Extension and Appication of Total Least Squares Method for the Identification of Bilinear Systems

Seokwon Han*, Jinyoung Kim*, and Koengmo Sung*

ABSTRACT

When the input-output record is available, the identification of a bilinear system is considered. It is assumed that the input is noise free and the output is contaminated by an additive noise. It is further assumed that the covariance matrix of the noise is known up to a factor of proportionality. The extended generalized total least squares (e-GTLS) method is proposed as one of the consistent estimators of the bilinear system parameters. Considering that the input is noise-free and that bilinear system equation is linear with respect to the system parameters, we extend the GTLS problem. The extended GTLS problem is reduced to an unconstrained minimization problem, and is solved by the Newton-Raphson method. We compare the GTLS method and the e-GTLS method in the point of the accuracy of the estimated system parameters.

Ⅰ. Introduction

There have been various studies on bilinear systems due to their simple structure, their similarity to linear systems and their applicability to real processes [1-5]. However, there is little research on the identification of bilinear systems in the presence of an additive noise. Such methods as the Gabr-Rao method and the Inagaki-Mochizuki method have been investigated[4, 5]. The Gabr-Rao method uses the likelihood function and the Newton-Raphson method. It is therefore computationally intensive. The Inagaki-Mochizuki method uses the Volterra representation transformed from the bilinear equation. Thus it is a complex method.

On the other hand, there are extensive studies on the identification of linear systems, and the literature on the identification of noisy systems is extensive[6]. Unfortunately, most of the identification methods applied to noisy linear systems are not applicable to the identification of noisy nonlinear systems. Recently, the generalized total least squares (GTLS) method was successfully applied to the identification of noisy linear systems[7,8]. It is possible to apply the GTLS method directly to the identification of bilinear systems through minor modifications to the covariance matrix of the errors in a row of the data matrix. However, the direct application of the GTLS problem neglects the structure of the data matrix of bilinear systems, which arises from only the output being corrupted by noise, and that the bilinear system equation is linear with respect to the system parameters.

In this paper, we extend the GTLS problem considering the special structure of the data matrix of bilinear system equations. We show that the extended GTLS(e-GTLS) problem is reduced to an unconstrained minimization problem using the method of Lagrange multipliers. The e-GTLS problem is then solved iteratively using Newton-Raphson method[12]. Through computer simulation we compare the performances of the e-GTLS method with those of the GTLS method.

Ⅱ. The structure of the data matrix of bilinear system Equations

We consider the bilinear system which is the single input, single output (SISO) and is also a time-invariant system. The bilinear system shown in Fig. 1 is described by the following Eqs (1) and (2). It is assumed that the order of the system and the delay of the system are known.

$$x_t = \sum_{i=1}^{p} a_i x_{t-i} + \sum_{j=0}^{q} b_j u_{t-d-j} + \sum_{k_1=1}^{l} \sum_{k_2=1}^{m} c_{k_1 k_2} x_{t-k_1} u_{t-d+1-k_2} \tag{1}$$

$$y_t = x_t + e_t \tag{2}$$

*Dept. of Electronics Engineering Seoul National University

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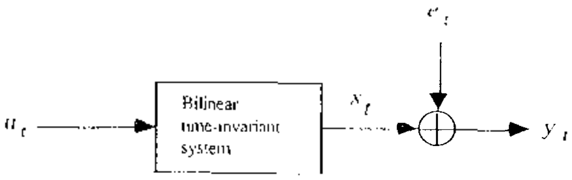


Fig 1. Bilinear system.

where $\{u_t\}$ is the input, $\{x_t\}$ is the uncorrupted output, $\{y_t\}$ is the corrupted output, $\{e_t\}$ is the additive noise and d is the delay of the system. We assume that the covariance matrix of the additive noise is known up to a factor of proportionality. For simplicity we also assume that l is less than p in Eq. (1).

In general the least square(LS) method is used for the estimation of the system parameters $\{a_i, b_j, c_{k_1k_2}\}$ when the input and the output y_t are known. However, the parameters estimated by the LS method are biased, for output is corrupted by the noise. Thus the LS method fails to estimate system parameters correctly.

