Korean Journal of Hydrosciences, Vol. 7, June, 1996

A Stochastic Model for the Prediction of Water Quality Variation in a River System

Han, Kun Yeun* · **Kim, Sang Hyun****

ABSTRACT : A stochastic model "STO-RIV" for the prediction of water quality variation in a river system has been developed. Extended Streeter-Phelps equation and Monte-Carlo simulation are used in the model. The model is applied to the reach of Waegwan to Mulkeum in the Nakdong River to compute the probability distribution of BOD and DO concentrations at Mulkeum site. As the strategies to attain the goal of the water quality, some alternatives considering the treatment effect of the Keumho River are discussed using the stochastic model. Application of stochastic analysis to water quality management is strongly recommended in this country.

1. Introduction

As the country has rapidly been grown and industrialized, the river has served as the principal source of water supply in nearby cities. In this study, stochastic water quality model as well as deterministic water quality model are developed for the basin-wide water quality management in the country. In water quality modeling, modelers should take into account the errors including in the model parameter. They should not account only for deterministic conditions of the model parameters, as any change in these parameters may cause a large error. For example, a slight variation in reaeration rate constant may result in an error in the predicted dissolved oxygen value, which then might result in a large difference in the cost of water quality management plan. This shows the importance of considering errors in the model parameter, initial conditions, temperatures and loading conditions.

The models developed in this study are applied to the Nakdong River, which contains a polluted tributary, the Keumho River in the midstream and the water supply intake, Mulkeum station in the downstream. Deterministic and stochastic simulations are performed to get the characteristics of water quality variations in the river.

---

* Professor, Kyongbuk National University

* * Post-Doc., University of Illinois at Urbana-Champaign, IL, USA

2. Hydrologic and Hydraulic Analysis for Water Quality Modeling

Water quality variations in a river are greatly influenced by discharge, velocity, depth and temperature. Since monthly discharge and temperature are significantly varied in a river in this country, hydrologic data based on monthly mean and minimum discharge have been analyzed. From the monthly mean and minimum discharge of the main river and tributaries, the discharges at each element in a river system can be determined by continuity equation as shown in Eq. 1. Through varied flow analysis, velocity, cross sectional area, depth and hydraulic radius can be computed by momentum equation as shown in Eq. 2.

$$Q_{i+1} = Q_i + q_i \Delta x_i \tag{1}$$

$$\left(\frac{Q^2}{A}\right)_{i+1} - \left(\frac{Q^2}{A}\right)_i + g\overline{A_i}(h_{i+1} - h_i + \Delta x_i \overline{S_{fi}}) = 0 \tag{2}$$

where

Q = discharge, A = cross section area, h = stage, x = distance, S_f = energy slope, and q = lateral inflow by tributaries.

Geometric configurations should be surveyed to obtain stage–top width data and Newton–Raphson method is applied to solve the momentum equation, shown in Eq. 3. Depth, area, velocity and hydraulic radius are computed at each element and these data will give the hydraulic informations for the water quality modeling in a river.

$$\chi^{k+1} = \chi^k - \frac{f(\chi^k)}{f'(\chi^k)} \tag{3}$$

where

$$f'(\chi^k) = \frac{df(\chi^k)}{dh_i} = \left(\frac{Q^2 B}{A}\right)_i + \frac{1}{2} gB_i(h_{i+1} - h_i + \overline{S_{fi}}\, \Delta x_i + gA_i(-1 + \Delta x_i(\frac{d\overline{S_{fi}}}{dh_i})) \tag{4}$$

$$\frac{dS_f}{dh_i} = 2\bar{S}_f\left(\frac{d\bar{n}/dh}{\bar{n}} - \frac{5\bar{B}}{6\bar{A}} + \frac{d\bar{B}/dh}{3\bar{B}}\right) \tag{5}$$

where n = Mannings roughness coefficient, and B = width of water level.

