

# Active Tonal Noise Canceller with Frequency Tracking

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Abstract

In this paper, we propose a novel adaptive digital filter for tonal noise cancellation, with a frequency tracking capability. The proposed algorithm not only estimates the magnitude and phase of the tonal disturbance but also tracks its frequency, which changes in quasi-static manner. The algorithm uses the steepest descent method and the instantaneous frequency approach for the phase/magnitude estimation and frequency tracking, respectively. A number of computer simulations have been carried out in order to demonstrate the feasibility of the proposed ANC algorithm under various conditions.

## I. Introduction

There are many different methods for cancellation of periodic noise and vibration cancellation. One of the most widely known methods of active noise/vibration cancellation is based on adaptive filtering. Burgess and Widrow introduced the most popular adaptive filtering method called filtered-x least mean square(LMS) algorithm for ANC systems[1, 2]. In many cases, the wave form of the acoustic field is nearly periodic, and the fundamental frequency is generally known. Simpler methods such as adaptive notch filter introduced by Glover and Widrow could be used to cancel the tonal noise[3]. In these previous works, the reference signal is measured or the accurate frequency of the tonal sound is assumed to be known *a priori*. The frequency tracking capability which can provide the estimation of tonal noise frequency is necessary in tonal ANC system because it is difficult to measure the precise frequency of noise and sometimes the frequency drifts in quasi-static manner.

In this paper, we propose a novel tonal ANC algorithm with a frequency tracking capability. The proposed algorithm not only estimates the magnitude and phase of the tonal noise but also track the frequency of the tone. Frequency tracking is based on the instantaneous frequency, which is obtained by Hilbert transform pairs[4]. The effectiveness of proposed tonal ANC algorithm is illustrated by simulations.

## II. Active Tonal Noise Canceller

The adaptive notch filter introduced by Glover and Widrow cannot be used directly for tonal ANC because of presence of the error path. Therefore, we modified an adaptive notch filter accordingly. The block diagram of tonal ANC system for the phase/magnitude estimation is shown in Figure 1.

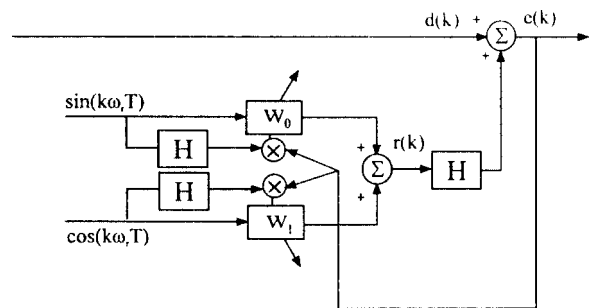


Fig 1. The block diagram of a tonal ANC system

Primary noise signal  $d(k)$  is assumed to be a pure sine wave with the frequency of  $\omega_d$  such as

$$d(k) = D \sin(k\omega_d T + \varphi) = a_0 \sin(k\omega_d T) + a_1 \cos(k\omega_d T) \quad (1)$$

where  $D = \sqrt{a_0^2 + a_1^2}$  and  $\varphi = \tan^{-1} \left( \frac{a_1}{a_0} \right)$ . The tonal noise is cancelled by adding a cancelling signal  $r(k)$ . The cancelling signal has the form of the following:

$$r(k) = w_0(k) \sin(k\omega, T) + w_1(k) \cos(k\omega, T) \quad (2)$$

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where  $T$  is a sampling time and  $w_0(k), w_1(k), k = 0, 1, 2, \dots$  are discrete time-varying Fourier coefficients updated on-line. The frequency  $\omega_r$ , the estimated frequency of  $\omega_d$ , is introduced to accommodate the real-world situation where the frequency  $\omega_d$  may not be identified exactly. The residual noise  $e(k)$  represents the result of acoustical interference as follows:

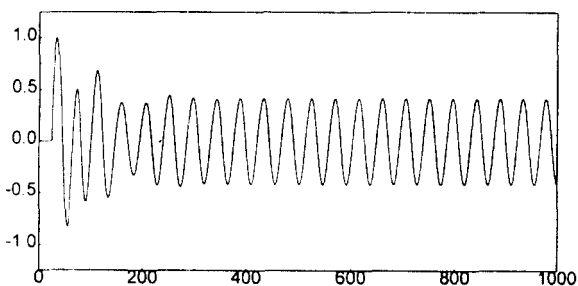
$$e(k) = d(k) - H \otimes r(k) \tag{3}$$

where  $H$  is a transfer function of the error path. The optimum set of filter coefficients required to minimize the residual can be adaptively obtained using a steepest descent method. The filter coefficients,  $w_0(k)$  and  $w_1(k)$ , are updated as follows:

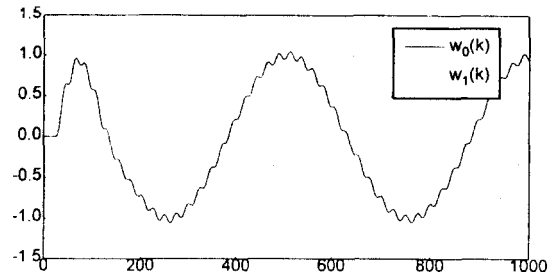
$$\begin{aligned} w_0(k+1) &= w_0(k) + 2\eta e(k) h \sin(k\omega_r T + \Phi) \\ w_1(k+1) &= w_1(k) + 2\eta e(k) h \cos(k\omega_r T + \Phi) \end{aligned} \tag{4}$$

where  $\eta$  is convergence factor which determines the speed of adaptation,  $h, \Phi$  are magnitude and delay of the error path, respectively. The method is easily extended to the case when periodic noise with arbitrary number of harmonics is needed to be cancelled.

We perform a simulation with the desired and the estimated frequencies of 0.02 Hz ( $f_d = \omega_d/2\pi$ ), 0.022 Hz ( $f_r = \omega_r/2\pi$ ) respectively. Figure 2 shows the resulting time history plot of the filter coefficients  $w_0(k), w_1(k)$  and residual noise  $e(k)$ . As expected, the coefficients do not converge to constants and oscillate. Figure 3 shows the spectrum of time history of the filter coefficients. The time varying filter coefficients oscillate with the fundamental frequency of  $f_d - f_r$ , (0.002 Hz). The other component with the frequency of two times of the true frequency  $f_r$ , (0.02  $\times$  2 Hz), i.e., carrier frequency is added to the fundamental. Therefore, time varying filter coefficients



(a) residual noise  $e(k)$



(b) filter coefficients  $w_0(k), w_1(k)$

Fig 2. The resulting time history plots of the filter coefficients  $w_0(k), w_1(k)$  and residual noise  $e(k)$

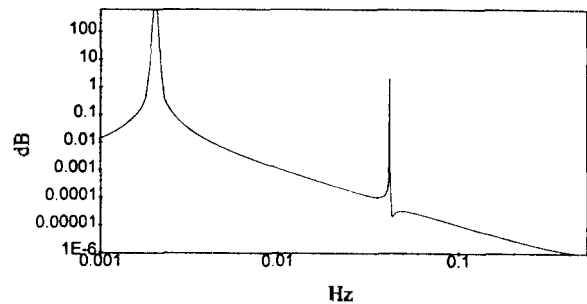


Fig 3. Spectrum of time varying coefficient

has useful information which can be utilized for frequency error estimation.

### III. Frequency Tracking Based on Instantaneous Frequency

The block diagram of tonal ANC system with frequency tracking capability is shown in Figure 4.

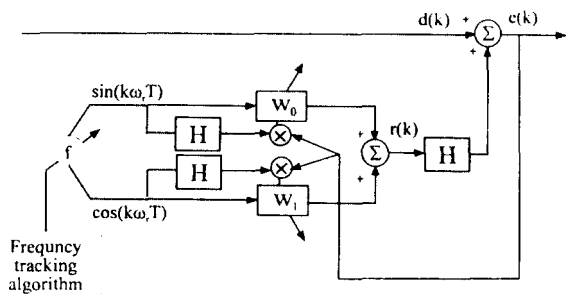


Fig 4. The block diagram of tonal ANC system with frequency tracking capability

The controller uses instantaneous frequency approach for frequency tracking. The fundamental instantaneous frequency, i.e., the difference of the frequency estimation  $f_d - f_r$ , can be obtained by differentiating the phase of analytic signal, which can be calculated from the Hilbert transform pairs of the time varying coefficients  $w_0(k)$  and  $w_1(k)$ . Let the analytic signal  $z(k)$  has the coefficient  $w_0(k)$  and its Hilbert pair,  $w_1(k)$  [4].