Let $\mathbf{h}$ be $[\mathbf{b}^T, \mathbf{a}^T, \mathbf{c}_1^T \cdots \mathbf{c}_m^T]^T$. By considering N over-determined bilinear equations, that is, t = 1, 2 ,...,N, the system equations are represented in the following matrix form.

$$[\mathbf{U}\ \mathbf{Y}\ \mathbf{Z}_1 \cdots \mathbf{Z}_m]\cdot \mathbf{h} \approx 0 \tag{3}$$

$$\text{where } \mathbf{U}=\begin{bmatrix}\mathbf{u}_1^r\\ \mathbf{u}_2^r\\ \vdots\\ \mathbf{u}_N^r\end{bmatrix}=\left[\mathbf{u}_1^c\ \mathbf{u}_2^c \cdots \mathbf{u}_{q+1}^c\right],\quad \mathbf{Y}=\begin{bmatrix}\mathbf{y}_1^r\\ \mathbf{y}_2^r\\ \vdots\\ \mathbf{y}_N^r\end{bmatrix}$$

$$=\left[\mathbf{y}_1^c\ \ \mathbf{y}_2^c \cdots \mathbf{y}_{p+1}^c\right],$$

$$\mathbf{u}_t^r=\begin{bmatrix}u_{t-d}\\ u_{t-d-1}\\ \vdots\\ u_{t-d-q}\end{bmatrix}^T,\quad \mathbf{a}=\begin{bmatrix}-1\\ a_1\\ \vdots\\ a_p\end{bmatrix},\ \mathbf{b}=\begin{bmatrix}-1\\ b_1\\ \vdots\\ b_q\end{bmatrix},\ \mathbf{c}_k=\begin{bmatrix}c_{1k}\\ c_{2k}\\ \vdots\\ c_{lk}\end{bmatrix},$$

$$\mathbf{y}_t^r=\begin{bmatrix}y_{t-d}\\ y_{t-d-1}\\ \vdots\\ y_{t-d-p}\end{bmatrix}^T,$$

$$\mathbf{Z}_k=\begin{bmatrix}u_{2-d-k}y_0 & u_{2-d-k}y_{-1} & \cdots & u_{2-d-k}y_{1-l}\\ u_{3-d-k}y_1 & u_{3-d-k}y_0 & \cdots & u_{3-d-k}y_{2-l}\\ \vdots & \vdots & \vdots & \vdots\\ u_{N+1-d-k}y_{N-1} & u_{N+1-d-k}y_{N-2} & \cdots & u_{N+1-d-k}y_{N-l}\end{bmatrix},$$

$\mathbf{U}\in R^{N\times(q+1)}$, $\mathbf{Y}\in R^{N\times(p+1)}$, $\mathbf{u}_t^r\in R^{1\times(q+1)}$, $\mathbf{u}_t^c\in R^{N\times 1}$, $\mathbf{y}_t^r\in R^{1\times(p+1)}$, $\mathbf{y}_t^c\in R^{N\times 1}$, $\mathbf{Z}_k\in R^{N\times l}$, $\mathbf{a}\in R^{(p+1)\times 1}$, $\mathbf{b}\in R^{(q+1)\times 1}$, $\mathbf{c}_k\in R^{l\times 1}$.

Let $\mathbf{D}$ be $[\mathbf{U}\ \mathbf{Y}\ \mathbf{Z}_1 \cdots \mathbf{Z}_m]$. In the data matrix $\mathbf{D}$, only $\mathbf{Y}$ and $\mathbf{Z}_k$ are subject to error due to additive noise. From the data matrix $\mathbf{D}$ it can be observed that there is a special relation between the matrices $\mathbf{Y}$ and $\mathbf{Z}_k$. Let $\mathbf{Y}$ be partitioned as $[\mathbf{Y}_1\ \mathbf{Y}_2\ \mathbf{Y}_3]$ where $\mathbf{Y}_1=[\mathbf{y}_1^c]\in R^{N\times 1}$, $\mathbf{Y}_2=[\mathbf{y}_2^c \cdots \mathbf{y}_{l+1}^c]\in R^{N\times l}$, $\mathbf{Y}_3=[\mathbf{y}_{l+2}^c \cdots \mathbf{y}_{p+1}^c]\in R^{N\times(p-l)}$. Also, define $\mathbf{W}_k$ as $[\mathbf{u}_k^c \cdots \mathbf{u}_k^c]\in R^{(N\times l)}$, that is, all the columns of $\mathbf{W}_k$ are the k_{th} column vector of $\mathbf{U}$. Then the following Equation is satisfied.

