Groundwater Fluxes in a Watershed with a Lake

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ABSTRACT : The purpose of this study is to investigate the influence of the position of lake upon groundwater fluxes on a lake watershed, and to provide guidance for the monitoring network design to survey the exchange relations between groundwater and lake water. Three kinds of hypothetical flow through lakes, which are located at the upper, middle, and lower portion of a watershed were considered. Groundwater flow for each case was numerically simulated under three-dimensional steady state conditions. The exchange rates of the groundwater, the amounts of recharge and discharge, and groundwater fluxes between lake and groundwater in a watershed system with a lake were clarified.

1. Introduction

There are more than 30,000 lakes in the upper parts of Michigan, Minnesota and Wisconsin in U. S.A (Born et al., 1979). In this region, lake water and groundwater influenced each other. For the investigation of hydrological phenomenon in the area where many lakes are located, it is important to the quantification of the groundwater fluxes which are in and around the lake. As the numerical modeling is helpful in analyzing the groundwater flow and the quantifying groundwater fluxes, many scientist use this method by various simulation for the analyzing of field data and quantifying of groundwater fluxes.

Winter (1976, 1978) examined the interactions between lake and groundwater flow systems by modeling for hypothetical problems. Bae (1989, 1990) investigated the interaction between groundwater in the Dejima Upland Area and Kasumigaura Lake in Japan using the three-dimensional groundwater model, and compared this with the analytical results obtained from water quality and isotopes data (Bae and Kayane, 1987a and 1987b).

Recently, Cheng and Anderson (1994) studied the effects of lake position on groundwater fluxes. They compared the differences of groundwater fluxes where the lake is located in different positions in a watershed. They designed a two-dimensional hypothetical model and simulated to three cases of

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lake positions in a watershed system under steady state and transient conditions. They got good results for providing guidence to the design of monitoring and sampling networks in a watershed with a lake. But they had over simplified the shape of the lake bottom and had a two-dimensional system so that their study are limited a figuring out a recharge-discharge relationship on the groundwater and lake. This was unsatisfactory in the quantification of groundwater fluxes occuring between groundwater and lake.

In this study, the basic concept of Cheng and Anderson (1994) was extended to three-dimension and a real lake bottom was designed to investigate the role of lake position on groundwater fluxes to lakes.

2. Basic equation

Transient three-dimensional groundwater flow for a nonhomogeneous, anisotropic medium, where the principal axes are in a Cartesian coordinate system is expressed by

$$\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_{yy}\frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_{zz}\frac{\partial h}{\partial z}\right)=S_s\frac{\partial h}{\partial t}+W(x,y,z,t) \tag{1}$$

where h is hydraulic head above a common datum, K_{xx}, K_{yy} and K_{zz} are saturated hydraulic conductivities, x, y and z are the Cartesian coordinates, S_s is specific storage, W is a volumetric flux per unit volume, and t is time.

Using the fully implicit and second-order approximation in time and space, the finite difference representation of Eq. (1) is given by

$$\begin{aligned}&K_{xx\,i,j+1/2,k}\cdot(h_{i,j+1,k}-h_{i,j,k})/\Delta x_j^2-K_{xx\,i,j-1/2,k}\cdot(h_{i,j,k}-h_{i,j-1,k})/\Delta x_j^2-K_{yy\,i+1/2,j,k}\cdot(h_{i+1,j,k}-h_{i,j,k})/\Delta y_i^2\\&-K_{yy\,i-1/2,j,k}\cdot(h_{i,j,k}-h_{i-1,j,k})/\Delta y_i^2-K_{zz\,i,j,k+1/2}\cdot(h_{i,j,k+1}-h_{i,j,k})/\Delta z_k^2\\&-K_{zz\,i,j,k-1/2}\cdot(h_{i,j,k}-h_{i,j,k-1})/\Delta z_k^2=S_{s\,i,j,k}\cdot(h_{i,j,k}-\hat{h}_{i,j,k})/\Delta t+W_{i,j,k}\end{aligned} \tag{2}$$

where Δx_j, Δy_i and Δz_k are the space increment in the x(j), y(i) and z(k) directions, respectively, $h_{i,j,k}$ and $\hat{h}_{i,j,k}$ are the hydraulic heads at the old and new time steps, respectively, Δt is the time step size and i, j and k are the indices in the y, x and z coordinates, respectively.

