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Effects of Hydraulic Variables on the Formation of Freshwater–Saltwater Transition Zones in Aquifers

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ABSTRACT : The location and the shape of a freshwater-saltwater transition zone in a coastal aquifer are affected by many hydraulic variables. To date most works to determine the effects of these variables are limited to qualitative comparison of transition zones. In this work characteristics of transition zones are analyzed quantitatively. The investigation is limited to a steady-state transition zones. Three dimensionless variables are defined to represent characteristics of steady-state transition zones. They are maximum intrusion length, thickness, and degree of stratification. Effects of principal hydraulic variables (velocity and dispersivity) on these characteristics are studied using a numerical model. Dimensional analysis is used to systematically analyze entire model results. Effects of velocity and dispersivity are seen clearly. From this study, increase in velocity is found to cause shrinkage of transition zones. This observation contradicts claims by some that, because dispersion is proportional to velocity, increase in velocity would cause expansion of transition zones.

1. Introduction

Degradation of surface water quality increases the value of groundwater as another water resource. Ground water is commonly accepted as high quality. However, ground water is also susceptible to various contaminants. Thus, many wells are being closed due to contamination. Once polluted, remediation of ground water and aquifers are much more difficult than treatment of surface water systems. Therefore, prevention of contamination is essential. For effective prevention, impact of ground water development must be assessed carefully using predictive tools such as numerical models.

Density difference between freshwater and saltwater is the most important single element in analyzing saltwater intrusion in coastal aquifers. For saltwater intrusion problems two types of numerical models are available. On one hand, the dispersion model can handle the transition zone between freshwater and saltwater. On the other hand, the sharp–interface model neglects the thickness of the

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transition zone and assumes an abrupt interface between two waters. Because of heavy computational requirements of the dispersion model, when a modeling under the influence of saltwater is of regional scale, use of the sharp-interface model is the only choice. However, the assumption of an abrupt interface makes the sharp interface model unsuitable for analyses of saltwater intrusion problems of small scales.

In this paper, effects of hydraulic parameters on the characteristics of transition zones are analyzed using a dispersion model. The ground water velocity and the dispersivity affect the advection and the dispersion, which have major impact on the formation of transition zones. Many researchers investigated responses of transition zones to changes of these parameters (for example: Volker and Rushton, 1982; Reilly and Goodman, 1986). However, previous analyses were limited to qualitative comparisons of transition zones for a variety of cases. The objective of this work is in the quantitative analysis of transition-zone characteristics. To that end three variables are defined. They are maximum intrusion distance of a saltwater wedge, the thickness of a transition zone, and the degree of stratification of ground water. The maximum intrusion distance indicates how far the saltwater intrudes, and the thickness and the stratification indicate concentration distribution in the horizontal and vertical directions, respectively.

To give numerical experiments of some practical significance, parameter values for the north eastern area of Cheju Island were used (Water resources development plan in cheju, 1993). Results of numerical experiments were normalized via the Buckingham's Pi theorem. Because dispersion coefficient is proportional to the velocity, some claim that higher velocity causes thicker transition zones. But, ground water flow transports freshwater from the upstream area to coastal area, pushing saltwater seaward. This role tends to shrink the transition zone. For lateral intrusion problems considered in this study, the net effects of velocity are found to be shrinkage of the thickness of the transition zone.

2. Dispersion Model

The governing equations are obtained from the mass conservation principles for ground water and solute. For ground water flow

$$\nabla \cdot \mathrm{K} \cdot (\nabla h + \varepsilon c e_z) = S_s \frac{\partial h}{\partial t} + \theta \varepsilon \frac{\partial c}{\partial t} \tag{1}$$

and for solute transport

$$\nabla \cdot (\mathrm{D} \cdot \nabla c) - \mathrm{v} \cdot \nabla c = \theta \frac{\partial c}{\partial t} \tag{2}$$

where ∇ is the gradient operator, c is the chloride concentration normalized by sea water concentration, $\varepsilon(=\Delta\rho/\rho^f)$ is the density difference ratio between sea water and freshwater, K is the hydraulic tensor, θ is the effective porosity, and e_z is the unit vector in the vertical direction, S_s is the specific storage. The primary unknown in Eq. (1) is the equivalent freshwater hydraulic head (Lusczynski, 1971):

$$h=\frac{p}{\rho^f g}+z \tag{3}$$

where p is the pressure, ρ^f is the freshwater density, z is the elevation. Velocity is obtained from the Darcy law:

$$\mathrm{v}=-\mathrm{K}\cdot(\nabla h + \varepsilon c e_z) \tag{4}$$

and the dispersion tensor is

$$\mathrm{D}=D_m\mathrm{I}+\alpha_T\,|\,\mathrm{v}\,|\,\mathrm{I}+(\alpha_L-\alpha_T)\frac{\mathrm{vv}}{|\,\mathrm{v}\,|} \tag{5}$$

where D_m is the molecular diffusion coefficient, I is the identity tensor, α_L and α_T are longitudinal and transverse dispersivities, respectively. In this work, a numerical code named DSTRAM (Huyakorn and Panday, 1990) is used. This model has been subject to extensive verification and has been applied to a number of field problems (Park, 1991).

3. Scope of the Numerical Experiments

Transition zones are affected by several hydraulic variables such as D, v, θ, H (thickness of the aquifer), L (length of the aquifer), S_s, i (vertical leakage), $\Delta\rho$, ρ_f, g, K, *and* H. However, the scope of this work is limited to a steady state flow and transport in a uniform confined aquifer. Under these conditions θ and S_s vanish from the governing equations, and K, α, i, and H are constants. Average sea water properties are used so that $\Delta\rho$ (=25 kg/m^3) and ρ^f (=1000 kg/m^3) are constants. Then, the remaining variables are L, α, and v. Among these three, the length of the aquifer (L) becomes irrelevant when the landward boundary condition does not affect the transition zone. Therefore, variables to be studied reduce to two, namely the dispersivity and the ground water velocity.

Values for hydrogeologic parameters used in this study represent the northeastern Cheju Island (Water resources development plan in cheju, 1993). In that area of the Island, average thickness of the aquifer is about 200 m and the average hydraulic conductivity is approximately 100 m/d. For numerical experiments, an aquifer of 2 km length is used. For dispersivities six values (100, 70, 50, 25, 10, 7 m) were used. The values used for the freshwater inflow rate at the upstream boundary are 3.0, 1.0, 0.7, 0.5, 0.3, 0.1, 0.07, 0.05, and 0.03 m/d. Values used in the previous numerical study (Lee, 1993) are 40m for the dispersivity and 0.3 m/d for the average freshwater inflow rate at the upstream boundary.

4. Boundary Conditions and Numerical Experiments

Boundary conditions used in this work are depicted in Fig. 1. The general pattern of the ground water flow in the aquifer is such that freshwater entered from the upstream boundary of the aquifer pushes saltwater intruding from the seaward side. Flow boundary conditions specified at the seaward side are standard. Salt water enters through the lower portion of the seaward boundary, and ground water of less chloride content leaves the aquifer through the upper portion of the side (Fig. 1). For the portion where sea water enters, $c=1$ condition can be used. For the outflow boundary, $c=1$ condition is commonly used assuming instantaneous mixing with sea water. However, a more realistic boundary condition for the outflow boundary is $\nabla c \cdot n = 0$ where n is the unit normal vector. The difficulty with this boundary condition is that the exact size of this outflow boundary is not known a priori. Therefore, a trial-and-error method is needed in applying this boundary condition.

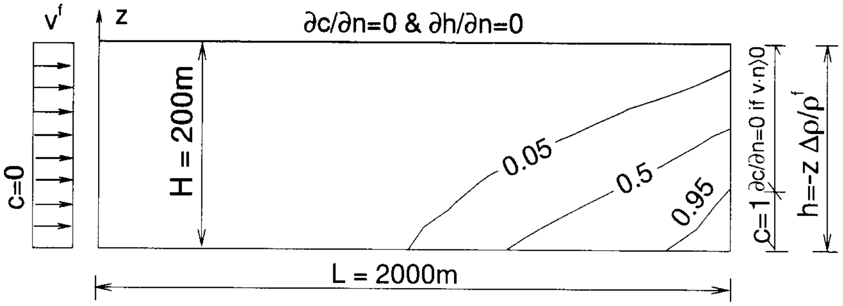

Fig. 1. Hypothetical Aquifer and Boundary Conditions Used in the Study

During the numerical experiments upstream weighting was not used to avoid numerical dispersion. For stability of numerical solutions near transition zones, grid sizes were adjusted so that grid Peclet numbers would not exceed 4 (Price et al., 1966; Voss and Souza, 1987). Two sets of meshes were used: 11 by 21 and 21 by 80. For small dispersivities, the fine mesh was used. The nonlinearities of the governing equations were treated using the Picard method (Huyakorn and Pinder, 1983). Sometimes, parameter continuation method (Rheinboldt, 1987) was also used for nonlinearities. Selected results ($v=3$, 0.3, 0.03 m/d and$=100$, 50, 10 m) from numerical experiments are depicted in Fig. 2.