$$\mathbf{Z}_k=\mathbf{W}_k\otimes\mathbf{Y}_2 \tag{4}$$

where $\mathbf{W}_k\otimes\mathbf{Y}_2$ is the direct or Hadamard product. Then Eq. (3) is represented as follows by using the relation of Eq. (4).

$$[\mathbf{U}\ \mathbf{Y}\ \mathbf{W}_k\otimes\mathbf{Y}_2 \cdots \mathbf{W}_m\otimes\mathbf{Y}_2]\cdot\mathbf{h}\approx \mathbf{0} \tag{5}$$

Ⅲ. Application of the GTLS method to the bilinear system identification

In this section the GTLS method is directly applied to estimation of the system parameters. Let the data matrix $\mathbf{D}$ be partitioned as $[\mathbf{D}_1\ \mathbf{D}_2]$, where $\mathbf{D}_1=[\mathbf{U}]\in R^{N\times(q+1)}$ and $\mathbf{D}_2=[\mathbf{Y}\ \mathbf{W}_1\otimes\mathbf{Y}_2 \cdots \mathbf{W}_m\otimes\mathbf{Y}_2]\in R^{N\times(p+lm+1)}$. Only the matrix $\mathbf{D}_2$ is subject to error. Then the GTLS problem for the bilinear system identification is described as minimization of $\|\Delta\hat{\mathbf{D}}_2 \mathbf{R}_{\Delta\mathbf{D}_2}^{-1/2}\|_F$ with respect to $\{a_i, b_j, c_{k_1k_2}\}$ and subject to $[\mathbf{D}_1\ \hat{\mathbf{D}}_2]\cdot\mathbf{h}=0$ and $a_0=-1$ where $\mathbf{R}_{\Delta\hat{\mathbf{D}}_2}$ is the covariance matrix of the errors in the data matrix, $\|\cdot\|_F$ is the Frobenius norm and $\Delta\hat{\mathbf{D}}_2$ is the error matrix of $\mathbf{D}_2$ i.e. $\Delta\hat{\mathbf{D}}_2=\hat{\mathbf{D}}_2-\mathbf{D}_2$. We assume that the covariance matrix of the additive noise is known up to a factor of proportionality. Substituting Eq. (2) in the matrix $\mathbf{D}_2$ and inspecting it, we observe that the error vector of the tth row $\Delta\hat{\mathbf{d}}_{2t}^r$ is represented as follows.

$$\Delta\hat{\mathbf{d}}_{2t}^r=\left[\Delta\hat{\mathbf{y}}_t^r\ \ \Delta\hat{\mathbf{z}}_{1t}^r\cdots\Delta\hat{\mathbf{z}}_{mt}^r\right]$$

where $\Delta\hat{\mathbf{y}}_t^r=\left[e_t \cdots e_{t-p}\right]$ and $\Delta\hat{\mathbf{z}}_{kt}^r$ $=\left[u_{t+1-d-k}e_{t-1} \cdots u_{t+1-d-k}e_{t-l}\right]$.

Let $\mathbf{R}_{ee}$ be the (p + 1) by (p + 1) covariance matrix of the additive noise. Then $\mathbf{R}_{\Delta\hat{\mathbf{D}}_2}$ is represented by Eq. (6).

$$\mathbf{R}_{\Delta\hat{D}_2}=\mathbf{E}\left[\left(\Delta\hat{\mathbf{d}}_{2t}^{r}\right)^{T}\Delta\hat{\mathbf{d}}_{2t}^{r}\right]=\begin{bmatrix}\Gamma_{aa} & \Gamma_{ac_1} & \cdots & \Gamma_{ac_m}\\ \Gamma_{c_1a} & \Gamma_{c_1c_1} & \cdots & \Gamma_{c_1c_m}\\ \vdots & \vdots & \vdots & \\ \Gamma_{c_ma} & \Gamma_{c_mc_1} & \cdots & \Gamma_{c_mc_m}\end{bmatrix} \qquad (6)$$

where $\Gamma_{aa}(i,j)=\mathbf{R}_{ee}(i,j)$,
$\Gamma_{ac_n}(i,j)=\Gamma_{c_na}(i,j)=\mathbf{R}_{ee}(i,j)\cdot m_u$,

$\Gamma_{c_nc_k}(i,j)=\Gamma_{c_kc_n}(i,j)=\mathbf{R}_{ee}(i,j)\cdot\mathbf{R}_{uu}(k,n)$, m_u is the mean of the input and $\mathbf{R}_{uu}(k,n)$ is the autocovariance matrix of the input. Using $\mathbf{R}_{\Delta D'_2}$ in Eq. (6) we can solved by the generalized singular value decomposition (GSVD) and the QR factorization methods[7].

