t) = kAG\Gamma(x, t) = kAG(v_{,x} - \theta) \quad (9)$$
$$Q_y(x, t) = kAG(w_{,x} + \phi) \quad (10)$$
$$M_z(x, t) = EI\Delta(x, t) = EI\theta_{,x} \quad (11)$$
$$M_y(x, t) = EI\phi_{,x} \quad (12)$$

where E is Young's modulus, $k$ is shear constant, $\Delta$ and $\Gamma$ mean shear and bending strains, respectively. From equations (4) to (12),

$$\rho A \ddot{v} = kAG(v_{,x} - \theta) \quad (13)$$
$$\rho A \ddot{w} = kAG(w_{,x} + \phi) \quad (14)$$
$$\rho I \ddot{\theta} - \rho I_p \Omega \dot{\phi} = EI\theta_{,xx} + kAG(v_{,x} - \theta) \quad (15)$$
$$\rho I \ddot{\phi} - \rho I_p \Omega \dot{\theta} = EI\phi_{,xx} + kAG(w_{,x} - \phi) \quad (16)$$
$$\rho I \ddot{\psi} = GI_p \psi_{,xx} \quad (17)$$

Equations (13) to (17) are the strong forms of the equations of motions. From the strong forms, we can form the weak forms mutiplying the strong forms by weight or trial functions, $S_i$, $i=1, 2, 3, 4, 5$, and partial integrating through the element.

$$\rho A\int_0^L(s_1\ddot{v}+S_2\ddot{w})dx+\rho I\int_0^L(S_3\ddot{\theta}+S_4\ddot{\phi})dx$$
$$+\rho I_p\int_0^L S_5\ddot{\psi}\,dx+\rho I_p\Omega\int_0^L(S_4\dot{\theta}-S_3\dot{\phi})dx$$
$$+EI\int_0^L(s_{3,x}\theta_{,x}+S_{4,x}\phi_{,x})dx+GI_p\int_0^L S_{5,x}\psi_{,x}\,dx$$
$$+kAG\int_0^L[(S_{1,x}-S_3)(v_{,x}-\theta)+(S_{2,x}+S_4)(w_{,x}+\phi)]dx$$
$$=S_1(L)Q_y(L,t)+S_2(L)Q_z(L,t)+S_3(L)M_z(L,t)+S_4(L)M_y(L,t)$$
$$+S_5(L)M_x(L,t)-S_1(0)Q_y(0,t)-S_2(0)Q_z(0,t)-S_3(0)M_z(0,t)$$
$$-S_4(0)M_y(0,t)-S_5(0)M_x(0,t) \tag{18}$$

Now we need to discretize the struture to get solutions which can be expressed as a linear combination of shape functions and nodal displacements.

$$v^h(x,t)=[N_v]\{q(t)\} \tag{19}$$
$$w^h(x,t)=[N_w]\{q(t)\} \tag{20}$$
$$\theta^h(x,t)=[N_\theta]\{q(t)\} \tag{21}$$
$$\phi^h(x,t)=[N_\phi]\{q(t)\} \tag{22}$$
$$\psi^h(x,t)=[N_\psi]\{q(t)\} \tag{23}$$

where

$$[N_v]=[N_1\ 0\ 0\ N_3\ 0\ N_2\ 0\ 0\ N_4\ 0]$$
$$[N_w]=[0\ N_1\ -N_3\ 0\ 0\ 0\ N_2\ -N_4\ 0\ 0]$$
$$[N_\theta]=[\overline{N}_3\ 0\ 0\ \overline{N}_1\ 0\ \overline{N}_4\ 0\ 0\ \overline{N}_2\ 0]$$
$$[N_\phi]=[0\ \overline{N}_3\ -\overline{N}_1\ 0\ 0\ 0\ \overline{N}_4\ -\overline{N}_2\ 0\ 0]$$
$$[N_\psi]=[0\ 0\ 0\ 0\ N_5\ 0\ 0\ 0\ 0\ N_6]$$

where $N_i$ and $\overline{N}_i$ are shape functions, superscript $h$ means discretized value. Shape functions developed by Nelson[3] were used (see Appendix), though any kind of good shape functions can be used.,

For isoparametric elements, the same shape functions are used to discretize the weight or trial functions.