3. Development of Deterministic Water Quality Model DET–RIV

3.1 Extended Streeter–Phelps Equation

Deterministic water quality model DET-RIV is developed based on the extended Streeter-Phelps equation. All reaction kinetics are assumed to be first-order. BOD, DO and NOD concentrations can be computed from the analytical solution of extended Streeter-Phelps equation as follows :

$$\mathrm{BOD}: L = L_0 a_3 + L_D\left(\frac{1-a_3}{k_1+k_3}\right) \tag{6}$$

$$\mathrm{NOD}: N = N_0 a_4 + N_D\left(\frac{1-a_4}{k_4}\right) \tag{7}$$

$$\mathrm{DO}: C = C_0 a_2 + C_S(1-a_2) - k_1 L_0 \frac{a_3-a_2}{(k_2-(k_3+k_1))} - k_4 N_0 \frac{(a_4-a_2)}{(k_4-k_2)} + k_1 L_D \frac{(a_2-a_3)}{(k_1+k_3)\ (k_1+k_3-k_1)} - k_1 N_D \frac{(1-a_2)}{k_2(k_1+k_3)} + N_D \frac{(a_2-a_4)}{(k_4-k_2)} - N_D \frac{(1-a_2)}{k_2} - R\frac{(1-a_2)}{k_4} + P(t)\frac{1-a_2}{k_4} \tag{8}$$

$$a_2 = \mathrm{Exp}(-k_2 \frac{X}{U}) \tag{9}$$

$$a_3 = \mathrm{Exp}(-(k_1+k_3)\frac{X}{U}) \tag{10}$$

$$a_4 = \mathrm{Exp}(-k_4 \frac{X}{U}) \tag{11}$$

where, L_0, N_0 are C_0 are initial concentrations. of BOD, NOD and DO, respectively.

3.2 Application and Discussion

DET-RIV contains two codes, DET-RIVH for hydraulic analysis and DET-RIVQ for water quality analysis. DET-RIV has been applied to the Nakdong River (Waegwan-Mulkeum reach, 167.8 km) which contains several tributary including the Keumho River and water supply intakes Mulkeum station as shown in Fig. 1. Fig. 2 indicates the BOD and DO concentration profile for the mean discharge in June. The computed BOD and DO concentrations are good agreements with the observed as presented in Table 1.

4. Uncertainty and Reliability Analysis

4.1 Uncertainty Analysis

Deterministic water quality models have become widely accepted tools for water quality management. These models, being no more than simplified description of the natural system, have some amount of error (i.e., uncertainty) associated with them. Many researches have been conducted to quantify uncertainties in model projections. The three widely used techniques are sensitivity analysis, first-order error analysis and Monte-Carlo analysis.

Sensitivity analysis consists of varying model coefficient one at a time. Sensitivity is determined by the change in model results corresponding to the change in model inputs. This technique has been quite popular in the past due to ease of application, but has been shown to produce only the crudest

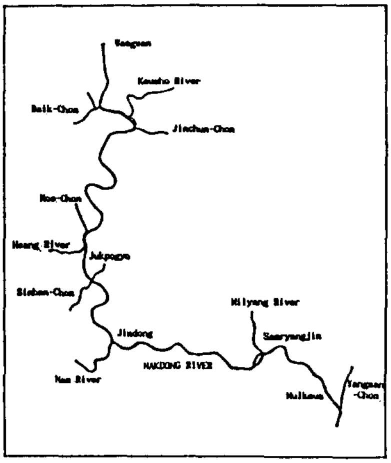

Fig. 1. Location Map of the Nakdong River
(Waegwan–Mulkeum Reach)

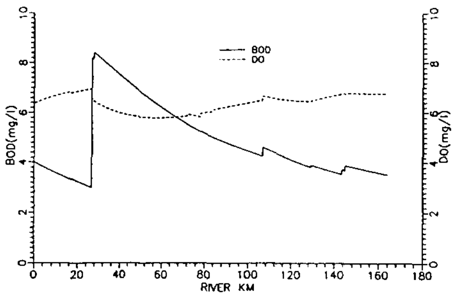

Fig. 2. Result of Deterministic Simulation
(1996, June)

Table 1. Comparison between Computed and Observed Value

Month	BOD Computed (mg/l)		BOD Observed (mg/l)
	Mean Discharge	Min Discharge	
1	5.85	6.58	N.A.
2	6.39	8.17	N.A.
3	5.33	6.81	5.80('85)
4	4.04	5.97	5.17('85)
5	3.98	6.03	4.81('85)
6	2.89	4.95	N.A.
7	1.99	3.71	N.A.
8	1.96	3.37	N.A.
9	2.05	3.14	2.20('84)
10	3.63	4.57	N.A.
11	4.94	5.99	5.16('84)
12	5.93	7.2	6.38('84)

estimate of model uncertainty by ignoring higher–order effects of interactions between multiple uncertain parameters.