Ⅳ. Extension of the GTLS problem for the bilinear system identification

4.1. The definition of the e-GTLS problem

The GTLS method, discussed in section 3, does not consider the special structure of the data matrix D. That is, the error matrices of $\mathbf{Y}$ and $\mathbf{Z}_k$ are treated independently. So, it is necessary to modify the GTLS problem so that the relation of Eq.(4) is considered. From Eq.(4) we observed that if the error matrix of Y, i.e., $\Delta\hat{\mathbf{Y}}$ is determined, the error matrix of $\mathbf{Z}_k$, $\Delta\hat{\mathbf{z}}_k$ is determined by $\Delta\hat{\mathbf{Z}}_k=\mathbf{W}_k\otimes\Delta\hat{\mathbf{Y}}_2$ where $\Delta\hat{\mathbf{Y}}_2$ is the error matrix of $\mathbf{Y}_2$. Then the GTLS problem is extended as minimization of $\left\|\Delta\hat{\mathbf{Y}}\ \mathbf{R}_{ee}^{-1/2}\right\|$ with respect to $\{a_i, b_j, c_{k_1k_2}\}$ and subject to $\left[\mathbf{U}\ \hat{\mathbf{Y}}\ \mathbf{W}_1\otimes\hat{\mathbf{Y}}_2\cdots\mathbf{W}_m\otimes\hat{\mathbf{Y}}_2\right]\cdot\mathbf{h}=0$ and $a_0=-1$ where $\hat{\mathbf{Y}}_i=\mathbf{Y}_i+\Delta\hat{\mathbf{Y}}_i$, $\Delta\hat{\mathbf{Y}}_i$, is the error matrix of $\mathbf{Y}_i$ and $\mathbf{R}_{ee}$ is the covariance matrix of the additive noise and $\mathbf{R}_{ee}\in R^{(p+1)\times(p+1)}$. We assume that is known up to a factor of proportionality. In the e-GTLS problem the weighting matrix $\mathbf{R}_{ee}^{-1/2}$ is used for the decorrelation of the error vector $\Delta\hat{y}_i$. It can be explained as follows. In the ideal case $\Delta\hat{y}_i=\left[e_i\ e_{i-1}\cdots e_{i-p}\right]^T$ and $\mathrm{E}\left[\Delta\hat{y}_i\ \Delta\hat{y}_i^T\right]=\mathbf{R}_{ee}$. That is, the estimated error vector is correlated. So, it is necessary to whiten the error vector by $\mathbf{R}_{ee}^{-1/2}$.

4.2. The e-GTLS method and the ML estimator

Considering $\left\|\Delta\hat{\mathbf{Y}}\ \mathbf{R}_{ee}^{-1/2}\right\|_F$ in detail, we observe that the following Equation is satisfied.

$$\left\|\Delta\hat{\mathbf{Y}}\mathbf{R}_{ee}^{-1/2}\right\|_F^2=\mathrm{tr}\left[\Delta\hat{\mathbf{Y}}\mathbf{R}_{ee}^{-1}\Delta\hat{\mathbf{Y}}^T\right]$$

$$=\sum_{i=1}^{N}\Delta\hat{y}_i^r\mathbf{R}_{ee}^{-1}\left(\Delta\hat{y}_i^r\right)^T=\sum_{i=1}^{N}\mathrm{tr}\left[\left(\Delta\hat{y}_i^r\right)^T\Delta\hat{y}_i^r\mathbf{R}_{ee}^{-1}\right]$$

($\because\ \mathbf{R}_{ee}^{-1}$ is symmetric matrix.)

$$=\mathrm{tr}\left[\sum_{i=1}^{N}\left(\Delta\hat{y}_i^r\right)^T\Delta\hat{y}_i^r\mathbf{R}_{ee}^{-1}\right]$$

Let $\sum_{i=1}^{N}\left(\left(\hat{y}_i^r\right)^T\hat{y}_i^r\right)$ be the estimated noise covariance matrix $\widetilde{\mathbf{R}}_{ee}$. It is then observed that the e-GTLS problem concerns maximization of the probability of the noise covariance $\widetilde{\mathbf{R}}_{ee}$, which is represented by the Wishart distribution of $p\left(\widetilde{\mathbf{R}}_{ee}\right)=C\left|\widetilde{\mathbf{R}}_{ee}\right|^{1/2(N-p)}e^{-tr\left(\mathbf{R}_{ee}^{-1}\widetilde{\mathbf{R}}_{ee}\right)}$[9].

