韓國水資源學會誌 論文
第29卷 第3號·1996年 6月
pp. 217~228

외부전단력 적용에 의한 균일대칭복단면에서의 하도추적

Flow Routing in Prismatic Symmetrical Compound Channels by Applications of Apparent Shear Force

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Abstract

A new routing computer model for the symmetric compound channel called the ASFMCS (Apparent Shear Force Muskingum-Cunge Method in Symmetry) is developed. The Muskingum-Cunge routing method is adapted. The Apparent Shear Force (ASF) between the deep main channel and shallow floodplain flow is introduced while the flow is routed. The nonlinear parameter method is applied. The temporal and spatial increments are varied according to the flow rate. The adaptation of above schemes is tested against the routed hydrographs using the DAMBRK model. The results of general routing practice of Muskingum-Cunge Method (GPMC) are also compared with those of the above two models. The results of the new model match remarkably well with those of DAMBRK. The routed hydrographs show smooth variation from the inflow boundary condition without any distortions caused by the difference of cross-section shape. However, the results of GPMC, showing earlier rising and falling of routed hydrograph, have considerable differences from those of the ASFMCS and DAMBRK.

요 지

Apparent Shea Force(ASF:외부전단력)을 적용 균일대칭복단면의 하도추적을 위한 새로운 컴퓨터 모델 ASFMCS가 개발되었다. 기본공식은 Muskingum-Cunge방법이 이용 되었으며 ASFMCS의 특징은 복합단면에서 하도추적이 수행되는 과정에서 중앙저수부와 홍수터 사이에 발생하는 ASF가 고려되었다. 비선형 매개변수를 적용하고 유량변화에 따라 시간과 공간의 차분의 크기가 변하도록(Variable Time Step and Spatial step법)모델링이 되었다. ASFMCS의 결과는 DAMBRK와 기존의 단면분리법을 사용한 GPMC모델에 의해 발생한 수문곡선과 비교되었다. ASFMCS와 DAMBRK의 결과는 같은 경계조건 및 입력조건에 대해 거의 같은 수문곡선을 발생시켰으나 GPMC의 결과와는 상당한 차이를 보였다.

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1. Introduction

Open channel flow routing has many uses for water resources projects, for flood control system, and for proper management and design of hydraulic structures. Many studies have been performed using routing schemes, especially for simple cross-sections such as rectangular, trapezoidal, and triangular shapes.

However, most natural channel cross-sections are composed of a compound cross-section having a deep main channel with associated shallow floodplain or berm on one or both sides.

The purpose of the research is to develop a new computer model called ASFMCS (Apparent Shear Force Muskingum-Cunge Method in Symmetry) to predict open channel flow when the flow is routed in a symmetric compound channel. The new computer model developed in the present work is based on the Muskingum-Cunge flow routing scheme introduced by Cunge (1969). The Muskingum-Cunge scheme is a hydraulic approach and uses the kinematic wave principle.

The major new addition of the ASFMCS model is the application of the Apparent Shear Force (ASF) which results from momentum exchange in the turbulent flow between the deep main channel and the shallow floodplain as investigated by Knight et al. (1983), Myers (1978), Zheleznyakov (1971), Ghosh and Jena (1971), and Sellin (1964).

In addition to the Apparent Shear Force, the new model has several other new characteristics in carrying out the flow routing. The flows are mixed and redistributed at each spatial and time increment point, instead of the separated flow routing scheme. The model also uses the nonlinear parameter method recommended by Ponce (1989). The test of the model is accomplished by comparing the flow-routing results at the downstream end of the prismatic channel with the equivalent hydrographs produced by Fread's (1988) DAMBRK (1988 version).

The above two results are also compared with Eli's (1993) general routing practices of the Muskingum-Cunge Method(GPMC). The overall hydrograph shapes, peak discharges, and peak times of the new model are matched remarkably well with those of DAMBRK. However, considerable differences appeared between the results of the new model and GPMC.