Combining coefficients and bringing terms containing unknown head values to the left, Eq. (2) can be written in the form:

$$Z_{i,j,k}h_{i,j,k-1}+B_{i,j,k}h_{i-1,j,k}+D_{i,j,k}h_{i,j-1,k}+E_{i,j,k}h_{i,j,k}+F_{i,j,k}h_{i,j+1,k}+H_{i,j,k}h_{i+1,j,k}+S_{i,j,k}h_{i,j,k+1}=q_{i,j,k} \tag{3}$$

where

$$Z_{i,j,k}=2K_{zz\ i,j,k}\cdot K_{zz\ i,j,k-1}/(K_{zz\ i,j,k}\varDelta z_{k-1}+K_{zz\ i,j,k-1}\varDelta z_k)/\varDelta z_k \tag{3a}$$

$$B_{i,j,k}=2K_{yy\ i,j,k}\cdot K_{yy\ i-1,j,k}/(K_{yy\ i,j,k}\varDelta y_{i-1}+K_{yy\ i-1,j,k}\varDelta y_i)/\varDelta y_i \tag{3b}$$

$$D_{i,j,k}=2K_{xx\ i,j,k}\cdot K_{xx\ i,j-1,k}/(K_{xx\ i,j,k}\varDelta x_{j-1}+K_{xx\ i,j-1,k}\varDelta x_i)/\varDelta x_i \tag{3c}$$

$$F_{i,j,k}=2K_{xx\ i,j,k}\cdot K_{xx\ i,j-1,k}/(K_{xx\ i,j,k}\varDelta x_{j+1}+K_{xx\ i,j-1,k}\varDelta x_j)/\varDelta x_i \tag{3d}$$

$$H_{i,j,k}=2K_{yy\ i+1,j,k}\cdot K_{yy\ i,j,k}/(K_{yy\ i,j,k}\varDelta y_{i+1}+K_{yy\ i+1,j,k}\varDelta y_i)/\varDelta y_i \tag{3e}$$

$$S_{i,j,k}=2K_{zz\ i,j,k+1}\cdot K_{zz\ i,j,k}/(K_{zz\ i,j,k}\varDelta z_{k+1}+K_{zz\ i,j,k+1}\varDelta z_k)/\varDelta z_k \tag{3f}$$

$$E_{i,j,k}=-(B_{i,j,k}+D_{i,j,k}+F_{i,j,k}+H_{i,j,k}+S_{i,j,k}+Z_{i,j,k}+S_{s\ i,j,k}/\varDelta t) \tag{3g}$$

$$q_{i,j,k}=W_{i,j,k}-(S_{s\ i,j,k}/\varDelta t)\hat{h}_{i,j,k} \tag{3h}$$

The set of equations (3) can be written in matrix form as:

$$[M]\ \{h\}=\{q\} \tag{4}$$

The direct solution of Eq. (4) require excessive computer storage and calculation time. To modify the Eq. (4) add [N] {h} to both sides of this equation giving:

$$[M+N]\ \{h\}=\{q\}+[n]\ \{h\} \tag{5}$$

Eq. (5) can be solved if the right hand side of the equation is known. Leading to the iterative scheme:

$$[M+N]\ \{h\}^n=\{q\}+[N]\ \{h\}^{n-1} \tag{6}$$

where n is an iteration index. To maintain precision during computation Eq. (6) is usually solved in residual form by adding and subtracting [M] $\{h\}^{n-1}$ to the right hand side giving:

$$[M+N]\ \{\xi\}^n=\{R\}^{n-1} \tag{7}$$

where

$$\{\xi\}=\{h\}^n-\{h\}^{n-1},\ \{R\}^{n-1}=\{q\}-[M]\ \{h\}^{n-1} \tag{7a}$$

When ξ^n_{max} is less than error criterion(ε), the desire results is obtained. Eq. (6) or (7) is the basic equation of Strongly Implicit Procedure (SIP; Stone, 1968).

3. Model design

In this paper, three hypothetical groundwater flow systems each containing a flow-through lake were analyzed. The region to be considered is set around the lake area of Wisconsin State in U.S.A.

Topography, geology and situation in this area do not have so much variety. The Trout River Basin in upper Wisconsin is located at the North Temperate Lakes site, which is one of 18 sites in the Long Term Ecological Research network in the United States (Magnuson and Bower, 1990). The hydrological studies have been conducted many years in this area, many data for groundwater study have been collected around here. For this study, the data of Trout River Basin were used. It was assumed that the lake has a lake morphometry of Crystal Lake (Fig. 1) located in Trout River Basin.