First–order error analysis is a significant improvement over sensitivity analysis. This analysis consists of expanding the differential equations into their respective Tayler series, and truncating all terms after the linear one. The terms of each equation are combined, and can then be used to determine the variance associated with model results as a function of the uncertainty in model inputs. This technique can determine the uncertainty including the interactions among the uncertain parameters. However, it has limitation that the uncertainty in model coefficients must fall within the linear range for each model equation. Another limitation of the first–order error analysis is the inability to determine if the technique is satisfactory without performing more detailed analysis.

Monte Carlo analysis is an iterative technique where many simulations are conducted with the model inputs chosen randomly during each iteration from specified statistical distributions. These distributions are selected to represent the uncertainty in each model parameter. Monte Carlo analysis in rapidly becoming the most popular technique for quantifying model uncertainty, due to its easily understood theory and application. The limitation of this technique in the past has been the large computational requirement, however, it has been eliminated as the rapid improvement of the computer technology. Review of the available error analysis techniques indicates that Monte Carlo analysis to be most promising technique for future development.

4.2 Reliability Analysis

The sources of uncertainty in a model output can be grouped into two main categories : model parameter uncertainty and structural uncertainty. Parameter uncertainty comes from errors in model parameter such as model coefficient, initial conditions, discharges and loadings. Small errors on model parameters can be propagated into much larger errors on the model results. Structural uncertainty is caused from the error resulting from the use of an incorrect model framework. This occurs when information is ignored or incorrect assumptions are made during model development.

The types of relationships between uncertainty and complexity in river water quality model are classified into three categories. From the analysis of available data for river water quality model in this country, it can be concluded the simple model will give the most reliable and accurate result by reliability analysis. Increasing model complexity may lead to less acurrate and reliable results, since the lack of data or inaccurate data will increase the model uncertainty. Therefore a stochastic model based on BOD–DO–NOD analysis has been developed in this study to predict water quality variation in a river.

5. Development of Stochastic Water Quality Model STO–RIV

5.1 Probability Distribution of Parameter

The probability distribution of parameters are to be determined from observed data, if available. This can be done by comparing various distributions with the data. If the data available are not sufficient for this, which is most likely the case, then approximate probability distibutions may be used. Many researches have been performed to estimate the probability distribution of model coefficients, initial conditions, discharges, temperature and loading conditions. If there exist no data available for some of the parameters, then the probability distribution and standard deviation can be selected from the existing literature. Monte–Carlo simulation technique is used in this study. In this technique, random value of variable parameters such as model coefficients, loadings, and initial conditions are generated from independent probability distributions that repensent the variability in these parameters. Normal, lognormal, triangular and uniform distributions can be selected from the observed data or previously analyzed data in literatures. Random numbers are generated by using multiplicative congruential generator as follows :

$$X_{i+1} = a\,X_i (\mathrm{mod}\ m) \tag{12}$$

$$U_i = \frac{X_i}{m} \tag{13}$$

where $a = 16807$ and $m = 2^{31} - 1$.

5.2 Stochastic Water Quality Model STO-RIV

A stochastic water quality model in a river STO-RIV has been developed based on BOD-DO-NOD analysis and Monte-Carlo technique. The choice of the probability distributions for the uncertain parameters depends on information and knowledge available about the uncertainty in these parameters. STO-RIV consists of the choice of optimum water quality model by reliability analysis, the calculation of stochastic water quality variations by chosen model and the stochastic water quality management planning. Fig. 3 shows the flow diagram of STO-RIV. The BOD and DO concentrations computed in probabilistic form within a given bounds, means, and standard deviations are more representative of the water quality in the natural river system than the deterministic results. The model is programmed in Fortran-77 and can be used in most IBM compatible PC's.