2. Basic Equations and Theories

Three types of unsteady open channel flow waves are commonly used in distributed routing schemes: kinematic, diffusion, and dynamic waves.

The simplest model is the kinematic wave model, which neglects the local acceleration, convective acceleration, and pressure terms in the momentum equation. It assumes the friction and gravity forces balance each other.

Since Cunge (1969) used the analogy between the finite different approximation of the conventional Muskingum difference scheme and the linear kinematic wave equation to relate the routing parameters K and X, the Muskingum-Cunge flood routing method has been studied extensively by Koussis (1980, 1983), Ponce and Yevjevich (1978), and Ponce and Theurer (1982) for the propagation of flood waves in open channels.

2.1 Governing Equations of Classical Muskingum Method and Muskingum-Cunge Method

Ponce (1989) has thoroughly summarized the theoretical differences between the classical Muskingum method and Muskingum-Cunge

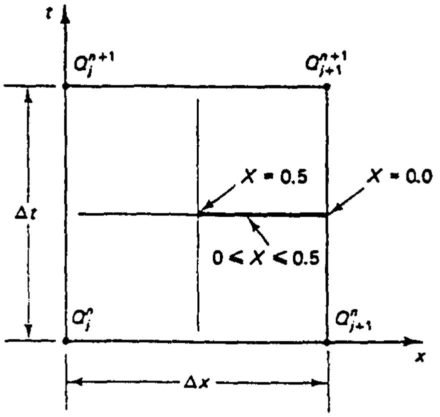

Fig. 1. Space–Time Discretization

method.

In the x–t plane of Fig. 1, the discretized form of the classical Muskingum method and Muskingum–Cunge method appears the same and is expressed as follows:

$$Q_{j+1}^{n+1}=C_1Q_j^n+C_2Q_j^{n+1}+C_3Q_{j+1}^n \tag{1}$$

in which C_1, C_2, and C_3 are the routing coefficients. The coefficients are expressed as follows:

$$C_1=\frac{(\Delta t/K)-2X}{2(1-X)+(\Delta t/K)} \tag{2}$$

$$C_2=\frac{(\Delta t/K)-2X}{2(1-X)+(\Delta t/K)} \tag{3}$$

$$C_3=\frac{2(1-X)-(\Delta t/K)}{2(1-X)+(\Delta t/K)} \tag{4}$$

in which K, X = routing parameters and Δt= time interval.

However, the origin of the two methods is different as follows.

2.2 Classical Muskingum Method

The classical Muskingum method is based on the differential equation of a storage which is expressed as follows:

$$I-O=\frac{dS}{dt} \tag{5}$$

where I = inflow, O = outflow, S = storage volume, and t = time.

The routing parameters K and X are related to flow and channel properties. K accounts for the translation portion while X accounts for the storage portion. The parameters in the Muskingum method are determined by calibration using recorded inflow and outflow hydrographs. According to Ponce (1989), the values of K and X determined are valid only for the given reach and flood event used in the calibration.

However, in the Muskingum–Cunge method, these parameters are computed by formulas derived by Cunge (1969) on the basis of physical condition of the stream channel.

2.3 Muskingum–Cunge method

The kinematic wave equation is expressed as follows:

$$\frac{\partial Q}{\partial t}+(mv)\frac{\partial Q}{\partial x}=0 \tag{6}$$

in which Q = flow rate, v = velocity, and m = exponent coefficient.

Cunge (1969) discretized Eq. (6) on the x–t plane of Fig. 1 in such a way that parallels the Muskingum method, centering the spatial derivative and off–centering the temporal derivative by means of a weighing factor X, and is expressed as follows:

$$\frac{X(Q_j^{n+1}-Q_j^n)+(1-X)(Q_{j+1}^{n+1}-Q_{j+1}^n)}{\Delta t} + C\frac{(Q_{j+1}^n-Q_j^n)+(Q_{j+1}^{n+1}-Q_j^{n+1})}{2\Delta x}=0 \quad (7)$$

where Δx = spatial interval.