$$S_1^h=[S][N_v]^T\quad S_{1,x}^h=[S][B_w]^T \tag{24}$$
$$S_2^h=[S][N_w]^T\quad S_{2,x}^h=[S][B_w]^T \tag{25}$$
$$S_3^h=[S][N_\theta]^T\quad S_{3,x}^h=[S][B_\theta]^T \tag{26}$$
$$S_4^h=[S][N_\phi]^T\quad S_{4,x}^h=[S][B_\phi]^T \tag{27}$$
$$S_5^h=[S][N_\psi]^T\quad S_{5,x}^h=[S][B_\psi]^T \tag{28}$$

where

$$[S]=[s_1\ s_2\ s_3\ s_4\ s_5\ s_6\ s_7\ s_8\ s_9\ s_{10}]$$
$$[B_k]=d/dx[N_k],\ k=v,w,\theta,\phi,\psi$$

Subsitituting equations (19) to (28) into equation (18) and factoring out the constant matrix [S], then we get finite element equations fo motion for the element as follows

$$[M]_{eq}\{\ddot{q}\}-\Omega[G]\{\dot{q}\}+[K]_{eq}\{q\}=\{R\}_{eq} \tag{29}$$

when we include the damping term, then the finite element equation becomes

$$[M]_{eq}^e\{\ddot{q}\}+[C]_{eq}^e\{\dot{q}\}+[K]_{eq}^e\{q\}=\{R\}_{eq}^e \tag{30}$$

where

$$[M]_{eq}=\rho A\int_0^L N_t^T N_t dx+\rho I\int_0^L N_Y^T N_Y dx+\rho I_p\int_0^L N_\psi^T N_\psi dx$$
$$[C]_{eq}=[C]-\Omega[G]$$
$$[G]=\int_0^L N_\theta^T N_\phi-N_\phi^T N_\theta dx$$
$$[K]_{eq}=EI\int_0^L B_Y^T B_Y dx+kAG\int_0^L B_s^T B_s dx+GI_p\int_0^L B_\psi^T B_\psi dx$$

$[N_t]$, $[N_Y]$, $[B_Y]$, and $B_s$ are 2 by 10 matrices, which are represented by

$$[N_t]=\begin{bmatrix}N_v\\N_w\end{bmatrix}$$
$$[N_Y]=\begin{bmatrix}N_\theta\\N_\phi\end{bmatrix}$$
$$[B_Y]=\begin{bmatrix}B_\theta\\B_\phi\end{bmatrix}$$
$$[B_t]=\begin{bmatrix}B_v-N_\theta\\B_w+N_\phi\end{bmatrix}$$

When the model is extended to include axial load, the equilibrium equations in x direction become

$$\sum F_x=\rho A\Delta x\ddot{u}=Q_x(x+\Delta x,t)-Q_x(x,t) \tag{31}$$
$$\rho A\ddot{u}=\frac{1}{\Delta x}(Q_x(x+\Delta x,t)-Q_x(x,t)) \tag{32}$$
$$\sum M_x=\rho I_p\Delta x\ddot{\psi}=M_x(x+\Delta x,t)-M_x(x,t) \tag{33}$$
$$\rho I_p\ddot{\psi}=\frac{1}{\Delta x}(M_x(x+\Delta x,t)-M_x(x,t)) \tag{34}$$

Then,

$$\rho I\ddot{u}=EIu_{,xx} \tag{35}$$
$$\rho I_p\ddot{\psi}=GI_p\psi_{,xx} \tag{36}$$

Adding equation (36) to the equilibriums of the system, the finite element equations will be easily formulated in the same way.

### 2.2 Unbalance disk element

A sufficiently stiff disk can be idealized as rigid. In this study, the disk is assumed to be thin and very stiff. Figure 3 shows a typical unbalanced rigid disk, with mass $m_d$, inertia moment $I_d$, and polar inertia moment $I_p$. The $(x, y, z)$ coordinate is an inertia reference with the x axis coinciding with the undeformed center line of the shaft element. The $(n_1, n_2, n_3)$ triad is a rotating body fixed reference with its $n_1$ coincedent with x axis. The $(n_1, n_2, n_3)$ triad rotates at a uniform rate $\Omega$ about x axis. Point O is geometrical center, point G is the center of mass, and e represents the eccentricity. Then the force due to unbalanced mass, $F_e$ can be written as

$$F_e = \begin{Bmatrix} F_{ey} \\ F_{ez} \end{Bmatrix} = m\Omega^2 e \begin{Bmatrix} n_2 \\ n_3 \end{Bmatrix} = \begin{Bmatrix} y_e \cos\Omega t - z_e \sin\Omega t \\ y_e \sin\Omega t + z_e \cos\Omega t \end{Bmatrix}$$