$$z(k) = w_0(k) + w_1(k) \quad (5)$$

The envelope and phase of  $z(k)$  can be written as

$$|z(k)| = \sqrt{w_0^2(k) + w_1^2(k)} \quad (6)$$

$$\varphi(k) = \tan^{-1} \left( \frac{w_1(k)}{w_0(k)} \right) \quad (7)$$

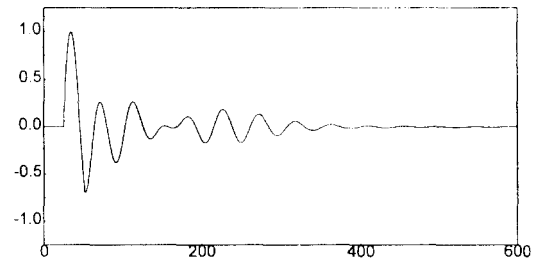
Therefore, the radial frequency  $\omega$  can be obtained from the derivative of eq.(7). The instantaneous frequency  $f(k)$ , the frequency estimation error  $f_d - f_r$ , is

$$f(k) = \frac{1}{2\pi} \frac{d\varphi(k)}{dk} = -\frac{\frac{d w_1(k)}{dk} w_0(k) + \frac{d w_0(k)}{dk} w_1(k)}{2\pi |z^2(k)|} \quad (8)$$

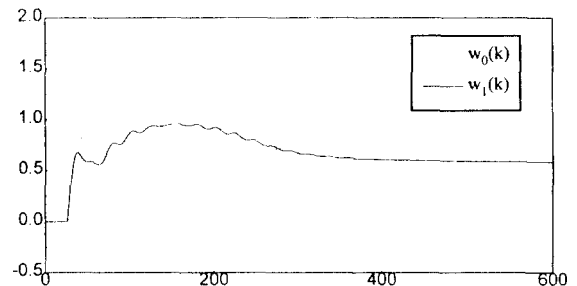
where  $\frac{d w_i(k)}{dk}$  is the time derivative of  $w_i(k)$ . We can calculate instantaneous frequency from the  $w_0(k)$ ,  $w_1(k)$  and their derivatives. There are two ways to calculate the derivatives of the functions[5]. One way of calculating the derivatives is to use the phase unwrapping algorithm. This method removes the discontinuity in the phase function. Another approach is to calculate the derivative of the function directly from the signal  $w_0(k)$  and  $w_1(k)$ . Since this approach does not includes the unwrapping procedure, the calculation is straight-forward, however, the output is sensitive to the window function and noise. In this paper, we take a direct derivative approach because of its simplicity. One requires a low pass filter which can minimize the effect of noise on the instantaneous frequency estimation.

In case of 0.002 Hz frequency error, we perform a simulation with the same condition of the previous section. ANC with frequency tracking is enabled at 0 step. Because of frequency tracking, the coefficients converge to constant steady state values as shown in Figure 5. After the ANC is on, the tonal noise is removed within 500th step. It is also interesting to investigate the frequency tracking capability in case

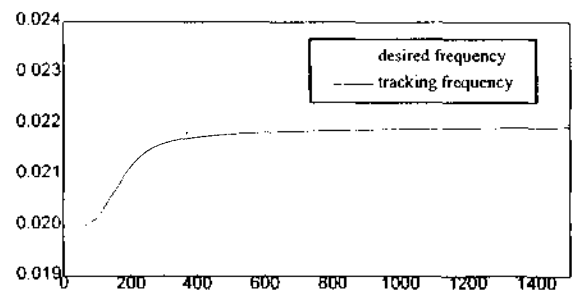
of slowly changing frequency. Figure 6 shows the stable tracking capability of the proposed ANC system. Drift of coefficients in Figure 6 (b) enables not only the estimation of the magnitude and phase of the tonal noise but also track the slowly changing frequency of the tone. Figure 7 shows the tracking capability of the ANC system when the uncorrelated input noise is added. Since the uncorrelated noise exist after control, the coefficients fluctuate around constant levels. But the frequency tracking capability is robust as shown in Figure 7(c).