The solution of Eq. (7) for unknown flow rate Q at (n+1), (j+1) gives the same as Eq. (1).

However, the routing coefficients C_1, C_2, and C_3 in Muskingum-Cunge method are expressed as follows:

$$C_1=\frac{c(\Delta t/\Delta x)+2X}{2(1-X)+c(\Delta t/\Delta x)} \quad (8)$$

$$C_2=\frac{c(\Delta t/\Delta x)+2X}{2(1-X)+c(\Delta t/\Delta x)} \quad (9)$$

$$C_3=\frac{2(1-X)-c(\Delta t/\Delta x)}{2(1-X)+c(\Delta t/\Delta x)} \quad (10)$$

The flood wave travel time K is defined as follows:

$$K=\frac{\Delta x}{c} \quad (11)$$

Therefore, Eqs. (8)-(10) are seen to be the same apparently as Eqs. (2)-(4). Cunge (1969) has shown that the parameter X can be related to the physical problem by matching the numerical diffusion of the kinematic wave and the hydraulic diffusivity of diffusion wave as follows:

$$\nu_n=c\Delta x(\frac{1}{2}-X) \quad (12)$$

$$\nu_h=\frac{Q_0}{2Ts_0}=\frac{q_0}{2s_0} \quad (13)$$

and therefore, X is expressed as follows:

$$X=\frac{1}{2}(1-\frac{q_o}{s_o c\Delta x}) \quad (14)$$

where ν_n = numerical diffusion coefficient, ν_h = hydraulic diffusion coefficient, T = width of channel, Q_o = reference flow, and q_o = reference flow per unit width.

With X calculated by Eq. (14), the Muskingum method is referred to as the Muskingum-Cunge method. The routing parameter X can be calculated as a function of the following numerical and physical properties: reach length, unit width discharge, wave celerity, and bed slope.

The second term in the parenthesis of Eq. (14) is the ratio of hydraulic diffusivity $q_o/2s_o$ to grid diffusivity $c\Delta x/2$ and is called cell Reynolds number by Roache (1972) and Weinman and Laurenson (1979). The cell Reynolds number D leads to the following:

$$D=\frac{q_o}{s_o c\Delta x}. \quad (15)$$

Therefore, Eq. (14) is simplified as follows:

$$X=\frac{1}{2}(1-D) \quad (16)$$

The ratio of wave celerity to grid celerity called Courant number is as follows:

$$C=c\frac{\Delta t}{\Delta x} \quad (17)$$

Substituting Eqs. (16) and (17) into Eqs. (8) and (9) leads to the following:

$$C_1=\frac{1+C-D}{1+C+D} \quad (18)$$

$$C_2 = \frac{-1+C-D}{1+C+D} \quad (19)$$

$$C_3 = \frac{1-C+D}{1+C+D} \quad (20)$$

The sum of Eqs. (18)-(20) is 1.0 because these routing coefficients C_1, C_2, and C_3 are the weighing factors in Eq. (1).

As explained in Eq. (1), the unknown flow rate is computed by the routing coefficients on the known values, and these coefficients are absolutely decided by the values of the cell Reynolds number D and the Courant number C as shown in Eqs. (18)-(20).

On the other hand, the cell Reynolds number and the Courant number depend on the flow properties such as discharge and velocity if the time interval Δt and the spatial interval Δx are selected by the user, and if the slope s_0 is known from the slope of the natural stream.

The flow properties of the simple cross-section can be computed by simple application of the various kinds of well known methods such as Manning equation, Chezy equation, Darcy-Weisbach formula, etc.,. In compound channels, however, there are too many flow complexities to apply these equations properly as will be explained in the next section.