The position of a lake in a watershed is different, but the size and permeability are the same in each groundwater system. The center of lakes 1, 2, and 3 are located at 0.75, 0.5, and 0.25 times the basin length from the left boundary of the system and are located at upper and groundwater recharge, middle, and lower and groundwater discharge area of the system (Fig. 2).

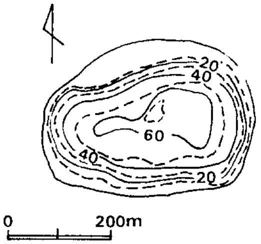
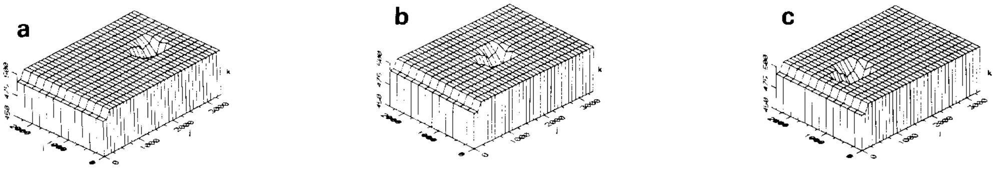


Fig. 1. Bottom Shape of the Crystal Lake (Depth, unit: ft)



(a) Lake 1 (b) Lake 2 (c) Lake 3

Fig. 2. Finite Difference Grid of Groundwater System

The two leftmost nodes are specified as constant head nodes to represent a stream. The length of i direction of the model basin is 2.6 km as the Crystal Lake Basin, and the length of j direction is 3.5km as the setting by Cheng and Anderson (1994). The model spacings are 200m and 100m in the i and j directions. In the Trout River Basin, the basic rock appears at a depth of 50 m (Attig, 1984). For the above reason, the aquifer depth in the model was assumed to be 52m. The vertical model spacing is 2m. The finite difference grid used in the simulation has 13 × 35 × 26 in the i, j, and k directions.

Table 1. Hydraulic Parameters Used in the Simulations

Horizontal hydraulic conductivity of the aquifer	1×10^{-4} m/sec
Ratio of horizontal to vertical hydraulic conductivity	10
Hydraulic conductivity of the lakebed sediments	2.9×10^{-10} m/sec
Thickness of the lakebed sediments	2m
Average annual groundwater recharge rate	263mm

The aquifer was assumed to be homogeneous but anisotropic (Krabbenhoft et al., 1990). Parameters values used in the simulations are shown in Table 1.

The hydraulic conductivity of the aquifer is 1×10^{-4}m/sec. The ratio of horizontal to vertical hydraulic conductivity is 10 (Krabbenhoft et al., 1990). The thickness of lakebed sediments is 2m and the hydraulic conductivity of the sediments is 2.9×10^{-10}m/sec (Kenoyer, 1986). The boundary conditions of the 4 vertical boundaries and the base of the system are specified as no-flow boundaries. An areally uniform groundwater recharge rate was specified across the water table boundary. An average annual precipitation is 790mm/year. An average annual evaporation is 660mm/year. An average annual groundwater recharge rate is 263mm/year (Cheng and Anderson, 1994).

The basic groundwater flow equation used is the steady state form of Richard's equation in which the right side of Eq. (1) vanishes.

Steady state three-dimensional groundwater flow was simulated with the above data and conditions. For the simulation, a three-dimensional finite-difference groundwater flow program developed following as SIP, and a lake package which would be calculating water balance of a lake in three dimension developed by Cheng and Anderson (1993), were used.

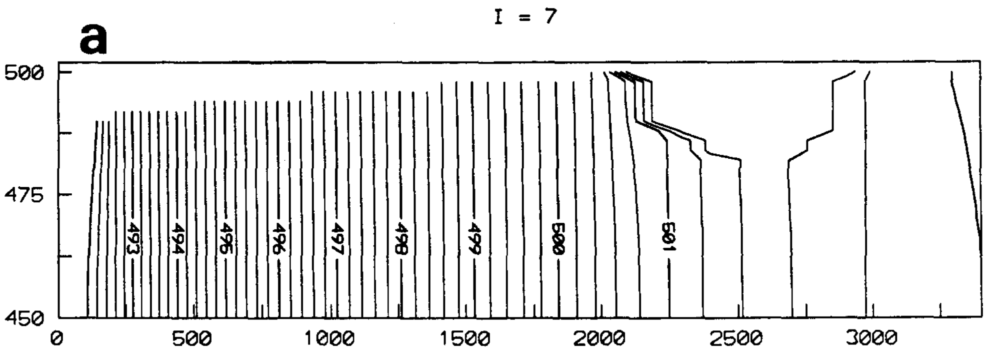
4. Results and Discussion

Among the simulating results the groundwater flow net of central vertical cross-section ($i=7$) of each lake position is represented in Fig. 3.