$F_{ey}$ and $F_{ez}$ are the forces due to unbalanced mass in $y$ and $z$ directions, $y_e$ and $z_e$ are the mass center eccentricties of the disk in $y$ and $z$ directions at $t=0$.

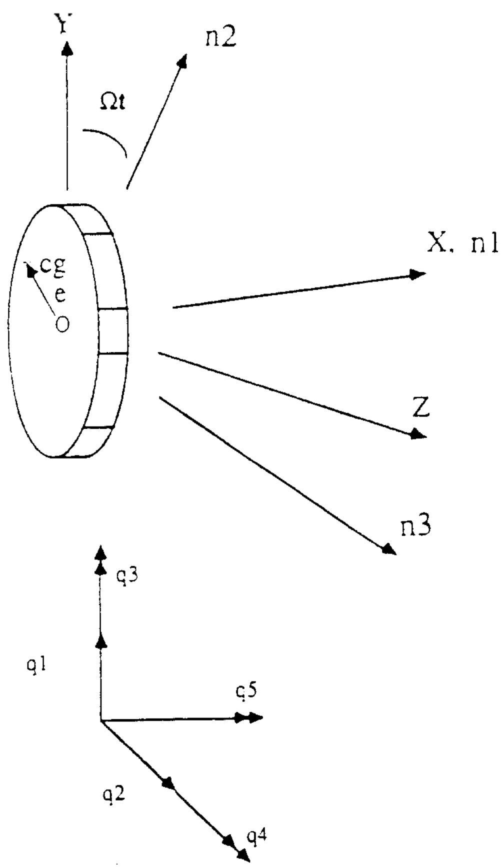


Figure 3 Unbalance disk element

From the equilibrium,

$$\sum F_y = m_d \ddot{y} = P_y + m\Omega^2 (y_e \cos\Omega t - z_e \sin\Omega t) \qquad (37)$$

$$\sum F_z = m_d \ddot{z} = P_z + m\Omega^2 (y_e \cos\Omega t - z_e \sin\Omega t) \qquad (38)$$

$$\sum M_z = I_d \ddot{\theta} = -I_p \omega \dot{\phi} \qquad (39)$$

$$\sum M_y = I_d \ddot{\phi} = I_p \omega \dot{\theta} \qquad (40)$$

$$\sum M_x = I_p \ddot{\psi} = T_x \qquad (41)$$

Then, the equilibrium equations are

$$m_d \ddot{y} = P_y + F_{ey} \qquad (42)$$

$$m_d \ddot{z} = P_z + F_{ez} \qquad (43)$$

$$I_d \ddot{\theta} + I_p \Omega \dot{\phi} = 0 \qquad (44)$$

$$I_d \ddot{\phi} - I_p \Omega \dot{\theta} = 0 \qquad (45)$$

$$I_p \ddot{\psi} = T_x \qquad (46)$$

$P_y$ and $P_z$ are applied forces in $y$ and $z$ directions, $T_x$ is the applied torque.

The rigid disk element has 5 DOF, 2 translations and 3 rotations, as shown in Figure 3. Then from the equilibrium, the finite element equation can be represented as

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} = \{F_d\} \qquad (47)$$

where

$$[M] = \begin{bmatrix} m_d & 0 & 0 & 0 & 0 \\ 0 & m_d & 0 & 0 & 0 \\ 0 & 0 & I_d & 0 & 0 \\ 0 & 0 & 0 & I_d & 0 \\ 0 & 0 & 0 & 0 & I_p \end{bmatrix}$$

$$[S] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -I_p & 0 \\ 0 & 0 & I_p & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\{F_d\} = \begin{Bmatrix} P_y + F_{ey} \\ P_z + F_{ez} \\ 0 \\ 0 \\ T_x \end{Bmatrix}$$