The likelihood function of $p\left(\widetilde{\mathbf{R}}_{ee}\right)$ is $\ln p\left(\widetilde{\mathbf{R}}_{ee}\right)=\ln C+\frac{1}{2}(N-p)\ln\left|\widetilde{\mathbf{R}}_{ee}\right|-tr\left(\mathbf{R}_{ee}^{-1}\widetilde{\mathbf{R}}_{ee}\right)$. The likelihood function of is. Then maximizing the likelihood function with respect to the parameters is the same as minimizing $tr\left(\mathbf{R}_{ee}^{-1}\widetilde{\mathbf{R}}_{ee}\right)-\frac{1}{2}(N-p)\ln\left|\widetilde{\mathbf{R}}_{ee}\right|$ with respect to the system parameters. This is the ML estimator. In the e-GTLS method the minimization is performed only to $tr\left(\mathbf{R}_{ee}^{-1}\widetilde{\mathbf{R}}_{ee}\right)$. Hence the e-GTLS method is suboptimal from statistical point of view.

4.3. The solution of the e-GTLS problem

In the e-GTLS problem there are two constraints. The second constraint $a_0=-1$ can be removed by the method of Lagrange multipliers. Define the function of

$$\mathrm{L}\left(\Delta\hat{\mathbf{Y}},\lambda\right)=\sum_{i=1}^{N}\Delta\hat{y}_i^r\mathbf{R}_{ee}^{-1}\left(\Delta\hat{y}_i^r\right)^T$$
$$+\lambda^T\left\{\left[\mathbf{U}\ \hat{\mathbf{Y}}\ \mathbf{W}_1\otimes\hat{\mathbf{Y}}_2\cdots\mathbf{W}_m\otimes\hat{\mathbf{Y}}_2\right]\cdot\mathbf{h}\right\} \qquad (7)$$

where $\lambda=[\lambda_1\ \lambda_2\cdots\lambda_N]^T$. Differentiating Eq. (7) with respect to λ_i and $\Delta\hat{y}_i^r$ we obtain Eqs. (8) and (9). Because $\mathbf{R}_{ee}^{-1}$ is a symmetric matrix,

$$\frac{\partial\mathrm{L}}{\partial(\Delta\hat{y}_i^r)}=2\Delta\hat{y}_i^r\mathbf{R}_{ee}^{-1}+\lambda_i(\mathbf{a}+\mathbf{s}_i)^T=0 \qquad (8)$$

$$\frac{\partial\mathrm{L}}{\partial\lambda_i}=\mathbf{u}_i^r\mathbf{b}+(\hat{y}_i^r)(\mathbf{a}+\mathbf{s}_i)=0 \qquad (9)$$

$$\text{where } \mathbf{s}_i=\begin{cases}s_{ij}=0 & \text{for } j=0\\ [s_{i2}\cdots s_{i(l+1)}]=\sum_{j=1}^{m}\mathbf{w}_{ji}^r\otimes\mathbf{c}_j^T \text{ and } \mathbf{w}_{ji}^r \text{ is the } i_{th} \text{ row vector of the matrix } \mathbf{W}_j. & \\ s_{ij}=0 & \text{for } l+2\le j\le p+1\end{cases}$$

From Eqs. (8) and (9) we obtain λ_i and $\Delta\hat{y}_i^r$ by some calculus. They are

$$\lambda_i=\frac{2\{\mathbf{y}_i^r(\mathbf{a}+\mathbf{s}_i)+\mathbf{u}_i^r\mathbf{b}\}}{(\mathbf{a}+\mathbf{s}_i)^T\mathbf{R}_{ee}(\mathbf{a}+\mathbf{s}_i)} \qquad (10)$$

and

$$\Delta \hat{y}_i^r = -\frac{2\{\mathbf{y}_i^{rT}(\mathbf{a}+\mathbf{s}_i)+\mathbf{u}_i^r\mathbf{b}\}}{(\mathbf{a}+\mathbf{s}_i)^T\mathbf{R}_{ee}(\mathbf{a}+\mathbf{s}_i)}(\mathbf{a}+\mathbf{s}_i)^T\mathbf{R}_{ee} \qquad (11)$$