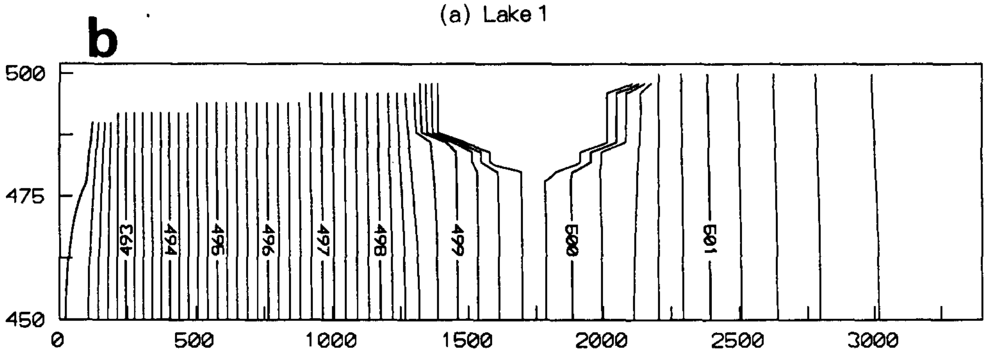
For the case of the lake 1, the following things were indicated. The interactions between lake and groundwater where the lake was located in upper reach are not active. The difference of the groundwater fluxes between the upper and lower part of the lake were very large. The groundwater in the upper area of the lake moved very slowly. But the lower part of the groundwater moves in a watershed system, the stronger groundwater fluxes were obtained.

The groundwater fluxes in the lowest area in the system were very strong compared with the groundwater fluxes in the vicinity of the lower part of the lake. But there are not so many differences of the groundwater fluxes for the depth of the aquifer except for there around the lake.

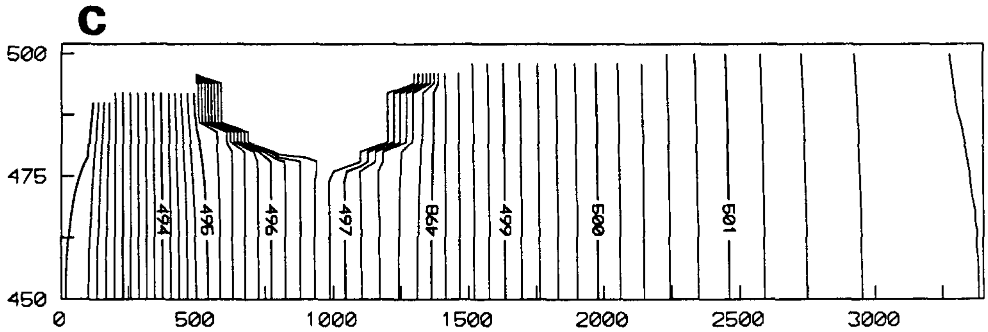
For the case of the lake 2, the following things were represented. Groundwater interactions between lake and groundwater were relatively active where the lake was located in the middle portion of the system. The differences of the groundwater fluxes between the groundwater of the lower part and that of the upper part of the lake were not large compared with the case of lake 1. In the upper