### 2.3 Fluid film journal bearing element

For the modelling of bearings for lateal motion, the eight bearing coefficient model as shown in Figure 4 is used and any inertial effects are assumed to be negligible. The bearing coefficients can be approximated based on the scheme of Chapter 6 in reference[11] or can be adapted from the bearing design handbook written by Lund[8]. For the torsional motion, damping coefficients can be approximated using the Pettloff's law, which is established to explain the phenomenon of bearing friction. The frictin torque $M_f$ is

$$M_t = \eta \frac{4N\pi^2 r^3 l}{60c} \quad (48)$$

where $\eta$ is viscosity, $r$ is radius of the journal, $c$ is radial clearance, $l$ is bearing length, and N is rpm. The inertial effect and torsional stiffness are assumed to be negligible. Then the equivalent torsional damping becomes

$$C_t \Omega = M_t \quad (49)$$

$$C_t = \frac{M_t}{\Omega} = f(\eta, r, l, c) = \eta \frac{4N\pi^2 r^2 l}{60c} \quad (50)$$

where $\Omega$ is rotational speed.

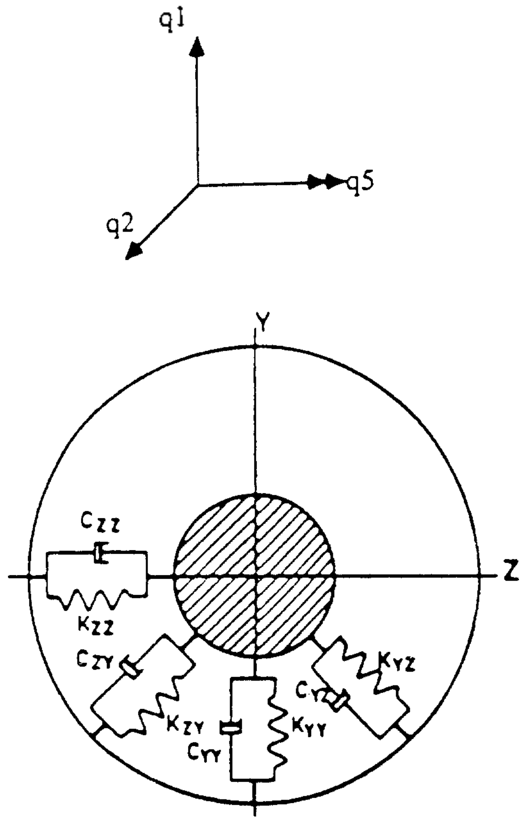


Figure 4.

In finite element analysis, each journal bearing can be modelled by using a set of spring and dash pot at the journal center, as a point element with 3 degrees of freedom, two translations and one rotation as shown in Figure 4. Then finite element equations for the bearing element become

$$[C]\{\dot{q}\} + [K]\{q\} = \{F_b\} \quad (51)$$

where

$$[C] = \begin{bmatrix} C_{yy} & C_{yz} & 0 \\ C_{zy} & C_{zz} & 0 \\ 0 & 0 & C_t \end{bmatrix}$$

$$[K] = \begin{bmatrix} K_{yy} & K_{yz} & 0 \\ K_{zy} & K_{zz} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\{\dot{q}\} = \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_5 \end{Bmatrix}$$

$$\{q\} = \begin{Bmatrix} q_1 \\ q_2 \\ q_5 \end{Bmatrix}$$

$$\{f_b\} = \begin{Bmatrix} f_1 \\ f_2 \\ f_5 \end{Bmatrix}$$

where $\{f_b\}$ is bearing force vector. In expanded form, fluid film journal bearing also can be idealized by two or three sets of springs and dash pots at the end points of the journal, or the three points located equidistantly along the journal axis respectively.

### 2.4 System equations

The finite element equations of motion of the complete system can be written as

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = \{f\} \quad (52)$$

where $[M]$, $[C]$, and $[K]$ are the mass, damping including gyroscopic moment, and stiffness matrices of the system obtained by assemblying the element matrices. Due to the bearing coefficients, $[K]$ and $[C]$ may be neither symmetric nor skew symjmetric, but they are highly banded in general.