2.4 Flow Properties of Compound Cross-Sections

When the water surface overtops the bank, the flow is modified considerably by the lateral and vertical interaction between regions of different depth as shown schematically in Fig. 2. According to Pache and Rouve (1985), Knight and Hamed (1984), and Sellin (1964), the secondary flow along the re-entrance corner between the floodplain and the main channel is usually marked in the region by strong vorticity in the vertical direction. Along this vertical interface, the vortices rotate about vertical axes and transfer momentum laterally between the floodplain and the main channel. These macro vortices also have the effect of flow resistance. This phenomenon is called the kinematic effect by Wormleaton et al. (1982), Myers (1978), Zhelenznyakov (1971), and Sellin (1964).

By this kinematic effect, the vertical intersection plane experiences appreciable turbulent force referred to as the apparent shear force (ASF) by Knight et al. (1983), Myers (1978), Zhelenznyakov (1971), Ghosh and Jena (1971), and Sellin (1964). Myers (1978) and Baird and Ervine (1984) also investigated the force effect overall is to produce on extra resistance to the main channel flow and an extra assistance to the floodplain. Other effects include a reduction in mean velocity and boundary shear stress in the main channel and an increase in velocity and boundary shear stress on the floodplain.

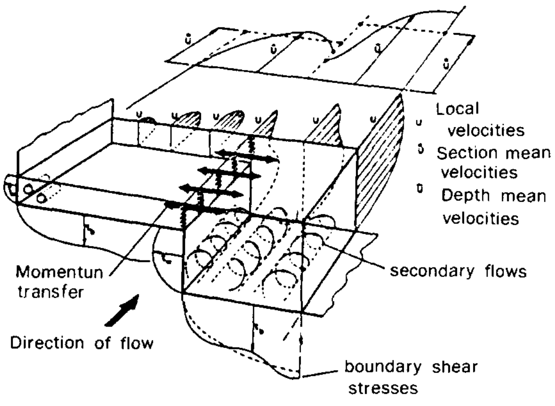

Fig. 2. Sketch of Hydraulic Aspects in Compound Cross-Section (Knight et al., 1984)

2.5 Velocity of Main Channel and Floodplain Sub-Cross Section

The actual average velocity equations for each sub-cross section are as follows:

$$v_m = \frac{1}{n_m\sqrt{g}} R_m^{1/6} \sqrt{\frac{\tau_m}{\rho}} \tag{21}$$

$$v_f = \frac{1}{n_f\sqrt{g}} R_f^{1/6} \sqrt{\frac{\tau_f}{\rho}} \tag{22}$$

in which v_m = actual average velocity of a main channel, v_f = actual average velocity of a floodplain, n_m = Manning roughness coefficient for a main channel, n_f = Manning roughness coefficient for a floodplain, R_m = hydraulic radius of a main channel, A_m/p_m, and R_f = hydraulic radius of a floodplain, A_f/p_f.

The value of shear stress must be obtained from the actual shear force of main channel and floodplain sub-cross section. In order to calculate the actual shear force on the floodplain, the apparent shear force is added to undisturbed shear force. On the other hand, the main channel shear force is decreased as much as the apparent shear forces on both intersections in the symmetric compound section.

Therefore, the actual average shear stress in the main channel and floodplain sub-cross section of a symmetric cross-section shown in Fig. 3 is expressed as follows:

$$\tau_m = \frac{SF_m - 2S_A}{2P_3 + P_4} \tag{23}$$

$$\tau_f = \frac{SF_f + S_A}{P_1 + P_2} \tag{24}$$

in which τ_m = actual average shear stress in main channel, τ_f = actual average shear stress in floodplain, SF_f = shear force in main channel, SF_f = shear force in floodplain, s_A = apparent shear force on a vertical intersection, p_1 = wetted perimeter of a floodplain wall, p_2 = wetted perimeter of a floodplain bed, p_3 = wetted perimeter of a main channel wall, and p_4 = wetted perimeter of a main channel bed.