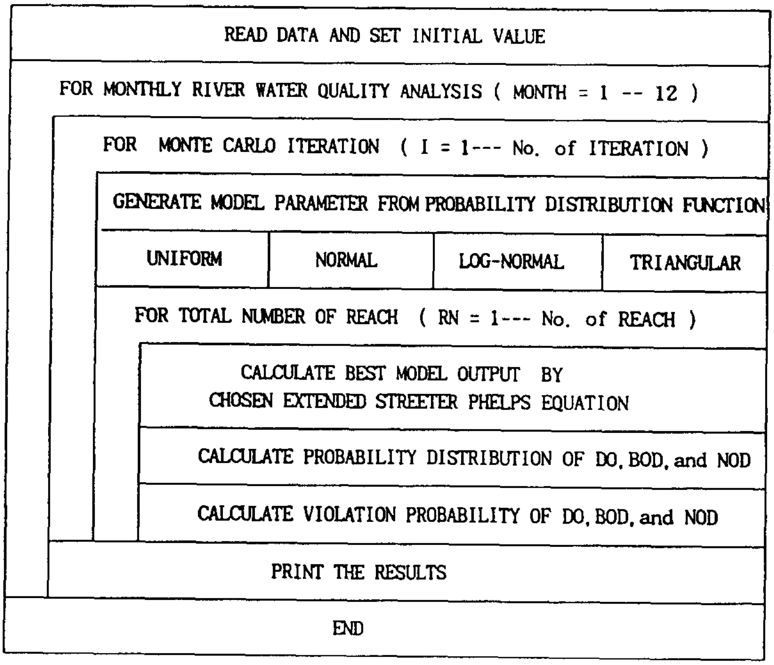

Fig. 3. Flow chart of STO-RIV

6. Water Quality Management Using Stochostic Model

6.1 Application and Discussion

Stochastic model STO–RIV is applied to the Nakdong River and water quality variation at Mulkeum station, where large scale water supply intake has been carried out for nearby cities (Fig. 1), is studied. The model is simulated three hundred times. In each simulation, new values for the various parameters in the model are randomly from the assumed probability distributions and substituted in model euqation to predict BOD and DO concertrations at several stations. Fig. 4–7 shows model simulation results of BOD and DO at Mulkeum station for monthly mean discharge in 1996.

Generally, the model simulation is not very sensitive to the choice of distributions for the individual parameters in the model since exponential function of the governing equation will diminish its effect of variability proceeding downstream in a river system. Fig. 8 shows the probability density distribution at each reach. From the figures it can be observed that the self–purification of a river is evident and water quality is improved as proceeding downstream. Fig. 9 shows the influence of the choice of probability distribution of discharge.

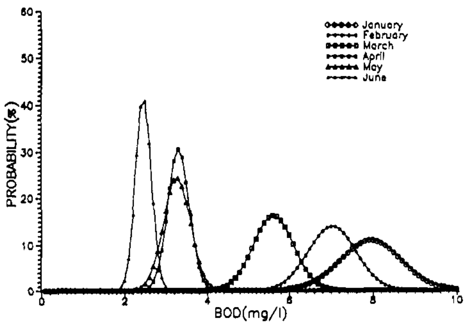

Fig. 4. Probability Distribution of BOD (1996, Jan–Jun)

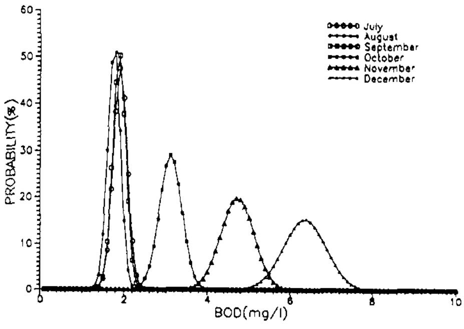

Fig. 5. Probability Distribution of BOD (1996, Jul–Dec)

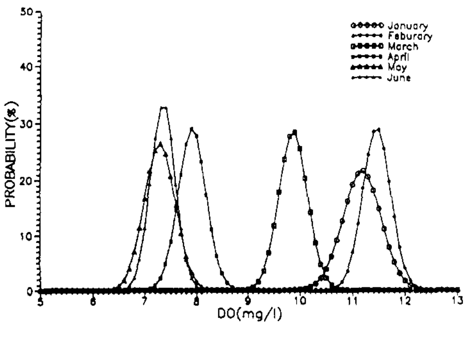

Fig. 6. Probability Distribution of DO (1996, Jan–Jun)