Generally speaking, dynamic problems can be classified by two broad classes. In one, we ask for the dynamic responses with time, under prescribed loads, impulses, or ground accelerations. In the other, we ask for the eigenvalues of the governing system equations which tell us the stability of the system, natural frequencies, and the corresponding mode shapes. In rotordynamics, eigenvalues are found in the form

$$\alpha = \lambda + i\omega \quad (53)$$

where $\omega$ is the whirl speed. $\lambda$ and $\omega$ are real. Logarithm decrement, $\delta$, is defined as

$$\delta = \frac{-2\pi\lambda}{\omega} \quad (54)$$

## Ⅲ. Dynamic responses

Equation (52) is a typical form of the system equations in dynamic problems. Since the order of the coefficient matrices $[M]$, $[C]$, and $[K]$ is large in finite element analysis, the procedure for the solution of the system equations can be very expensive unless the special charateristics of the systme matrices are taken advantage of. The solution prodcedures are considered as two methods, direct integration and modal method or mode superposition method. In direct integration, the equations are integrated using a numerical step by step procedure. The term direct means that no transformation of the equations into different forms is performed prior to the numerical integration. In modal method or mode superposition method, matural frequencies and mode shapes are extracted by solving eigenvalue problems and the dynamic responses are expressed as the sum of normal modes in appropriate portions. Systems that are subjected to arbitrary loads become extremely difficult to analyze in the physical domain. These difficulties can be avoided using modal method in modal or natural domain.

Direct integration equations are either explicit or implicit. Explicit methodes find the responses at time $t+\Delta t$ by use of the equations of motion written at time $t$, while implicit methods find the responses at $t+\Delta t$ from the equations of motion written at time $t+\Delta t$. Usually, explicit methods allow a small time step but produce equations that are cheap to solve, while implicit methods allow a large time step but produce equations that are expensive to solve. Use of different values of time step in different parts is also possible. Most explicit methods are conditionally stable. Most unconditionally stable methods are implicit. In unconditionally stable methods, the size of time step is decided by accuracy rather than stability. Many algorithms for dynamic responses are abailable in text books [9, 10]. In 1959, Newmark generalized certain direct numerical integrations that had been used up to that time, which is still a popular method for dynamic responses. Often mathematicians recommend the fourth order Runge-Kutta method. In this study, to solve the system equations, Mewmark's method is used.
s method is based on the assumption

$$q_{t+\Delta t}=d_{t+\Delta t}+\beta(\Delta t)^2\ddot{q}_{t+\Delta t} \tag{55}$$

$$\dot{q}_{t+\Delta t}=\bar{v}_{t+\Delta t}+\gamma\Delta t\,\ddot{q}_{t+\Delta t} \tag{56}$$

$$d_{t+\Delta}T=q_t+\Delta t\,\dot{q}_t+0.5(\Delta T)^2(1-2\beta)\ddot{q}_t \tag{57}$$

$$v_{t+\Delta t}=\dot{q}_t+\Delta t(1-\gamma)q_t \tag{58}$$

$\beta$ and $\gamma$ are the numbers that the analyst can choose. Substituting equations (55) to (58) into equation (52), and rearranging for q, then

$$([M]+\gamma\Delta t[C]+\beta(\Delta t)^2\ddot{q}_{t+\Delta}t=\{f\}_{t+\Delta t}-[K]d_{t+\Delta t} \tag{59}$$

The algorithm operates as follows. We starts at $t=0$. initial conditions prescribes $q_0$ and $\dot{q}_0$. From these and equation (52), $\ddot{q}_0$ can be found. Then equations (55) to (58) are solved for $q_{\Delta t}$ and $\dot{q}_{\Delta t}$, and equation (59) is solved for $\ddot{q}_{\Delta t}$. With $q_{\Delta t}$, $\dot{q}_{\Delta t}$, and $\ddot{q}_{\Delta t}$, we can find $q_{2\Delta t}$, $\dot{q}_{2\Delta t}$, and $\ddot{q}_{2\Delta t}$, in the same way, and so on. If $\Delta t$ is not changed, the coefficient matrix needs to be reduced only once. A good choice of parameters for an implicit method that is unconditionally stable in linear problems is $\beta=0.25$ and $\gamma=0.5$. Then the method is also called the constant average acceleration method or the trapezoidal method.

