# Design of Subband Codecs Using Optimized Vector Quantizer

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#### ABSTRACT

This paper provides an approach for representing an optimum vector quantizer by a scalar nonlinear gain-plus-additive noise model. The validity and accuracy of this analytic model is confirmed by comparing the calculated model quantization errors with actual simulation of the optimum Linde-Buzo-Gray(LBG) vector quantizer. Using this model we form MSE measure of an M-band filter bank codec in terms of the equivalent scalar quantization model and find the optimum FIR filter coefficients for each channel in the M-band structure for a given bit rate, given filter length, and given input signal correlation model. Specific design examples are worked out for 4-tap filters in the two-band paraunitary case. These theoretical results are confirmed by extensive Monte Carlo simulation.

#### I. Introduction

The technique of Subband Coding(SBC) has become popular for low rate speech coding, still image, video, and High Definition Television(HDTV) signal coding[1]. The basic idea of SBC is to split up the frequency band of the signal into a number of subbands and them to encode each subband separately. Usually PCM or DPCM coder is used to encode the subbands, where the bit rate of each subband is determined by a bit allocation procedure. Thus, in the actual system, the signals are quantized before transmission at the receiver side and reconstructed by the synthesis filter bank. Recently, Haddad[9] provided a thorough analysis of the quantization effects in general M-band subband coding systems using the polyphase approach. According to Shannon's rate distortion theory, better results are always obtained when vectors rather than scalars are encoded. Most researchers have focused on the error in the quantizer, but not on the overall reconstruction error and its dependence on the filter bank. This purpose of this paper is to provide a thorough analysis of the vector quantization effects in general M-band subband coding systems using the polyphase approach. This paper demonstrates that the scalar nonlinear gain-plus-additive noise quantization model can be used to represent each vector quantizer in an M-band subband codec. The validity and accuracy of this analytic model are confirmed by comparing the calculated model quantization errors with actual simulation of the optimum LBG vector quantizer. We computer the mean squared reconstruction error(MSE) which depends on N the number of entries in each codebook, k the length of each codeword, and on the filter bank coefficients. We form this MSE measure in terms of the equivalent scalar quantization model and find the optimum FIR filter coefficients for each channel in the *M*-band structure for a given bit rate, given filter length, and given input signal correlation model. Specific design examples are worked out for 4-tap filters in the two-band paraunitary case. These theoretical results are confirmed by extensive Monte Carlo simulation.

## I. Requirements for Optimized Scalar Quantizer

For the Lloyd-Max pdf-optimized scalar quantizer, Fig. 1(a) shows the block diagram representation of the pdfoptimized quantizer where v is the signal to be quantized,  $\hat{v}$  is the quantized output, and the  $\tilde{v}$  is the quantization error. It can be shown that the quantization error is unbiased and that the error is orthogonal to the quantizer output[6]

$$E[\tilde{v}] = 0, \qquad E[\tilde{v}\,\hat{v}] = 0. \tag{1}$$

But the quantization error  $\tilde{v}$  is correlated with the input v so that the variance of the quantization error is

$$\sigma_{\bar{\nu}}^2 = \sigma_{\bar{\nu}}^2 - \sigma_{\bar{\nu}}^2 \tag{2}$$

Thus the simple input-independent-additive noise model

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Fig. 1. (a)Pdf-optimized quantizer, (b)gani-plus-additive noise model.

is only an approximation to the pdf-optimized quantizer. Fig. 1(b) shows the gain-plus-additive noise model representation advanced by Jayant and Noll[6]. For this model we can impose the conditions in (1), and force the fictitious random noise error r and v to be uncorrelated by choosing the gain and the equivalent noise variance as

$$\boldsymbol{\alpha} = 1 - \frac{\sigma_{\nu}^2}{\sigma_{\nu}^2} , \qquad \sigma_r^2 = \boldsymbol{\alpha}(1 - \boldsymbol{\alpha}) \, \sigma_{\nu}^2. \tag{3}$$

Note that both  $\alpha$ , and  $\sigma_r^2$  are new input signal dependent. From rate distortion theory[6], the quantization error variance  $\sigma_{\theta}^2$  for the pdf-optimized quantizer is

$$\sigma_{\tilde{\nu}}^2 = \beta(R) \, 2^{-2R} \, \sigma_{\nu}^2 \,, \tag{4}$$

where  $\beta(R)$  depends on the pdf of the input signal v and R is the number of bits used in the quantizer.