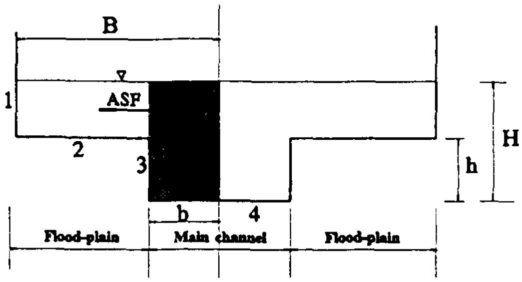

Fig. 3. Typical Symmetric Compound Cross-Section

2.6 Apparent Shear Force (ASF)

Knight et al. (1983), and Knight and Hamed (1984) studied the influence of differential roughness between the floodplain and main channel on the momentum transfer process, and presented a practical equation concerning the apparent shear force on the basis of roughness and geometry condition of a symmetrical compound cross-section.

In a symmetric cross-section as shown in Fig. 3, the percentage of apparent shear force may be expressed as follows by Knight and Hamed (1984):

$$\%s_A = \frac{50}{(\alpha-1)\beta+1} - \frac{1}{2}[100 - \%2(SF_1 + SF_2)] \tag{25}$$

in which $\%s_A$ = percentage of apparent shear force on the vertical interface, $\%[SF_1 + SF_2]$

= shear force percentage in a floodplain, SF_1 = actual shear force on a floodplain wall, SF_2 = actual shear force on a floodplain bed, α = non-dimensional width parameter, B/b, and β = non-dimensional depth parameter, (H − h)/H.

Therefore, the absolute value of apparent shear force s_A in kg/m may be obtained simply by multiplying the total shear force of whole cross-section, SFT, by $\%s_A$ as follows:

$$s_A = SFT\ X\ (\%s_A/100) \tag{26}$$

where SFT = ρgAS_f = total shear force of whole cross-section (kg/m), A = total cross-section area (m^2), and S_f = energy gradient.

On the other hand, Knight's (1984) experimental equation developed by physical hydraulic tests is applied to compute the shear force percentage on the both floodplains, $\%2[SF_1 + SF_2]$ of Eq. (25). It is as follows:

$$\%2(SF_1+SF_2)=48.0(\alpha-0.8)^{0.283}(2\beta)^{1/\omega}(1+1.02\beta^{1/2}\log_{10}\gamma) \tag{27}$$

in which $\omega = 0.75\ e^{0.38\alpha}$, $\gamma = n_f/n_m$, n_m = Manning's roughness coefficient for a main channel, and n_f = Manning's roughness coefficient for a floodplain.

As shown in Eq. (27), the percentage of the total shear force carried by floodplains, $\%2[SF_1 + SF_2]$, is increased with the floodplain width, water depth, and floodplain roughness.

As previously stated, the apparent shear force is an important factor in computing the velocity, and can be estimated by given geometric conditions of a cross-section.

3. Model Development

3.1 Nonlinear Parameters

The nonlinear method of computation recommended by Ponce and Yevjevich (1978) is applied because this method increases the computational accuracy. The nonlinear method allows the varying of the routing parameters as a function of discharge change for every computational cell. The updated values of the unit width discharge q_o and wave speed c are used to compute the routing parameters instead of peak flow or other reference values as in linear solution methods.

3.2 Application of Temporal and Spatial Increment

A variable computational time increment scheme is applied. Initially, a large time increment DT is selected. Then the time increment DT is further refined according to the rate of upstream inflow change into the channel reach. As shown in Fig. 4 the time increment in each computational cell may be different. If an abrupt flow change rate is sensed, the current time increment is reduced; if a gradual change is found, the size of time increment is increased. The flow calculations are dependent only on the current space and time increment, and they are, therefore, independent of any of computational cell.

The minimum size of Δt selected is larger than DT/100 in the present model because of the limit of computer capacity and CPU time. For the selection of the spatial increment, the model follows the Ponce's (1982) accuracy criteria on —x expressed as follows:

$$\Delta x \le \frac{1}{\lambda}(\Delta x_C + \Delta x_D) \tag{28}$$

where Δx_c= minimum subreach length, $\Delta x_C \geq q_o/(S_f c)$, Δx_D= maximum subreach length, $\Delta x_D \leq c\Delta t$, and λ = accuracy parameter by Ponce and Theurer (1982).