Generally, modal and implicit direct methods are more economical in inertial problems, while explicit direct methods are more economical in shock loading and wave propagation problems. In linear problems, modal method is fabored if only a few modes are needed to describe the response. With mode superposition, loading histories after the first are analyzed cheaply, but with direct integration, especially the explicit methods, the second and subsequent load histories do not reduce cost as greatly. Modal method is suitable if the nonlinearities are absent or small and confined to a few regions of the structure. The explicit approach is often best for nonlinear problems. An efficient and versatile computer program should be able to change automatically from an explicit method at an early time, where the time step is small to follow transient, to an implicit method at a later time, where a large time step is sufficiently accurate.

## Ⅳ. Results

To confirm the results of this study, simple examples are employed as shown in Figures 5 and 6. Figures 7 and 8 show the whirling and torsional responses at the location of the disk for the rigid bearing model. Figures 9 shows the whirling responses at the location of the disk for the flexible bearing model. Solid lines represent responses from the method of this study, and circle points represent the analytical solutions for the steady state responses. Since the results agree well, the basic procedure is confirmed by such correlation.

In this study, a finite element model of a rotor bearing system is presented. The major advantage of the model is that it includes tae effects of rotary inertia, gyroscopic moment, internal and viscous dampings, and axial torque in the same model. The developed computer program gives accurate predictions and will be a valuable tool for the analysis and design of a high speed rotor bearing system.

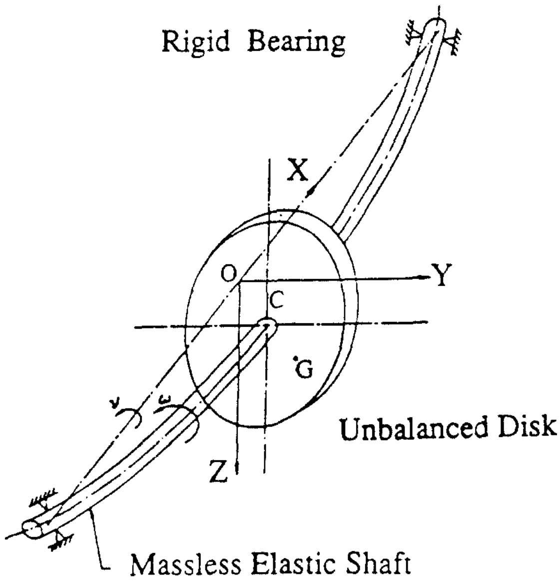


Figure 5.

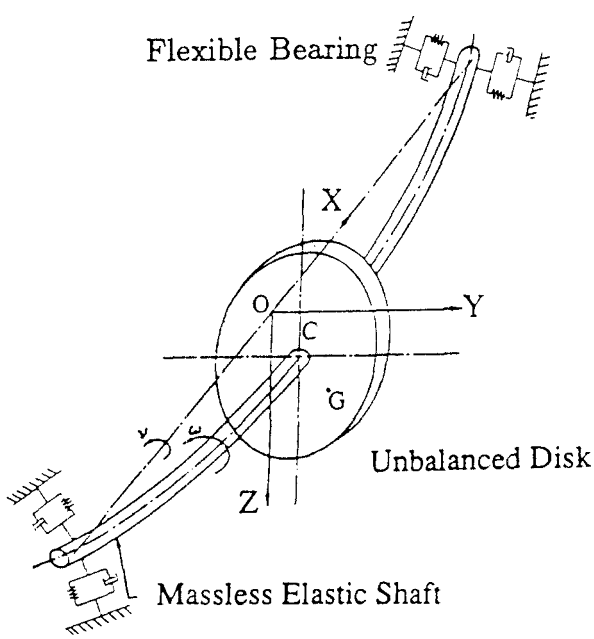


Figure 6.

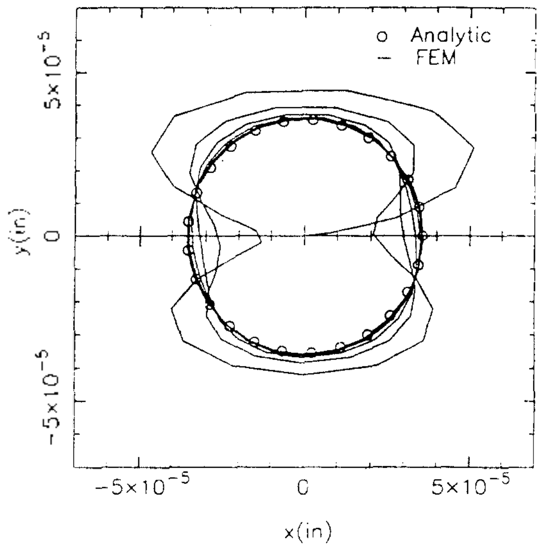


Figure 7.