(a) residual noise  $e(k)$

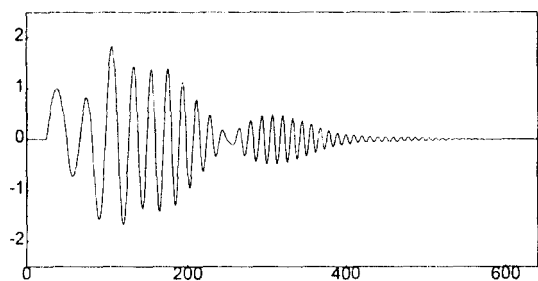


(b) filter coefficients  $w_0(k)$ ,  $w_1(k)$

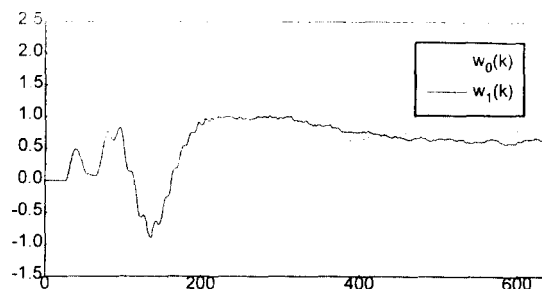


(c) tracking capability

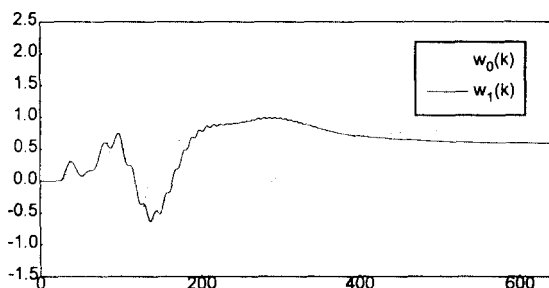
Fig 5. The resulting time history plots of the filter coefficients  $w_0(k)$ ,  $w_1(k)$  and residual noise  $e(k)$  in case of the tonal ANC system with frequency tracking capability



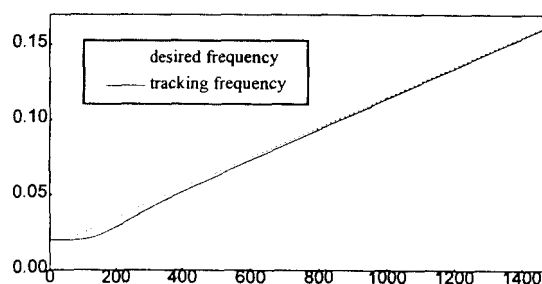
(a) residual noise  $e(k)$



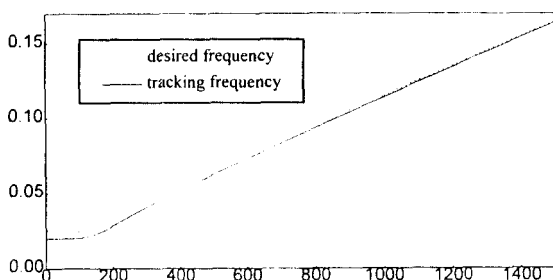
(b) filter coefficients  $w_0(k), w_1(k)$



(b) filter coefficients  $w_0(k), w_1(k)$

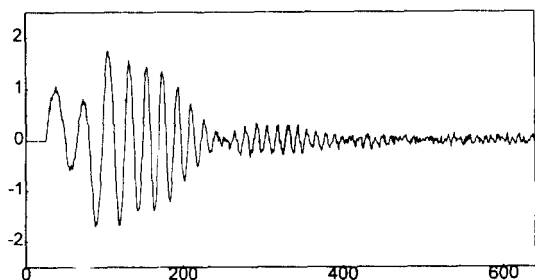


(c) tracking capability



(c) tracking capability

Fig 6. The resulting time history plots of the filter coefficients  $w_0(k), w_1(k)$  residual noise  $e(k)$  and frequency tracking for slowly changing frequency in case of the tonal ANC system with frequency tracking capability



(a) residual noise  $e(k)$

Fig 7. The resulting time history plots of the filter coefficients  $w_0(k), w_1(k)$  residual noise  $e(k)$  and frequency tracking for slowly changing frequency in case of the tonal ANC system with frequency tracking capability when the uncorrelated input noise is added

#### IV. Conclusion

A novel tonal ANC system with a frequency tracking capability is proposed for the real-world situation. For the estimation of the magnitude and phase, the adaptive notch filter is used. Time varying coefficients which act as the Hilbert pair is used to get the instantaneous frequency. Simulation results demonstrate the effectiveness of proposed tonal ANC system.

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