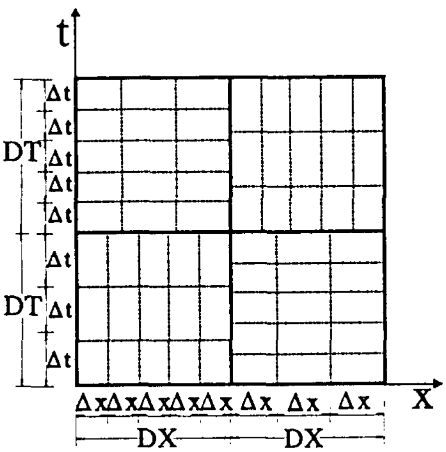

Fig. 4. Definition Sketch of Variable Time and Spatial Increment

4. Validation of the Model

4.1 Input Data

For the testing and validation of the ASFMCS, several channel cross-sections with the length of 30,480 m have been hypothesized. The geometries of the cross-section are shown in Fig. 5.

The Gamma Function is used to produce the inflow boundary condition (BC) at the upstream end. It is expressed as follows:

$$Q_t = Q_b + (Q_P - Q_b)\left(\frac{t}{t_P}\right)^M exp\left(\frac{t_P - t}{t_c - t_P}\right) \tag{29}$$

in which Q_t = inflow at time t, Q_b = base flow, Q_p = peak flow, t_p = time from beginning of direct runoff to peak of hydrograph, t_c = time from beginning of direct runoff to center of gravity of hydrograph, and M = $t_p/(t_c - t_p)$.

The Q_p in Eq. (29) is 5,095 m3/sec. The values of t_p and t_c are 240 minutes and 330 minutes, respectively. The base flow Q_b are 1,274 m^3/sec.

The selected time step DT is 15 minutes for every cross-sections. The models have shown numerical stability when the values are applied to them. The ASFMCS and DAMBRK have the same DT values; this makes the models produce the same time step outputs, which makes smoother hydrographs. In the ASFMCS, these values are refined again as necessary to a minimum of DT/100, according to the flow rate as explained in the section of Application of Temporal and Spatial Increment. However, in DAMBRK, the values are applied directly to

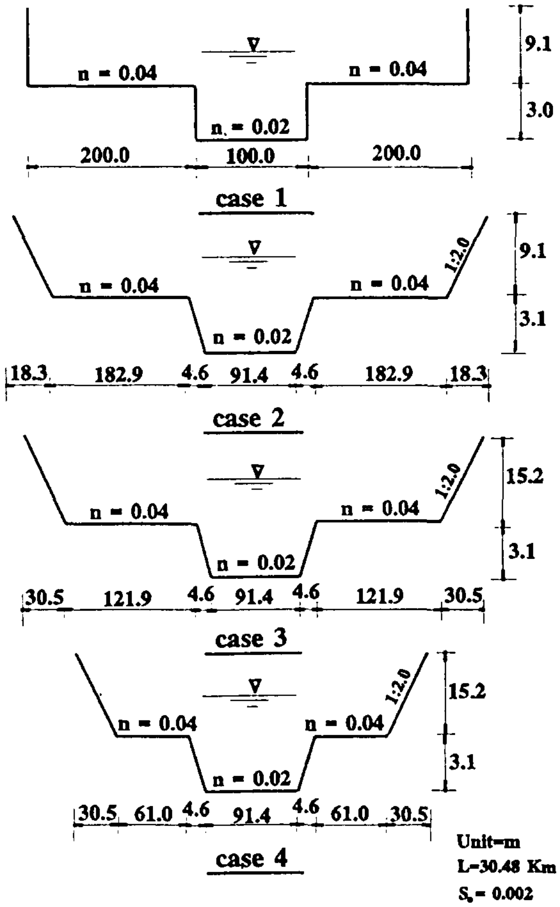

Fig. 5. Schematic of Test Cross-Sections

compute various parameters. The time step of the GPMC model is fixed in 1.0 hr. because the internal functions of the model are designed to use a 1.0 hr time step.