Substituting Eq.(13) into the e-GTLS formulation we obtain the e-GTLS problem as Minimization of

$$\sum_{i=1}^{N}\Delta\hat{y}_i^r\mathbf{R}_{ee}^{-1}(\Delta\hat{y}_i^r)^T=$$

$$\sum_{i=1}^{N}\frac{\{\mathbf{y}_i^{r}(\mathbf{a}+\mathbf{s}_i)+\mathbf{u}_i^{r}b\}^T\{\mathbf{y}_i^{r}(\mathbf{a}+\mathbf{s}_i)+\mathbf{u}_i^{r}b\}}{(\mathbf{a}+\mathbf{s}_i)^T\mathbf{R}_{ee}(\mathbf{a}+\mathbf{s}_i)} \quad \text{with}$$

respect to $\{a_i, b_j, c_{k_1k_2}\}$ and subject to $a_0=-1$. Defining the numerator and denominator in the e-GTLS problem as $\mathbf{h}^T\mathbf{N}_i\mathbf{h}=\{\mathbf{y}_i^{r}(\mathbf{a}+\mathbf{s}_i)+\mathbf{u}_i^{r}\mathbf{b}\}^T\{\mathbf{y}_i^{r}(\mathbf{a}+\mathbf{s}_i)+\mathbf{u}_i^{r}\mathbf{b}\}$ and

$\mathbf{h}^T\Delta_i\mathbf{h}=(\mathbf{a}+\mathbf{s}_i)^T\mathbf{R}_{ee}(\mathbf{a}+\mathbf{s}_i)$ where

$$\mathbf{N}_i=\begin{bmatrix}(\mathbf{u}_i^r)^T\\(\mathbf{y}_i^r)^T\\(\mathbf{w}_{1i}^r\otimes\mathbf{y}_{2i}^r)^T\\ \vdots\\(\mathbf{w}_{mi}^r\otimes\mathbf{y}_{2i}^r)^T\end{bmatrix}\begin{bmatrix}(\mathbf{u}_i^r)^T\\(\mathbf{y}_i^r)^T\\(\mathbf{w}_{1i}^r\otimes\mathbf{y}_{2i}^r)^T\\ \vdots\\(\mathbf{w}_{mi}^r\otimes\mathbf{y}_{2i}^r)^T\end{bmatrix}^T,$$

$$\Delta_i=\begin{bmatrix}0&0&0&0&\cdots&0\\0&0&0&0&\cdots&0\\0&0&\hat{\Gamma}_{aa,i}&\hat{\Gamma}_{ac_1,i}&\cdots&\hat{\Gamma}_{ac_m,i}\\0&0&\hat{\Gamma}_{c_1a,i}&\hat{\Gamma}_{c_1c_1,i}&\cdots&\hat{\Gamma}_{c_1c_m,i}\\ \vdots&\vdots&\vdots&\vdots&\vdots&\vdots\\0&0&\hat{\Gamma}_{c_ma,i}&\hat{\Gamma}_{c_mc_1,i}&\cdots&\hat{\Gamma}_{c_mc_m,i}\end{bmatrix},$$

$$\Gamma_{aa,i}=\mathbf{R}_{ee}\in R^{(p+1)\times(p+1)},$$

$$\hat{\Gamma}_{ac_n,i}=\hat{\Gamma}_{c_na,i}=\begin{bmatrix}\mathbf{r}_{ee22}^r\otimes\mathbf{w}_{ni}^r\\ \cdots\\ \mathbf{r}_{ee2(p+1)}^r\otimes\mathbf{w}_{ni}^r\end{bmatrix}\in R^{(p+1)\times l},$$

$$\hat{\Gamma}_{c_{n_1}c_{n_2},i}=\hat{\Gamma}_{c_{n_2}c_{n_1},i}=\begin{bmatrix}\mathbf{r}_{ee22}^r\\ \cdots\\ \mathbf{r}_{ee2(l+1)}^r\end{bmatrix}\otimes\left(w_{n,i}^{rT}w_{n_2i}^{T}\right)\in R^{l\times l},$$

