Prony based Multipath Channel Parameter Estimation not Requiring the Number of Received Rays

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ABSTRACT

This paper presents an algorithm for multipath channel parameter estimation by an improved Prony method. This algorithm applies a modified regularized spectral estimation to the conventional SVD Prony method. This method requires no *a priori* information on the number of multipath. The performance of the proposed algorithm is almost the same as that of the SVD based multipath channel parameter estimation algorithm.

I. Introduction

Multipath propagation has been of interest to engineers in communication, radar, underwater telemetry and others for decades. The demand for higher bit rates in digital telecommunication, requiring wider bandwidth and advanced modulation schemes, makes detailed modeling crucial since the multipath may induce degradation. For example, the reception of several differently delayed replicas of the transmission causes intersymbol interference in the time domain and deep selective fades, known as "nulls" or "notches" in the frequency domain.

For the multipath channel parameter estimation, many different methods have been studied, e.g. adaptive techniques, the Prony model based method and others[1, 2]. The Prony model based method uses the SVD to solve the Prony model. In solving by the SVD, however, the exact Prony model can not be derived if there is no a priori information about the number of received rays which is the same as the number of principal eigenvalues of Prony model matrix. A method to circumvent the shortcoming of the SVD was proposed, which is the multiple regularization inverse method[3]. The method does not require a priori information of the number of principal eigenvalues. Unfortunately performance of the multiple regularization inverse method degrades in the case of short time recorded noisy data. This degradation was improved by the modified regularized spectral estimation method(MORSE)[4].

In this paper, we propose two algorithms based on the MORSE to estimate multipath channel parameter estimation without a priori information referring to the number of received rays. One of the algorithms directly applies the MORSE algorithm to the Prony based multipath parameter estimation and the other is a simplified one of the above algorithm.

In Section II, the Prony based multipath channel parameter estimation algorithm is described. In Section III, multipath channel parameter estimation algorithms based on the MORSE is proposed. The performance is demonstrated in Section IV and we conclude in Section V.

II. Prony based multipath channel parameter estimation

We assume that a multipath signal is the sum of the finite number of delayed and attenuated copies of transmission signal. The impulse response of the multipath channel is eqn 1[1, 2].

$$h(t)=\sum_{k=1}^{N} a_k\delta(t-\tau_k). \tag{1}$$

where a_k is the attenuation factor of the kth path and τ_k is the delay factor of the kth path. In the frequency domain, eqn. 1 is transformed to

$$\begin{aligned} H(\omega_n)=H_n&=\sum_{k=1}^{N} a_k e^{j\omega_n\tau_k}=\sum_{k=1}^{N}\{a_k e^{j\omega_0\tau_k}e^{-jn\Delta\omega\tau_k}\} \\ &=\sum_{k=1}^{N} b_k z_k^n, \end{aligned} \tag{2}$$

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where $\omega_n = \omega_0 + n\Delta\omega$ and $\Delta\omega$ frequency sampling distance. Eqn. 2 has AR relationship as eqn. 3 [1].

$$H_n = -\sum_{m=1}^{N} \lambda_m H_{n-m} \text{ or } H_n^* = -\sum_{m=1}^{N} \lambda_m H_{n+m}^*. \tag{3}$$

From eqn.3 we can derive the prediction polynomial $P_N(z)$ as

$$P_N(z) = z^N + \sum_{i=1}^{N} \lambda_i z^{N-i}, \tag{4}$$

where z_k is the kth zero of eqn. 4. The delay factor can be derived from the zeroes and eqn. 2. To estimate the z_k, FBLP method is usually used.