The distance step DX in the ASFMCS is selected as 3,048m for all cross-sections. This value is refined again in the process of computation according to the size of the computational time step Δt. The value of DX in the DAMBRK is selected as 402 m because this value is within the accuracy criteria of the model, and the model shows numerical stability at this value for the given values of DT. The GPMC model is capable of internal subdivision of Δx in order to meet stability and accuracy criteria.

4.2 Results and Discussions

The corresponding inflow and outflow hydrographs are plotted in Fig. 6 for visual interpolation. As shown in the figure the results of ASFMCS routings are matched remarkably well with those of the equivalent routings of DAMBRK. The consistency of the peak time and peak discharge has been demonstrated in the comparative testing. The routing results of GPMC, however, are shown to be considerably different from those of the above two models.

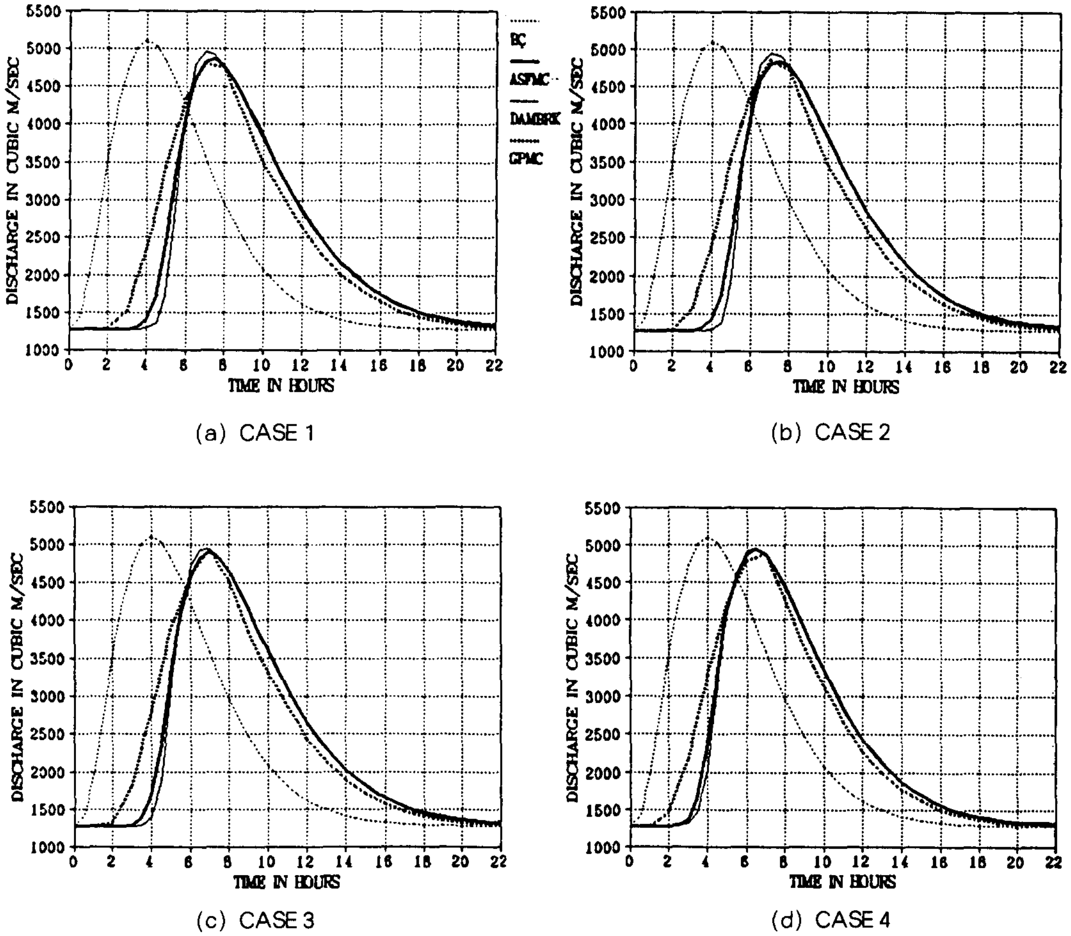

Fig. 6. Comparisons of Routed Hydrograph on the Test Cross-Section

In order to quantitatively compare the results, the root mean square(RMS) errors in percentage are computed for each set of routed hydrographs. The RMS (%) is calculated as follows:

$$RMS(\%)=[\sum_{j=1}^{N}\frac{D_j^2}{N}]^{1/2} \qquad (30)$$

$$D_j=(\frac{A_j-G_j}{A_j})\times 100 \qquad (31)$$

in which A_j = routed flow of ASFMCS at time step j, G_j = routed flow of DAMBRK or GPMC at time step j, and N = total number of time steps.

The RMS (%) errors are also listed in Table 1 for the cross-sections. The maximum errors between the ASFMCS and DAMBRK on the routed hydrograph are 5.07 % for Case 1, 4.57 % for Case 2, 3.93% for Case 3, and 2.85% for Case 4, respectively. However, as shown in the plotted hydrographs of Fig. 6, all errors between the ASFMCS and DAMBRK are mainly from the disparities at the rising limb where the abrupt flow changes are occurring as explained above.

Table 1. RMS Error (%) between Model Results

CASE	DAMBRK	GPMC	CASE	DAMBRK	GPMC
1	5.07	16.10	3	3.93	15.87
2	4.57	17.24	4	2.85	14.36