5. Quantization of Transition-Zone Characteristics

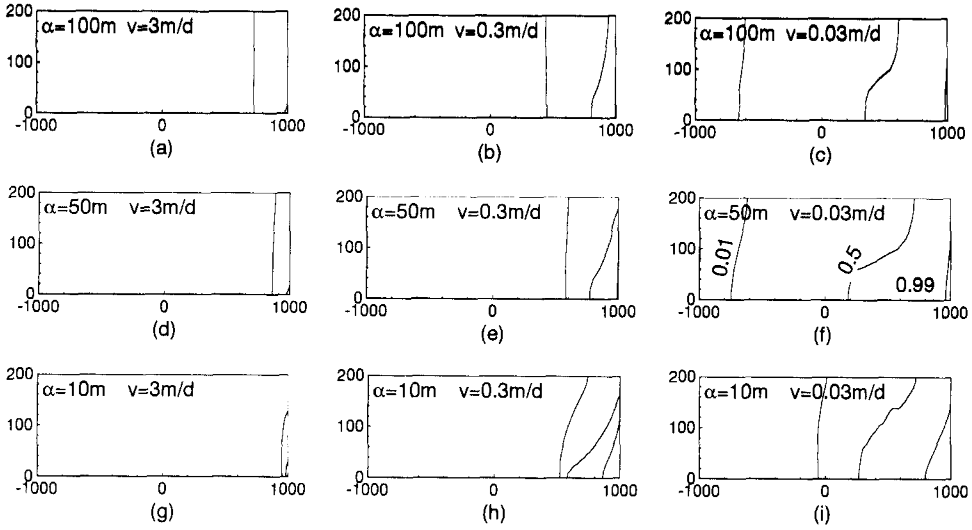

Fig. 2. Isochlors(0.01, 0.5, 0.99) from Some Numerical Experiments

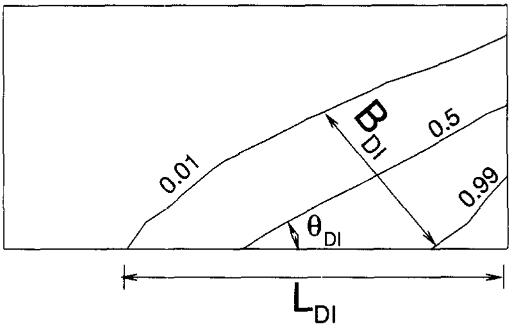

Fig 3. Definititions of Quantized Transition-Zone Characteristics

Characteristics of transition zones observed from numerical experiments were quantized as follows. The thickness of transition zones (B_{DI}) is defined as the distance between pure freshwater and pure saltwater. However, for practical purposes an isochlor of 0.01 (normalized concentration) is assumed to represent the limit of pure freshwater, and an isochlor of 0.99 is to represent the limit of saltwater. This distance clearly is not a constant for a transition zone. Therefore, the thickness must be defined in an average sense. It is defined to be the area, enclosed by 0.01 and 0.99 isochlors, divided by the length of the 0.01 isochlor.

As the second characteristic property, the maximum intrusion length (L_{DI}) is characterized by the distance between the coastline and the position where the 0.01 isochlor intersects the bottom of the aquifer (Fig. 3).

As the third characteristic, the degree of stratification of the transition zone can be rigorously defined as change of concentration per unit vertical distance. However, one can find that when the ground water is more stratified, the isochlors become more slanted from the vertical. Thus, the degree of stratification is conveniently represented by the angle (θ_{DI}) between the 0.5 isochlor and the aquifer bottom.

6. Analysis of Numerical Experiments via Dimensional Analysis

The dimensional analysis (Buckingham's Pi theorem) was used to analyze the numerical experimental results. There are ten variables of which three are dependent variables (B_{DI}, θ_{DI}, L_{DI}) and seven are independent variables (D, v, H, $\Delta\rho$, ρ^f, g, L). When v, H, ρ^f are used as repeating variables, the following relations are obtained among seven dimensionless variables:

$$\frac{B_{DI}}{H}, \frac{L_{DI}}{H}, \frac{2\theta_{DI}}{\pi} = f\left(Pe, Fr; \varepsilon, \frac{L}{H}\right) \tag{6}$$

where $Pe = Hv/D$ is the Peclet number, $Fr = v/(g'H)^{1/2}$ is the Froude number, $g' = \varepsilon g$ is the adjusted gravitational constant of acceleration. Eq. (6) indicates that the dependent variables are functions of four nondimensional variables. However, in this work, ε and L/H are constants as they were mentioned previously. Therefore, the dependent variables are functions of two dimensionless numbers Pe and Fr. Note that θ_{DI} is normalized by $\pi/2$. This variable varies between 0 and 1 where larger number indicates less stratification.