## I. Requirements for Optimized Vector Quantizer(VQ)

The properties of a VQ optimized for mean squared distortion error over a frame are[5]

$$E\{\hat{v}\} = 0 \qquad E\{\hat{v}^{t}|\hat{v}\} = 0 \tag{5}$$

where  $\hat{v} = Q(v)$  is the vector quantizer output,  $\underline{v} = \underline{v} - \hat{v}$ . Eqn.(5) demonstrates that optimal quantization can focus on zero mean random input vectors without loss of generality and the quantized vector is orthogonal to the quantization error vector. The consequence of this is that the quantization error  $\tilde{v}$  is always correlated with the input  $\underline{v}$  if the vector quantizer is optimum. This implies that a model of vector quantization as the addition of an independent "noise" vector to the input vector cannot be valid or at least cannot be strictly correct.

## **IV. Derivation of Formula for Distortion in** Optimal Vector Quantizer

The performance for an N-level k-dimensional vector

quantizer can be measured by the mean square distortion,  $D = \frac{1}{k} E \|\underline{v} - Q(\underline{v})\|^2$ , where  $\|\cdot\|$  denotes the usual  $l_2$  norm. We wish to choose  $\hat{v}_1, ..., \hat{v}_N$  to minimize D. The k-dimensional  $2^{nd}$  power distortion-rate function of an optimal vector quantizer in high resolution is given in [4] by

$$D_{FQ}^{k}(R) = C(k, 2) 2^{\frac{1}{2}(2/k)R} \left[ \int [p(\underline{v})]^{k/(2+k)} d\underline{v} \right]^{(2+k)/k}.$$
 (6)

The constant C(k, 2) is a function of the vector dimension k and represents how well cells can be packed in k-dimensional space. The density function  $p(\underline{v})$  is the k-dimensional joint pdf of the vector process.

1. Approximate Optimized Vector Quantizer Model

According to Jayant and Noll[6], the short-time pdf of a speech segment can be approximated by a Gaussian pdf. The vector to be quantized is assumed from consecutive samples of an autoregressive stationary random signat. This model is a good model for speech-like signals. The mean squared quantization error averaged over a frame in optimized vector quantizer coding can be computed approximately using the asymptotic distortion-rate function for high-rate quantization derived for a Gaussian random signal[3],

$$D_{\nu\rho}^{k} \approx \tau 2^{-2R/k} (det \ \Gamma)^{1/k} \stackrel{\circ}{=} \sigma_{\nu}^{2}$$
(7)

where k, R and  $\Gamma$  denote respectively the vector dimension, the number of bits allocated to the quantizer, and the covariance matrix of the input signal;  $\sigma_{\underline{k}}^2$  denotes the mean squared quantization error averaged over a frame in optimized vector quantizer, and  $\tau$  is a correction factor which, for a Gaussian pdf, is

$$\pi = 2\pi ck (1 + \frac{2}{k})^{k/2 + 1}$$
(8)

where c is the quantization coefficient for the VQ[2]. The correction factor r depends on the coefficient of quantization for VQ and on the dimension k. There are a number of approximations based on lower or upper bounds. The results in this paper are based on the values given by the Voronoi lattice upper bound[2]. It is computationally burdensome to directly estimate  $det \Gamma$ . However, using the Toeplitz distribution theorem[6],

$$\lim_{k \to 1} \det \Gamma^{1/k} = \exp \left[ \frac{1}{2\pi} \int_{-\pi}^{\pi} \log_e S_{w}(e^{jw}) dw \right] = \sigma_{e,\min}^{2}$$
(9)

where  $S_{vv}(e^{iw})$  is the power spectral density of the random



Fig 2. Vector quantizer.



Fig 3. A finite memory optimal predictor.

signal  $\{v(n)\}$  and  $\sigma_{e,min}^2$  the energy of the iminimum prediction error. When the vector dimension k and the predictor order are reasonably large, the quantization error in Eqn.(7) can be further simplified to

$$D_{FQ}^{k} \approx \tau 2^{-2R/k} \sigma_{e}^{2} \tag{10}$$

where  $\sigma_e^2 = E\{|\tilde{v}_p(n)|^2\}$  is the variance of the prediction error sequence using a finite memory optimal predictor in mean square sense[6]. A finite memory optimal predictor is shown in Fig. 3. The weights  $\{a_i\}$  are linear predictor coefficients and *M* is the predictor order.