(a) Lake 1



(b) Lake 2



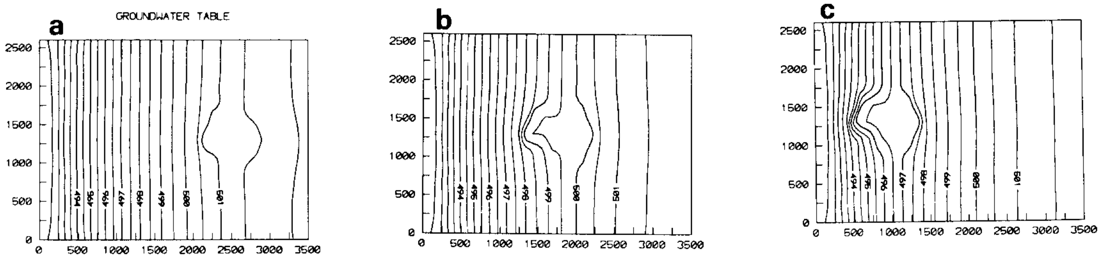
(c) lake 3

Fig. 3. Cross-Sectional Potential Lines in i=7 from Steady State Three-Dimensional Conditions (unit:m)

part of the lake, the further the more groundwater further from the lake, the weaker the groundwater fluxes were. In the lower part of the lake, the more the groundwater further from the lake, the stronger the groundwater fluxes obtained. But the differences of the groundwater fluxes for the distance from the lake were not so large.

The differences of the groundwater fluxes for the depth of the aquifer is very small like as the case of the lake 1.

For the case of the lake 3, the results show the following. The groundwater interactions between lake and groundwater occurred very actively where the lake was located in the lower portion of a



(a) Lake 1 (b) Lake 2 (c) Lake 3

Fig. 4. Groundwater Table under Steady state Three-Dimensional Conditions (unit: m)

Table 2. Groundwater Inflows and Outflows for the Three Simulated Lakes

Lake	inflow(m^3/day)	outflow(m^3/day)
Lake 1	221.92	367.27
Lake 2	527.64	657.22
Lake 3	771.58	923.83

watershed. The groundwater fluxes of the upper part of the lake were weak. The groundwater fluxes become stronger to move near the lake. The groundwater fluxes in the vicinity of the upper area of the lake were nearly the same as that in the vicinity of the lower part of the lake. The groundwater fluxes in the lower area of the lake were very strong. The differences of the groundwater fluxes for the different depths of the aquifer are small except around the lake, as in the other cases of the lake portion system.

Water table maps in each lake position are represented in Fig. 4. The results obtained from these Figures indicate very good agreement with those of Fig. 3. This Figure shows that groundwater flow is not affected by the lake shape because the interaction between lake and groundwater is not active where the lake is located upper portion of the watershed. But groundwater flow systems near the lake become complicated and are affected by the lake shape very much where the lake positions are in the middle and lower portion of the system. Groundwater flow systems where the lake is located at a lower part of a watershed are more complicated than where the lake is located at the middle.

This figure appeared that groundwater of the longitudinal side of the lake is very much affected by the lake shape. But the groundwater of the lateral side of the lake is not affected by the lake shape very much and the influencing distance from the lake is limited to 500m.

Table 2. shows the inflow and outflow rate of groundwater flow to and from the lake for the lake positions. This table indicates that interaction between groundwater and lake occurred more actively where the lake is located in the lower portion of the watershed. The recharge rate from the lake to groundwater is always larger than the discharge rate from groundwater to the lake regardless of

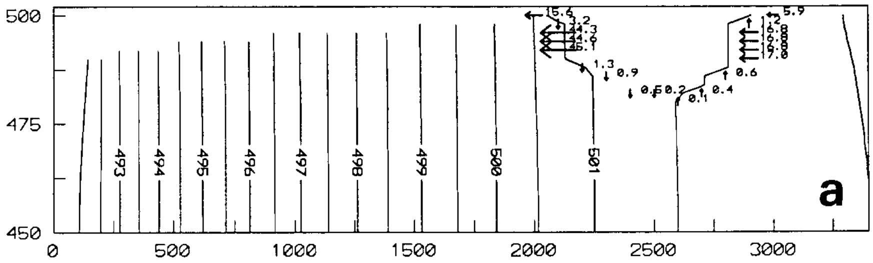
the lake position. This result shows the same tendency of the results of Kenoyer and Anderson (1989), and Anderson and Cheng (1993).

The difference rates between inflow and outflow of the groundwater become smaller for the lowest lake (lake 3) where differences of flow rate decreased to 1.2 compared with 1.25 and 1.65, respectively, for lakes 2, and 3.