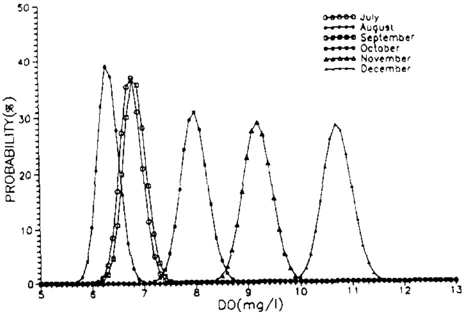

Fig. 7. Probability Distribution of DO (1996, Jul–Dec)

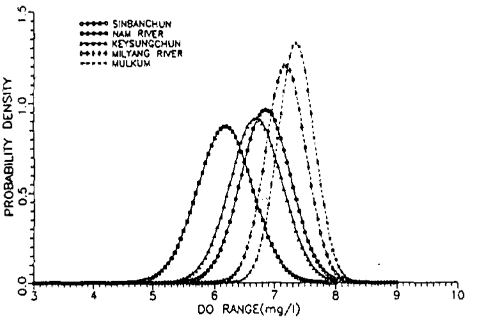

Fig. 8. Probability Density Distribution of DO at Each Confluence Proceeding Downstream

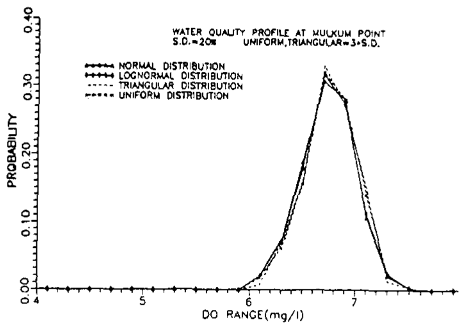

Fig. 9. Probability Distribution of DO Considering Variability of Discharge

6.2 Water Quality Management Planning

BOD load of the Keumho River and Jinchun Stream will be decreased to 19504 kg/day and 3257 kg/day respectively in 1996 by the wasteload abatement plan. Figs. 10 and 11 show probability distribution of BOD for mean discharge in January and DO for mean discharge in August respectively. Using Monte-Carlo analysis and considering variations in model parameters results in a probability distribution describing the variability at an interested location in a river. Monte-Carlo analysis is found to be useful for finding the probability distribution for BOD and DO concentrations from which the probability of violation can be easily calculated. Application of uncertainty analysis to water quality modeling ia strongly recommended in order to estimate the characteristics of water quality variation in a river system.

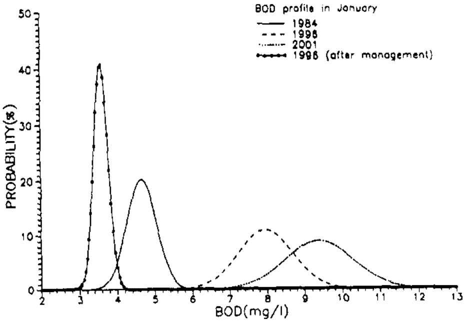

Fig. 10. Probability Distribution of BOD after Wasteload Abatement Plan

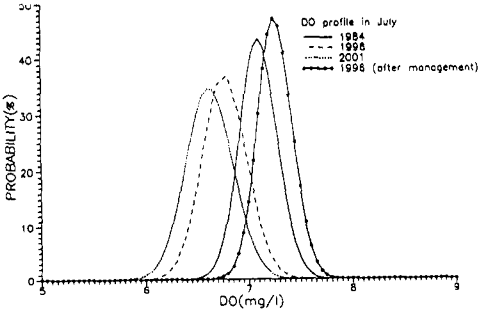

Fig. 11. Probability Distribution of DO after Wasteload Abatement Plan

7. Conclusion

(1) Deterministic water quality model DET–RIV and stochastic water quality model STO–RIV are developed for the prediction of water quality variations in a river.