### 2. Gain-Plus-Additive Noise Model for VQ

We show that Eqns.(1) and (3) for the pdf optimized scalar quantizer can also be used to represent the optimized VQ. The distortion per frame in the LBG classification algorithm is

$$\frac{1}{k} \sum_{i=n-(k-1)}^{n} |v(i) - \hat{v}(i)|^2.$$
(11)

We show that this distortion measure equals  $D_{VQ}^{*}$  of Eqn. (10). Assume  $E\{|v(n-i)-\hat{v}(n-i)|^2\}$  is the same for all iin that block. Can we use  $D_{VQ}^{\star}$  of Eqn.(10) as this measure? Is it true that  $\tilde{v}(n-i)$  is orthogonal to  $\hat{v}(n-i)$  as required by Eqn.(5)?, where  $\tilde{v}(i) = v(i) - \hat{v}(i)$ ? Thus, we calculate  $E\{\hat{v}(i)\}$  and  $E\{\hat{v}(i)|\hat{v}(i)\}$  as follows. First, 500,000 samples of an AR(1), input signal v(n) is concatenated into a sequence of vectors. Then, we create a codebook using the LBG algorithm. Second, the quantized signal  $\hat{v}$ (n) is made from VQ encoding method. Third, the error signal  $\tilde{v}(n)$  is observed. Then we calculate the following items:  $E\{\tilde{v}(i)\} \rightarrow \frac{1}{k} \sum_{i=0}^{k-1} \tilde{v}(i)$  for each block; then average over all blocks.  $E\{\hat{v}(i)|\hat{v}(i)\} \rightarrow \frac{1}{k} \sum_{i=0}^{k+1} \hat{v}(i)|\hat{v}(i)|$  for each block; then average over all blocks. The results of this simulation are shown in Table 1. Therefore we can conclude that  $E\{\tilde{v}(i)\} \simeq 0$ ,  $E\{\tilde{v}(i), \tilde{v}(i)\} \simeq 0$ . So, we can use  $D_{VQ}^{k} = \tau 2^{-2R/k} \sigma_{e}^{2} = \frac{1}{k} \sum_{i=n-(k-1)}^{n} |v(i) - \hat{v}(i)|^{2}$  in the pdf-optimized vector quantizer. Comparing Eqns.(1) and (3) for the scalar quantizer with Eqn.(10) for VO, we see that if  $\sigma_{\underline{\hat{v}}}^2$  of VQ obtained from  $\sigma_{\underline{\hat{v}}}^2 = \frac{1}{k} \sum_{i=0}^{k-1} \sigma_{\underline{\hat{v}}_i}^2$  per block and averaged over all blocks equals  $\sigma_{\mu}^2$  of the scalar quantizer, we can say

$$\alpha = 1 - \frac{\sigma_v^2}{\sigma_v^2} = 1 - \frac{\tau 2^{-2R/k} \sigma_e^2}{\sigma_v^2}$$
(12)

where  $\tau$ , which depends on k, the vector dimension, is given in [2]. Also, from the theory of linear optimum prediction[6],  $\sigma_e^2 = E\{(\hat{v} - v)^2\}$  and the optimal prediction error is represented as

$$\sigma_e^2 = \gamma_v^2 = \sigma_v^2 \,. \tag{13}$$

Thus



Fig 4. Optimum M-channel FB for simulation with AR(1) gaussian input.

$$\alpha = 1 - \tau 2^{-2R/k} \gamma_{\nu}^2 \tag{14}$$

where  $\gamma_{v}^{2}$  is the "spectral flatness measure" which is the reciprocal of the maximum prediction gain

$$\gamma_{\nu}^{2} = \min\{\infty \sigma_{\nu}^{2}\} / \sigma_{\nu}^{2} = [\max\{\infty G_{\nu}\}]^{-1}$$
(15)

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as predictor order becomes  $\infty$ , where  $G_p$  is the prediction gain of the predictor. We calculate  $\Upsilon_v^2$  in the following way. Consider a zero-mean process  $\{x(n)\}$  with power spectral density  $S_{xx}(e^{jw})$ . This signal is filtered by  $H(e^{jw})$ . Its filtered signal spectral density is  $S_{vv}(e^{jw}) = |H(e^{jw})|^2 S_{xx}(e^{jw})$ and  $\Upsilon_v^2$  is

$$Y_{\nu}^{2} = \frac{exp\{\frac{1}{2\pi} \int_{-\pi}^{\pi} \log_{e} S_{\nu\nu}(e^{jw}) dw\}}{\frac{1}{2\pi} \int_{-\pi}^{\pi} S_{\nu\nu}(e^{jw}) dw}$$
(16)