If we measure M data values which contain N_0 rays, N th order $(M > N > N_0)$ linear prediction can be used as eqn. 5 [1, 2].

$$h = H\lambda, \tag{5}$$

where h is $2(M-N)\times 1$ vector, H is $2(M-N)\times N$ matrix and λ is $N\times 1$ vector. Eqn 5 is solved as eqn 6.

$$\lambda = (H^H H)^{-1} H^H h, \tag{6}$$

where H^H is the complex transpose of H. In high SNR, $H^H H$ has N_0 rank which is the same as the number of received rays and then eqn. 6 is also expressed as eqn. 7 by SVD [1, 2].

$$\lambda = \sum_{i=1}^{N_0} \left\{ \frac{u_i^H r}{\sigma_i} \right\} u_i, \tag{7}$$

where u_i is the i-th eigenvector of $H^H H$, σ_i is the i-th eigenvalue of $H^H H$, and $r = H^H h$.

The estimates of the attenuation factors come from eqn. 8 and eqn. 9.

$$\hat{h} = \hat{W} b, \tag{8}$$

$$b = (\hat{W}^H \hat{W})^{-1} \hat{W}^H \hat{h}, \tag{9}$$

where $\hat{h} = [\hat{H}_0, \hat{H}_1, \cdots, \hat{H}_{M-1}]$, $b = [b_1, b_2, \cdots, b_N]$

and $\hat{W} = \begin{vmatrix} 1 & \cdots & 1 \\ z_1 & \cdots & z_N \\ \vdots & & \vdots \\ z_1^{M-1} & \cdots & z_N^{M-1} \end{vmatrix}$

Ⅲ. Multipath channel parameter estimation based on MORSE

If the measurements are noiseless and correspond exactly to the model in eqn. 2, $H^H H$ has rank N_0 and only the first N_0 eigenvalues of $H^H H$ are non zero. In practice, the correct model order N_0 is not known *a priori*, so it should be estimated. In this case, the MORSE can be applied effectively [4]. The method derives the regularization coefficient, μ_i, iteratively for each eigenvalue by eqn. 10.

$$\mu_i = \begin{cases} \dfrac{\sigma_i}{2}\left(\gamma_i - 2 - \sqrt{\gamma_i(\gamma_i - 4)}\right) & \gamma_i > 4 \\ \infty & \gamma_i \le 4, \end{cases} \tag{10}$$

where $\gamma_i = |u_i^H r|^2/\sigma_w^2$, in which u_i is the i-th eigenvector of $H^H H$ and σ_w^2 is the variance of noise.

The regularization coefficients become large for noise eigenvalue and small for signal eigenvalues. If the regularization coefficient is applied to eqn. 11, the effect of noise eigenvalues is selectively reduced so considerably that the results of the estimation is almost equivalent to that of the result of SVD based method with known number of the model order[3, 4].

$$\lambda = \sum_{i=1}^{N} \left\{ \frac{u_i^H r}{\sigma_i + \mu_i} \right\} u_i. \tag{11}$$

The MORSE based estimation procedure is summarized in table1. In the table 1, the procedure is composed of the initial conditioning part, i)~iv), and the estimation part, v)~x).

In this paper a simplified MORSE based procedure is also summarized in table 2. In the simplified procedure, initial noise variance is derived from the least eigenvalue in the SVD of data matrix H because the real noise variance has the relationship as eqn. 12 [5]. This makes it possible to calculate the regularization factors without the initial noise estimation procedure in [4].

$$\sigma_w^2 \gtrsim \frac{1}{2\times(M-N)}\ (\textit{the smallest } \sigma_i)^2. \tag{12}$$

where σ_w^2 is the noise variance and σ_i is ith eigenvalue of data matrix H.

The coefficients estimates of b_i are found as ordinary least square solution of eqn. 9 given the zero estimates $\{z_i\}_{i=1}^{N}$.

The simplified MORSE based estimation enables us to skip initialization steps in table 1, from step (i) to step (iv), and replace them with the single equation of eqn. 12.

Finally we compare the MORSE based Prony algorithms with the SVD based Prony algorithm in table 3. In table 3 the SVD based Prony algorithm is a special case of the MORSE based Prony algorithm.