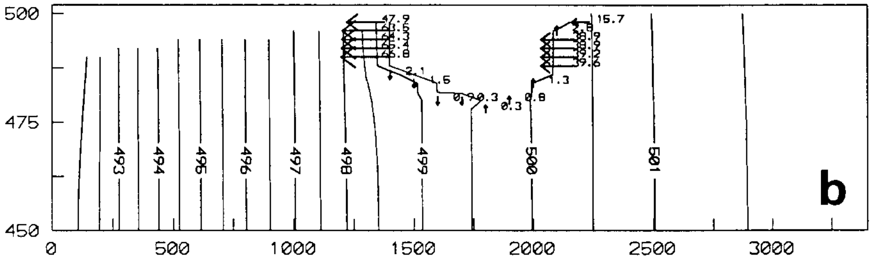
The three vertical cross-section in each centeral vertical cross-section (i=7) in the i direction for the different lake position in the watershed shown in Fig. 5. represents recharge quantity of groundwater from the lake and discharge quantity of groundwater to the lake.

(a) Lake 1

I = 7



(b) Lake 2



(c) Lake 3

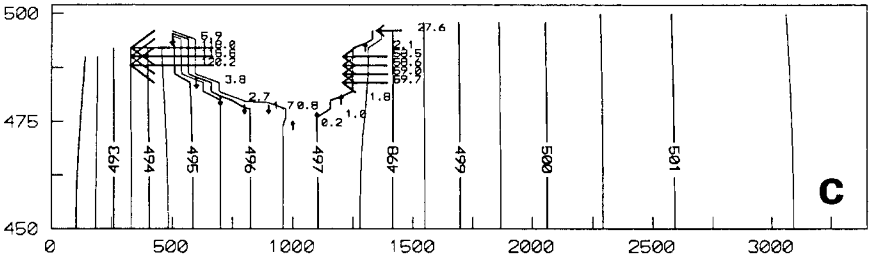


Fig. 5. Groundwater Recharge and Discharge Rates under Steady State Three-Dimensional Conditions (unit: m^3/day)

For the case of lake 1, groundwater recharge and discharge occurred in the lake bottom are very small and the rates are 2.9 m^3/day and 1.1 m^3/day. But in this figure it appears that groundwater recharge and discharge rates occurring in the lake side are 149.6 m^3/day and 73.3 m^3/day. Comparing the groundwater recharge and discharge, groundwater inflow and outflow of the lake side, these are 55.7 times larger than these occurring in the lake bottom. Groundwater recharge occurred in the lake side is two times larger than the groundwater discharge in the lake side.

For the case of lake 2, groundwater inflow and outflow occurring in the lake bottom is 7.2 m^3/day and small. Groundwater recharge and discharge rates occurring in the lake side are 307.9 m^3/day and 172.3 m^3/day. These results represent that groundwater inflow and outflow occurring in the lake side is 66.7 times larger than that occurring in the lake bottom. Groundwater recharge rate in the lake side is 1.8 times larger than the discharge rate of it.

For the case of lake 3, groundwater inflow and outflow rate occurring in the lake bottom is 13.0 m^3/day and small. Groundwater recharge and discharge rates in the lakeside are 357.0 m^3/day and 263.4 m^3/day. These results showed that groundwater inflow and outflow occurring in the lake side is 44.7 times larger than that occurring in the lake bottom. Groundwater recharge rate occurring in the lake side is 1.4 times greater than the discharge rate of it.

These results suggest that groundwater inflow and outflow ratio occurring in the lakeside is tens of times larger than that of the lake bottom. It is considered that groundwater recharge and discharge rate occurring in the lakeside is large but that of the lake bottom is small. Also, the difference rates between groundwater recharge and discharge occurring in the lakeside where a lake located in the upper portion of a watershed tend to be larger than for the lakes located in the lower portion of the watershed.

5. Conclusions

In order to investigate the role of lake position on groundwater fluxes, and to quantify the recharge and discharge relationship between groundwater and lake water on a watershed with a different lake portion, and to provide guidance for the monitoring network design to survey the exchange relations between groundwater and lake water steady state three dimensional simulation was undertaken.

The results are summarized as follows:

(1) The exchange between lake and groundwater in the case where a lake is located at a lower portion on watershed shows more active than that for a lake located at an upper portion.

(2) The differences of groundwater fluxes depend on the aquifer depth except where the vicinity of the lake are small.

(3) The amounts of inflow from groundwater to a lake are less than the amounts of discharge to groundwater in a target lake watershed.

(4) The rate of inflow and outflow of groundwater to a lake is increased as the lake is located at

the upper portion of a watershed.

(5) The horizontal flux of groundwater occurring on the lake bed is more significantly active than the vertical flux.

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