Eqn.(14) gives us a theoretical value for  $\alpha$ . Then use of Eqn.(12) gives  $\sigma_{\tilde{\nu}}^2 = (1 - \alpha)\sigma_{\nu}^2$ . This theoretical value is compared with simulated quantization error variance. In our case, *R*(bit rate) is small and k = 8(vector dimension of VQ) is also small. Therefore to improve the accuracy of the model we introduce an empirically determined correction factor  $\delta$  which depends on *R* and *k*. Hence

$$\alpha = 1 - \tau 2^{-2(R/k-\delta)} \gamma_v^2. \tag{17}$$

The optimized VQ mean squared error is now

$$\sigma_{\mu}^{2} = \tau 2^{-2(R/k-\delta)} \gamma_{\mu}^{2} \sigma_{\mu}^{2}.$$
(18)

### V. Model Validation

1. Codebook Design; An input AR(1)( $\rho=0.95$ , mean=0, var=1.0) signal is passed through a 4-tap Binomial Quadrature-Mirror-Filter(QMF)[1]. This filtered signal is used as a training signal for codebook design using a LBG classification algorithm with a mean squared error criterion. This algorithm converges iteratively toward a locally optimal codebook. We choose  $k \approx 8$  for vector dimension, N = 32, and 64 for codebook addresses and n = 500,000 samples for training sequences.

2. Simulations; 1) A test sequence of 500,000 samples of AR(1) gaussian input signal is used for simulation. Since this test sequence is generated by different seed, this sequence is not the same as that used in the codebook design. These test sequences are passed through a 4-tap Binomial QMF, and then arranged as a sequence of k-

dimensional vectors. The quantized signal is obtained using the LBG codebook. So, we can calculate  $E\{\tilde{v}(i)\}$ ,  $E\{\hat{v}(i)|\tilde{v}(i)\}$ , and the quantization error  $E\{\|\tilde{v}(i)\|^2\}$  as described previously. Simulation results are shown in Table 1. 2) And compared with the theoretical quantization error calculated using gain-plus-additive noise model. Eqn.(14) gives us a theoretical value for  $\alpha$ . 3) We compare  $E\{|\tilde{v}(i)|^2\}$ from simulations with  $\sigma_{b}^{2}$  from theoretical scalar gain-plusadditive noise model Eqn.(18). Thus, we obtain  $\delta$ . These results are shown in Table II with the correction factor  $\delta$ equal to zero. An even closer match can be found by selecting  $\delta$  from the empirically obtained table, as shown in Table III. From these simulations we conclude that the optimum vector quantizer in an M-channel subband coder can be modeled by the scalar gain-plus-additive noise scalar model.

Table I. Simulation results using AR(1) (n = 500,000 samples,  $\rho = 0.95$ ) for LBG vector quantizer.

Bit rate	Codebook	$E\{\tilde{v}(i)\}$	$E\{\tilde{v}(i) \hat{v}(i)\}$	$E\{v^2(i)\}$	$E\{\ \tilde{v}(i)\ ^2\}$
0.625	N = 32, k = 8	-3.57E-4	8.97E-4	1.9651	0.1187
0.75	N = 64, k = 8	-2.25E-4	2.32E-3	1.9651	0.0861

Table B. Comparision  $E(|\tilde{v}(i)|^2)_{sim}$  from test on VQ experimentally with  $\sigma_{\tilde{v}}^2$  from equivalent scalar gain-plusadditive noise model theoretically.

Bit rate (R)	$E\{\ \tilde{v}(i)\ ^2\}_{sim}$	$\sigma_{i}^2$
0.625	0.1187	0.1152
0.75	0.0861	0.0969

Table II. Values of  $\delta$  for AR(1) ( $\rho = 0.95$ ) gaussian input. R is the VQ rate in bit/sample, k is the VQ dimension.