Table 1. Multipath channel parameter estimation procedure based on modified regularized spectral estimation method

i)	initialize μ_i with any small value, and create FBLP data matrix H and h vector in eqn. 5.
ii)	estimate noise variance σ_w^2 with H, h and μ_i by eqn. T1a.
iii)	calculate regularization factor μ_i by eqn. T1b with noise variance σ_w^2, eigenvector u_i and r for all eigenvalues.
iv)	go to v) if regularization factors converge, otherwise repeat ii) and iii).
v)	inverse matrix with the regularization factors by eqn. 11 and estimate spectrum $P_M(\omega)$ in eqn. 4.
vi)	increase σ_w^2 to $\sigma_w^2 = \sigma_w^2 + \delta\sigma_w^2$, estimate μ_i by eqn.10 for all eigenvalues and update spectrum $P_M(\omega)$.
vii)	test convergence between past log spectrum and updated log spectrum.
viii)	repeat vi) and vii) until the log spectrum converges.
ix)	estimate zeros, z_i, from $P_M(\omega)$, and calculate delay factor from eqn. 2.
x)	estimate attenuation factors from eqn. 9 with the estimated zeros in ix)

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$$\sigma_w^2 = \frac{1}{m}\left\{ \| h \|^2 - \sum_{i=1}^{n} \frac{\sigma_i \, | u_i^H h |^2}{\sigma_i + \mu_i} \right\} \tag{T1a}$$

$$\mu_i = \sigma_w^2 (\sigma_i + \mu_{ip})^2 / \sigma_i \, | u_i^H y |^2 \tag{T1b}$$

where m is the row dimension of H, n is total number of eigenvalues larger than zero, u_i is the ith eigenvector of H, μ_{ip} is μ_i in previous iteration and σ_w^2 is the variance of noise.

Table 2. Multipath parameter estimation procedure using simplified MORSE

i)	create FBLP data matrix H and h vector in eqn. 5. and calculate the SVD of H as $V\Sigma U^H$.
ii)	estimate noise variance σ_w^2 from the least eigenvalue larger than zero in the eigenvalues of data matrix H as eqn 12.
iii)	calculate regularization factor μ_i by eqn. 10 with eigenvector u_i and r for all eigenvalues.
iv)	inverse matrix with the regularization factors by eqn. 11 and estimate spectrum $P_M(\omega)$.
v)	increase σ_w^2 to $\sigma_w^2 = \sigma_w^2 + \delta\sigma_w^2$, estimate μ_i for all eigenvalues and update spectrum $P_M(\omega)$.
vi)	test convergence between past log spectrum and updated log spectrum.
vii)	repeat v) and vi) until the log spectrum converges.
viii)	estimate zeros z_i, from $P_M(\omega)$, and calculate delay factor from eqn 2.
ix)	estimate attenuation factors from eqn. 9 with the estimated zeros in viii)

Table 3. Comparison between the SVD-Prony based method and the MORSE based method

	MORSE based method	SVD-Prony based method
governing equation	$h = H\lambda$	$h = H\lambda$

solution	$\lambda = \sum_{i=1}^{N_s}\left\{\frac{u_i^H r}{\sigma_i+\mu_i}\right\}u_i$	$\lambda = \sum_{i=1}^{N_s}\left\{\frac{u_i^H r}{\sigma_i}\right\}u_i$
eigenvalue truncation	The μ_i for signal eigenvalue is much smaller than the corresponding eigenvalue. The μ_i for noise eigenvalue is much larger than the corresponding eigenvalue or infinite.	This is a special case of the proposed method: The μ_i for signal eigenvalue is '0'. The μ_i for noise eigenvalue is infinite.
remarks	This method can truncate eigenvalues by observed data.	This method can truncate eigenvalues by *a priori* information of the number of receiving rays

Ⅳ. Simulation

To examine the performance of the proposed algorithms, we compare the performance of these algorithms with that of SVD based algorithm with known number of rays. In the experiments we have considered a four-path channel with $A_1=1$, $D_1=2T_s$, $A_2=0.9$, $D_2=6T_s$, $A_3=0.8$, $D_3=11T_s$, $A_4=0.7$, $D_4=16T_s$, where A_i is the attenuation factor of the i-th path, T_s is sampling interval and D_i is the delay factor of the i-th path. In the experiments, we assumed that $M=64$ and $N=24$.