$$\mathbf{r}_{ee2n}^r=[r_{ee}(n,2)\cdots r_{ee}(n,l+1)],$$

$$\mathbf{R}_{ee}=[\mathbf{R}_{ee2}\ \mathbf{R}_{ee2}\ \mathbf{R}_{ee3}]=$$

$$\begin{bmatrix}r_{ee}(1,1)&\mathbf{r}_{ee21}^r&r_{ee}(1,l+2)&\cdots&r_{ee}(1,p+1)\\ \vdots&\vdots&\vdots&\vdots&\vdots\\ r_{ee}(p+1,1)&\mathbf{r}_{ee2p}^r&r_{ee}(p+1,l+2)&\cdots&r_{ee}(p+1,p+1)\end{bmatrix}.$$

We get the e-GTLS problem as minimization of $\rho(\mathbf{h})$ with respect to $\{a_i, b_j, c_{k_1k_2}\}$ and subject to $a_0=-1$ where $\rho(\mathbf{h})=\sum_{i=1}^{N}\frac{\mathbf{h}^T\mathbf{N}_i\mathbf{h}}{\mathbf{h}^T\Delta_i\mathbf{h}}$.

The above e-GTLS problem can be viewed as an unconstrained minimization problem, for a_0 can be fixed to -1. The e-GTLS problem is a nonlinear optimization problem. So, it is impossible to solve the e-GTLS problem in closed form. To solve the problem, we adopted the Newton-Raphson method with the first and second derivatives of. $\rho(\mathbf{h})$ with respect to h_i and h_ih_j[12].

V. Simulation results

In this section, we examine the effectiveness of the e-GTLS method and its recursive algorithm through computer simulation. The following bilinear model is used in our simulation.

$$x_t=1.5\,x_{t-1}-0.7\,x_{t-2}+0.8\,u_t+0.5u_{t-1}+0.24\,x_{t-1}u_t \qquad (12)$$

$$y_t=x_t+\varepsilon e_t \qquad (13)$$

where e is a constant for varying SNR. The input series $\{u_t\}$ is generated from AR(1) model

$$u_t=0.5\,u_t+\eta_t \qquad (14)$$

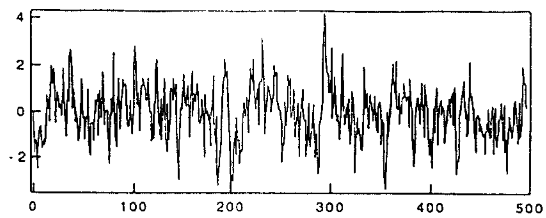
where $\{\eta_t\}$ is a gaussian white noise N(0, 1). The additive noise $\{e_t\}$ satisfies ARMA (2, 1)

$$e_t=0.9\,e_{t-1}+0.4\,e_{t-2}+\nu_t+0.6\nu_{t-1} \qquad (15)$$

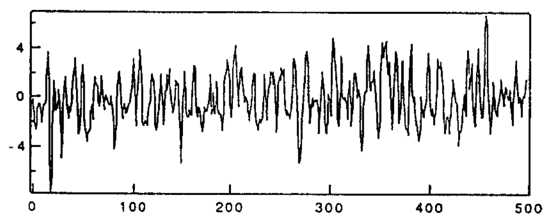
where $\{\nu_t\}$ is a gaussian white noise N(0, 1) as well as $\{\eta_t\}$. Fig. 2 shows the examples of the series $\{u_t\}$, $\{e_t\}$ and. In Fig. 2c the dotted line represent the time series generated by ARMA(2, 1) model which neglects the term of $x_{t-1}u_t$ in Eq.(12). The e-GTLS and GTLS methods were used to estimate the parameters of the bilinear system described by Eq. (12) and (13) with the given noise covariance matrix. The covariance matrix of the noise is given by the following matrix.

$$\mathbf{R}_{ee}=\begin{bmatrix}1.&0.738&0.262\\0.738&1.&0.738\\0.262&0.738&1.\end{bmatrix}$$

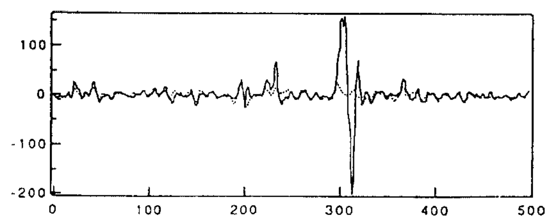
where the diagonals are scaled to 1 for simplicity.