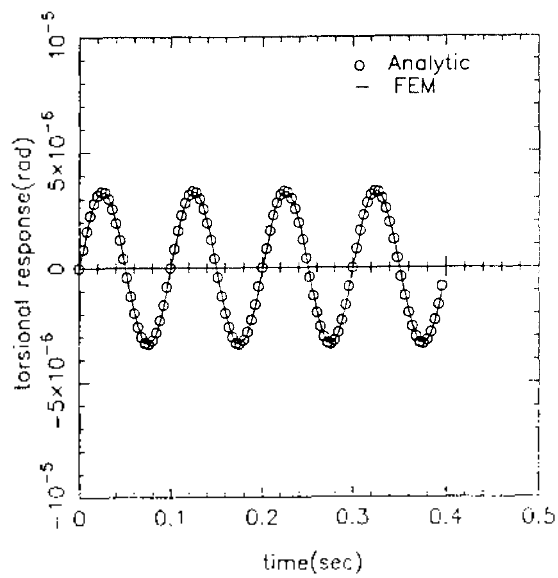


Figure 8.

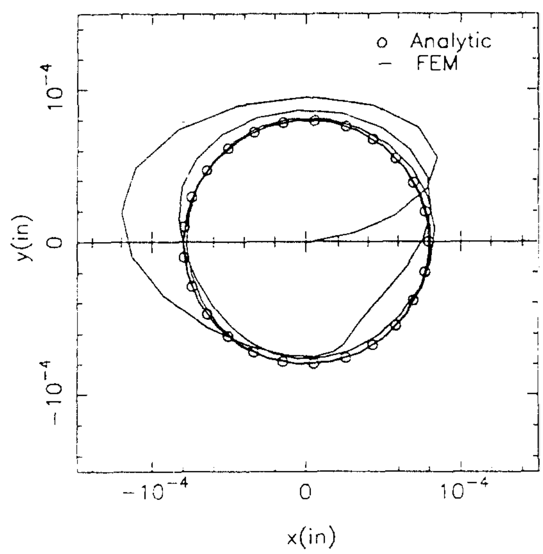


Figure 9.

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## Appendix

Shape Function for High Speed Shaft Element

$$N_i = \frac{1}{1+\Phi}(\alpha_i + \Phi\beta_i)$$

where $i$ = 1, 2, 3, 4

$$v = x/l \qquad \Phi = 12EI/kAGl^2$$

$$\alpha_1 = 1-3v^2 = 2v^3 \qquad \alpha_2 = 3v^2 \quad 2v^3$$

$$\alpha_1 = 1-3v^2+2v^3 \qquad \alpha_4 = 0.5l(-v+v^2)$$

$$\beta_1 = 1-v \qquad \beta_2 = v$$

$$\beta_3 = 0.5l(v-v^2) \qquad \beta_4 = 0.5l(\quad v+v^2)$$

$$\overline{N_i} = \frac{1}{1-\Phi}(\varepsilon_i + \Phi\delta_i)$$

$$\varepsilon_1 - \frac{1}{l}(6v^2-6v) \qquad \varepsilon_2 - \frac{1}{l}(-6v^2+6v)$$

$$\varepsilon_3 = 1+3v^2-4v) \qquad \varepsilon_4 = 3v^2-2v$$

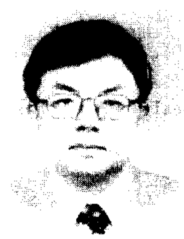
$$\delta_1 = 0 \qquad \delta_2 = 0$$

$$\delta_3 = 1-v \qquad \delta_4 = v$$

$$N_5 = 1-v$$

$$N_6 = v$$


▲Myung-Jin Choi



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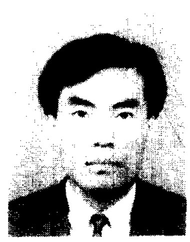

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