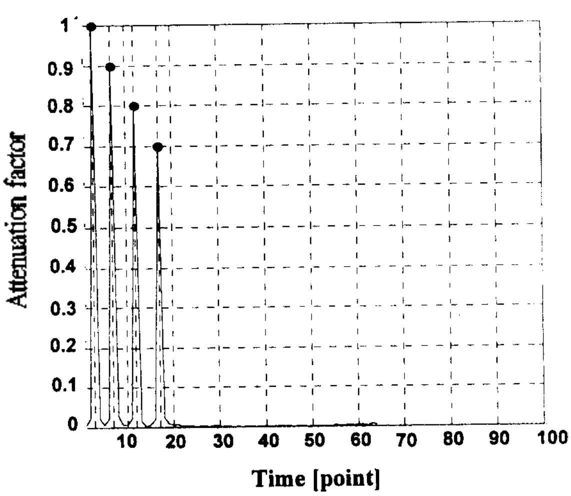

Fig. 1. Attenuation factors and time delay estimates by the proposed algorithm in 15dB SNR

Figure 1 presents the result referring to single trial of SNR = 15dB. To carry out a statistical performance analysis of the considered algorithms, we refer now 50 trials of the previous experiments under the different SNR. The performances are evaluated in terms of the sample normalized mean integrated square error ε on the reconstruction of the multipath channel transfer function [2]:

$$\varepsilon = \frac{\hat{E}\left\{\int_0^{1/2T_s}|H(f)|^2\,df\right\}}{\int_0^{1/2T_s}|H(f)|^2\,df}, \qquad (13)$$

where $H(f)$ is the true transfer function, $H(f)$ is the reconstructed transfer function in the band $|f| \leq$

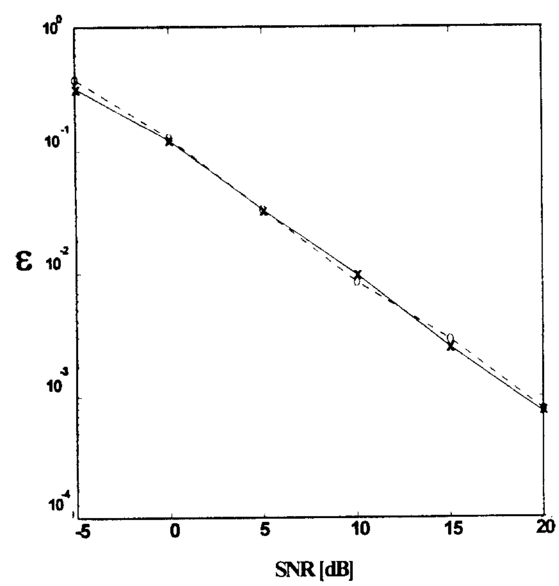

Fig. 2. Sample normalized mean integrated square error ε versus SNR

------- the MORSE based method

—x---- the simplified MORSE based method

-----0----- the SVD based method

$1/2T_s$, in which T_s is sampling interval, and $\hat{E}\{\}$ denotes sample mean. Figure 2 presents the behavior of ε versus SNR for the proposed algorithms and the SVD based algorithm respectively. The performance of the proposed algorithms is equivalent to that of the SVD based algorithm with the known number of rays, even though the proposed algorithm has no information of the number of rays. Figure 2 also shows that the simplified MORSE algorithm does not degrade the performance of the MORSE based algorithm.

V. Conclusions

The problem of estimating multipath channel parameters has been considered. Two MORSE based algorithms have been presented. It exploits the modified regularization to obtain channel estimates without any prior information of the number of received rays.

The performance of the considered algorithms has been evaluated in terms of sample mean integrated square error on the reconstruction of the multipath channel transfer function. It is shown that the performance of the proposed algorithms without any prior information of the number of rays is almost equivalent to that of SVD based algorithm with prior information of the number of rays.

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