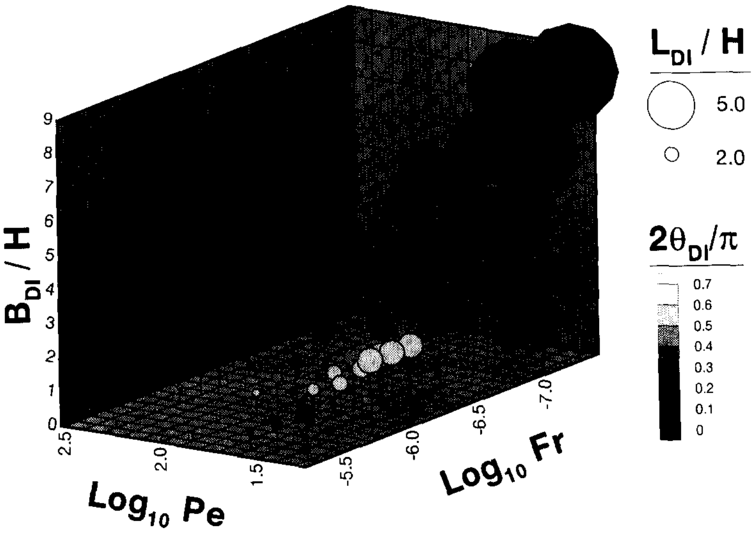

Fig. 4. Responses of Transition-Zone Characteristics to Changes of Pe and Fr Numbers

The three transition-zone characteristics from all numerical results are plotted against *Pe* and *Fr* (Fig. 4). The horizontal axes represent values of *Pe* and *Fr*. Note that the common logarithm is used. The vertical axis represents normalized transition zone thickness (B_{DI}/H), the size of circles represents the maximum intrusion length (L_{DI}/H), and the shade of circles represents the degree of stratification (θ_{DI}).

Fig. 4 reveals that the maximum intrusion length (L_{DI}/H) decreases as the Froude number(i.e., velocity) increases. The same trend is observed when the Peclet number increases except when the Froude number is small. When the Froude number is small, the intrusion length appears independent of the Peclet number. However, this apparent independence is due to the effects of the landward boundary condition ($c=0$). The $c=0.01$ isochlor would have shifted further inland if the landward boundary (where $c=0$ condition was specified) were located further inland. This would have caused the maximum intrusion length to increase.

The thickness of the transition zone (B_{DI}/H) also displays a similar trend, i.e., the thickness decreases as the Froude number increases. The increase in velocities makes the dispersion coefficient large. But, because the effects of advection dominate the effects of dispersion, the thickness decreases.

The degree of stratification becomes severe as the Froude number decreases and as the Peclet number increases. This trend is consistent with the observation that the slope of a sharp interface ($\alpha=0$) is milder than a dispersed ($\alpha>0$) transition zone for the given set of conditions (Volker and Rushton, 1982; Park, 1994).

7. Conclusions

The development of transition zones formed between freshwater and saltwater in a coastal aquifer is affected by a number of hydraulic variables. In this study we have investigated effects of two primary variables, namely velocities and dispersivities. A numerical model was employed to perform experiments. Three parameters which represent the characteristics of a transition zone are defined as: the maximum intrusion length, the thickness of a transition zone, and the degree of stratification. Results are analyzed using the Buckingham's Pi theorem. The following conclusions were made: The increase in the dispersivity values would increase the intrusion length and the thickness of a transition zone. However the degree of stratification would decrease. Larger velocities tend to decrease both intrusion lengths and degrees of stratification. Higher velocities would increase the dispersion coefficients which, in turn, tend to increase the thickness of transition.zones. However, this effect is offset by the increased freshwater advection. The net effect of increased velocity is to decrease the transition zone thickness.

The conclusions were obtained based on the numerical simulation of lateral intrusion of saltwater. Therefore, we cannot extend above conclusions to the cases of saltwater intrusion in the vertical direction. This phenomenon is commonly called upconing. Investigations need to be extended to cases of saltwater upconing and unsteady–state responses of transition zones.

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