(2) DET–RIV contains two codes, DET–RIVH for hydrologic and hydraulic analysis and DET–RIVQ for water quality analysis. Monthly mean and minimum discharges in the Nakdong River (Waegwan–Mulkeum reach) are estimated. A varied flow analysis is made based on the geometric data of channel and DET–RIVH. The computed concentration of BOD and DO at several stations have good agreements with the observed.

(3) STO–RIV is developed for the choice of optimum water quality model by reliability analysis, calculation of stochastic water quality analysis by chosen optimum model and stochastic water quality management planning.

(4) The stochastic model is applied to Nakdong River, which contains a polluted tributry, the Keumho River, in the midstream and the water supply intake Meulkeum station in the downstream. The probability distributions of BOD and DO concentration in the future are computed.

(5) As the strategies to attain the goal of the water quality, some alternatives considering the treatment improvement effect of Keumho River have been discussed using the stochastic model. The stochastic water quality model presented in this study will contribute to the water quality management in the country.

References

Bauer, D.P., Jennings, M.E., and Miller, J.E. (1979). "One–dimensional steady state stream water quality model." *US Geological Survey Water Resources Investigation 79–45.*

Bowie, G.L. et al. (1985). "Rates, constants, and kinetics formulations in surface water quality modeling," 2nd Ed., *EPA/600/3–85/040,* US EPA.

Brown, L.C., and Barnwell, T.O (1987). "Computer program documentation for the enhanced stream water quality model QUAL2E–UNCAS." *EPA/600/3–87/007,* US EPA.

Chadderton, R.A., Miller, A.C., and McDonnell, A.J. (1982). "Uncertainty analysis of dissolved oxygen model." *J. of Env. Eng. Div.,* ASCE, Vol. 108, No. EE5, pp. 1003–1011.

Dewey, R.J. (1984). "Application of stochastic dissolved oxygen model." *J. of Env. Eng. Div.,* ASCE, Vol. 110, No. 2, pp. 412–429.

Ditoro, D.M. (1984). "Probability model of stream quality due to runoff." *J. of Env. Eng. Div.,* ASCE, Vol. 110, No. 3, pp. 607–628.

Duke, J.H. (1973). *Provision of a steady state version of the stream model.* QUAL. Water Resources Engineers.

Gardner, R.H. et al. (1981). "Comparative error analysis of six predator–prey models." *Ecology,* Vol. 62, No. 2, pp. 323–332.

Han, Kun-Yeun et al. (1993). "Water quality management using deterministic and stochastic models in the Nakdong River." *Proc. of IAHR,* Paper No. D-7-1, pp. 203-210, Tokyo, Japan.

Hornberger, G.M. (1980). "Uncertainty in dissolved oxygen prediction due to variability in algal photosynthesis." *Water Research,* Vol. 14, pp. 355-361.

Martin, J.L. (1986). "Simplified, steady state temperature and dissolved oxygen model : User's guide." *Instruction Report E-86-4,* US Army Engineer Waterways Experiment Station.

Survey data of the Nakdong River. (1981-1983). Ministry of Constuction.

Padgett, W.J. et al. (1977). "A Random differential equation approach to the probability distribution of BOD and DO in streams." *J. of Applied Mathematics,* Vol. 32, No. 2, pp. 467-483.

Qaisi, K. (1985). "Uncertainty analysis of stream dissolved qxygen model," Ph. D dissertation, Univ. of Michigan.

Thomann, R.V., and Mueller, J.A. (1987) *Principles of surface water quality modeling and control.* Harper and Row.

Tiwari, J.L., and Hobbie, J.E. (1976). "Random differential equations as models of ecosystems Monte Carlo simulation approach." *Math Biosci,* pp. 25-44.

Tiwari, J.L. et al (1978). "Some stochastic differential equation models of an aquatic ecosystem." *Ecol. Modeling* , Vol. 4, pp. 3-27.

Walker, W.W. (1982). "A sensitivity and error analysis framework for lake eutrophication modeling." *Water Resources Bulletin,* Vol. 18, No. 1, pp. 53-60.

Wilem, F.B. (1975). "Water quality modeling of Monte-Carlo simulation." *Water Resources Bulletin,* Vol. 11, No. 2.