R	0.25	0.5	0.75	1.0
k = 8	0.5450	0.1499	-0.0853	-0.2434
k = 12	0.1323	-0.2855	-0.5371	[
<b>k</b> = 16	-0.1476	-0.5780		

#### VI. Sample Optimum Filter Bank Design

Our design problem is to find the optimal filter bank which minimizes the MSE for an AR(1) input signal with correlation coefficient  $\rho$  and a total bit allocation constraint. In [9][10], it is shown that the MSE for a Perfect Reconstruction(PR) filter bank is

$$MSE \triangleq \sigma_d^2 + \sigma_n^2$$
  
$$\sigma_d^2 = \frac{1}{M} \sum_{i=0}^{M-1} (\alpha_i s_i - 1)^2 \sigma_{v_i}^2, \quad \sigma_n^2 = \frac{1}{M} \sum_{i=0}^{M-1} s_i^2 \sigma_{r_i}^2.$$
(19)

These terms  $\sigma_d^2$ ,  $\sigma_n^2$  are called the signal distortion and the

random noise component of the MSE respectively. And  $s_i$  is called optimal compensator,  $\sigma_{\nu_i}^2$  is the variance of the fictitious random noise in the *i*th channel,  $\sigma_{\nu_i}^2$  is function of FIR filter coefficient and  $\sigma_{\nu_i}^2$ . Under perfect reconstruction constraints,  $\sigma_d^2$  measures the deviation from perfect reconstruction due to the quantizer and compensator. This decomposition of the total MS quantization error allows us to analyze each component error separately. For the paraunitary solution,  $s_i = 1$ . In our example, M = 2, and a filter with 4 taps {a, b, c, d} is used. Here,

$$\sigma_{v_1}^2 = 1 + 2\rho(ab + bc + cd) + 2\rho^3(ad), \quad \sigma_{v_1}^2 = 2 - \sigma_{v_2}^2$$
(20)

$$\sigma_{\tau_{\bullet}}^{2} = \alpha_{0}(1 - \alpha_{0})\sigma_{\tau_{\bullet}}^{2}, \quad \sigma_{\tau_{1}}^{2} = \alpha_{1}(1 - \alpha_{1})\sigma_{\nu_{1}}^{2}.$$
(21)

Each quantizer takes only integer bits per channel, the high-pass receiving at least 1 bit, and the low-pass channel at most 11 bits, for two codebooks of 2,2048 words each. Our optimization algorithm tests for all possible bit combinations for the given average bit rate R bits/sample, calculates the optimal filter coefficients, compensators, and MSE. It chooses the one with the minimum MSE among them. This is implemented by using (MSL Library(DNCONF). These anaytical results were confirmed by inside training simulation using 64,000 samples of AR (1) gaussian input.

Our calculation procedure is as follows:

- (a) Fix R, k,  $\rho$ , codebooks; calculate  $\tau$ .
- (b) Choose initial  $h_0(n)$  using 4-tap Binomial QMF.
- (c) Calculate  $\gamma_{\nu}^2$ ,  $\alpha$ ,  $\sigma_{\nu_i}^2$ ,  $\sigma_{\gamma_i}^2$  from Eqns.(16), (17), (20), (21) respectively.
- (d) Express MSE(Eqn.(19)) in term of 4 filter parameters;
   Calculate h<sub>0</sub>(n) using DNCONF algorithm. Calculate MSE.
- (e) If  $(MSE)_i \leq (MSE)_{i-1}$ , go to step (c). If not, stop.

The analysis and simulation results for the paraunitary FB are shown in Table VI(a), (b) for the input correlation  $\rho = 0.95$ , 0.75. From our simulation[7] we know that the correction factor  $\delta$  affects variation of  $MSE_{sim}$  within 0.5 percent and the optimal filter coefficients are quite insensitive to changes in  $\delta$  value. So, we ignore the correction factor  $\delta$ . Table VI(a), (b) lists the optimum integer bits allocated to each channel  $R_0$ ,  $R_1$ , MSE, the theoretical calculation(analysis) of the output MSE based on (19), and the simulation results,  $MSE_{sim}$ . The corresponding optimal filter coefficients are shown in Table V(a)(b). As seen from Table V, the optimal filter coefficients are quite insensitive to changes in average bit rate R although the output MSE is highly dependent on them. We note that the random noise  $\sigma_{\pi}^2$  is the dominant component in the total output MSE when compared with  $\sigma_{d}^2$ . The simulation results closely match the theoretical ones.

Table N. Optimum bit allocations, theoretical and simulation results of the output *MSE* for the paraunitary FB at (a)  $\rho = 0.95$ , (b) $\rho = 0.75$ .

R	R <sub>0</sub>	R <sub>1</sub>	MSE	MSEsim
0.5	7	1	0.088010	0.087281
0.625	9	I	0.063701	0.063123
0.75	11	1	0.046211	0.045602
1.0	11	5	0.038583	0.038246

R	R <sub>0</sub>		MSE	MSEsim
0.5	7	t	0.317896	0.292531
0.625	9	1	0.247525	0.229685
0.75	11	I	0.197778	0.178358
1.0	11	5	0.145828	0.138982

Table V. Optimum paraunitary filter coefficient at  $(a)\rho = 0.95$ , (b) $\rho = 0.75$ .