(a) input sequences.



(b) additive noise sequences.



(c) the output sequences of the bilinear and linear system.

Fig 2. The input, noise and the output of the bilinear system.

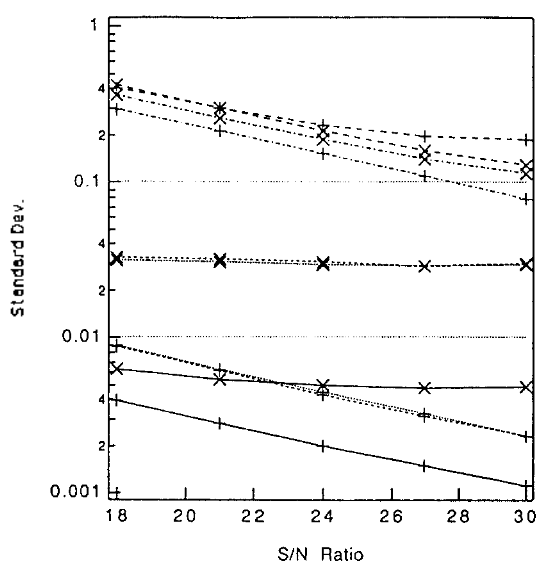


Fig 3. Standard deviation

+ : e-GTLS Method × : GTLS Method
b1
b2
a1
a2
beta

One hundred Monte Carlo trials were performed at each SNR.

The mean and standard deviation of the estimated system parameters by the both method were obtained for SNR's ranging from 18 dB to 30 dB at every 3dB intervals. 500 samples were used for each trial. Figs. 3 and 4 show the bias and standard deviation of the estimated parameters through both methods. From the figures it is observed that the bias and the variance of the estimated parameters via the e-GTLS method are smaller that those via the GTLS method. Especially, the standard deviations of the AR and are reduced from one quarter to one tenth of those via the GTLS method.

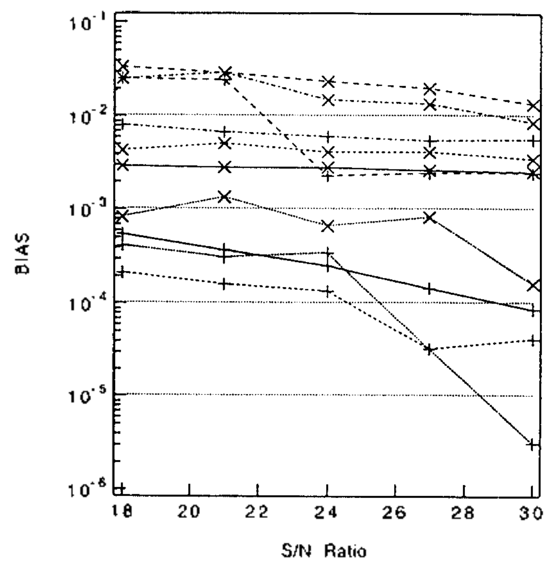


Fig 4. Bias

+ : e-GTLS Method × : GTLS Method
b1
b2
a1
a2
beta

Ⅵ. Conclusions

In this paper we have presented a new method of estimation of the parameters of bilinear systems. Considering the structure of the data matrix of the bilinear system equations, we extended the GTLS problem and defined the e-GTLS problem. We proposed the solving method of the e-GTLS problem using the Newton-Raphson method. Through the computer simulation we showed that the e-GTLS method is an unbiased estimator of the bilinear system parameters, and that the performances of the e-GTLS method are better than those of the GTLS method.

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▲Seokwon Han

Seokwon Han was received the B.S. and M.S. degree in electronics engineering at Seoul National University in 1990 and 1992. He is currently working on his Ph.D degree in electronics engineering at Seoul National University. His research interest is in DSP and room acoustics.

▲Jinyoung Kim

Jinyoung Kim was received the B.S. and M.S. degree in electronics engineering at Seoul National University in 1986 and 1988. From 1993 to 1994 he was in KTA as a research engineer. He received Ph.D degree in electronics engineering at Seoul National University in 1994. Since 1995 he has been with Cheon-nam university where he is a associate professor in the department of electronics engineering. His research interest is in DSP and speech signal processing.

▲Koengmo Sung

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