R	h <sub>0</sub> (0)	$h_0(1)$	h <sub>0</sub> (2)	h <sub>0</sub> (3)
0.5	0.488441	0.832218	0.226277	-0.132805
0.625	0.488484	0.832219	0.226200	-0.132772
0.75	0.488485	0.832219	0.226199	-0.132771
1.0	0.488501	0.832216	0.226180	-0.132765

(a)

R	$h_0(0)$	$h_0(1)$	$h_0(2)$	$h_0(3)$
0.5	0.50929806	0.81527467	0.23372086	-0.14600427
0.625	0.50929805	0.81527467	0.23372088	-0.14600428
0.75	0.50929806	0.81527467	0.23372088	-0.14600428
1.0	0.50930305	0.81527319	0.23371601	-0.14600293

#### Performance comparison of the filter banks

Optimum design of biorthogonal filter banks with similar AR(1) input and bit constraints can be found on [7]. We observed that optimum paraunitary and biorthogonal filter coefficients are insensitive to changes in average bit rate R and in input correlation  $\rho$  although the output MSE is highly dependent on them. The MSE in biorthogonal filter bank is lower than that for the paraunitary filter bank at input correlation  $\rho = 0.95$ ; but almost equal at input correlation  $\rho = 0.75$ . We next compare the optimum vector quantizer with the optimum scalar quantizer in term of robustness, compression and MSE.

- Robustness: For the scalar case the paraunitary filter coefficients are robust and the biorthogonal filter coefficients are very sensitive to changes in input statistics and bit rate. But for VQ both the paraunitary and the biorthogonal filter coefficients are insensitive to changes in bit rate for fixed input signal statistics.
- 2. Compression and MSE: For  $\rho = 0.95$  and average bit rate of 1 bit/sample, we found that in the scalar case the  $MSE_{sim}$  of 6-tap paraunitary FB is 0.3521838 and for VQ the  $MSE_{sim}$  of 4-tap paraunitary FB is 0.038144. So we conclude that the optimum vector quantizer is superior to optimum schalar to optimum scalar quantizer in terms of compression and MSE.

### M. Conclusions

This paper has presented an approach for modeling of optimum vector quantizer by a scalar nonlinear gain-pulsadditive noise model. The validity and accuracy of this analytical model is confirmed by comparing the calculated quantization errors with actual simulation of the optimum LBG vector quantizer. We have presented specific design examples for 4-tap paraunitary filters in two-band case using this model.

#### References

- A. N. Akansu ana R. A. Haddad, Multiresolution Signal Decomposition: Transforms, Subbands, and Wavelets. Academic Press, 1992.
- J. H. Conway and N. J. A. Sloane, "A lower bound on the average error of vector quantizers," in *IEEE Trans. on Inf. Theory*, vol. IT-31, pp. 106-109, Jan. 1985.
- V. Cuperman, "Joint bit allocation and dimension optimization for vector transform quantization," in *IEEE Trans. on Inf. Theory*, vol. 39, No. 1, pp. 302-305, Jan. 1993.
- A. Gersho, "Asymptotocally optimal block quantization," in *IEEE Trans. on Information Theory*, vol. IT-25, No. 4, pp. 373-380, Jul. 1979.
- A. Gersho and R. Gray, Vector Quantization and Signal Compression. Kluwer Academic Publishers, 1992.
- N. S. Jayant and P. Noll, Digital Coding of Waveforms. Englewood Cliffs, NJ, Prentice-Hall, 1984.
- Innbo Jec, "Vector-quantized M-channel subband codecs: modeling and analysis," Ph.D. Thesis, Dept. of Elect. Eng. Polytechnic Univ, New York, USA, 1995.
- Y. Linde, A. Buzo and R. M. Gray, "An algorithm for vector quantizer design," in *IEEE Trans. Commun.*, vol. COM-28, pp. 84-95, Jan. 1980.
- 9. R. A. Haddad and K. Park, "Modeling, analysis and optimum design of quantized *M*-band filter banks," in *IEEE*

Trans. on Signal Processing, Nov. 1995.

 Innho Jee and R. A. Haddad, "Modeling and analysis of vector-quantized *M*-channel subband codecs," in Proc. *ICASSP-95*, pp. II 1320-1323.

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