The plots of routed hydrographs of the GPMC are also shown in the same figures. According to the figures, the hydrographs produced by the GPMC model rise and fall earlier than those of the other two models. This phenomenon means that the flows of the GPMC reach the downstream end of the channel earlier than the flows of other models. Theoretically, this phenomenon is reasonable because the GPMC model, in the course of computation, is not considering the Apparent Shear Force which is caused by the vortices at the interface plane between the main channel and floodplain. As explained in the previous section, the macro vortices contain the major portion of the turbulence energy and thus of the flow resistance. Therefore, it is concluded that the hydrographs from the ASFMCS respond later than the hydrographs from the GPMC since the flows of the ASFMCS see more apparent resistance than those of the GPMC.

5. Conclusions

A new computer model called ASFMCS (Apparent Shear Force Muskingum-Cunge Method in Symmetry) is developed for the computation of flow routing in compound channels. The Apparent Shear Force (ASF) at the intersection between the main channel and floodplain is applied while the flow is routed by the Muskingum-Cunge Method.

The adaptation of the ASFMCS to the model-flow routing scheme is tested using the DAMBRK model as bench mark. The DAMBRK model uses a momentum coefficient for velocity distribution adjustments in a one-dimensional full dynamic equation. The several symmetric cross-sections with different geometric conditions are investigated.

From the present investigations, the following conclusions have resulted:

(1) The hydrographs produced by the ASFMCS are matched closely to those of the DAMBRK.

(2) Comparing the hydrographs produced by the ASFMCS and DAMBRK models, the consistency of the hydrograph characteristics, such as peak discharge and peak time, are also demonstrated with reasonable hydraulic responses according to the geo-metric conditions.

(3) The routed hydrographs have smooth variation, comparing with the inflow hydrographs showing no distortions or abrupt changes due to overbank flows. This contradicts Garbret and Brunner (1991) who show a distortion (two peak times) from routed hydrographs due to the presence of floodplain width.

(4) The routed hydrographs of the GPMC show earlier rising and falling limbs and have significantly greater differences as compared to those from ASFMCS. This is a reasonable hydraulic phenomenon, because the flows of the ASFMCS are resisted more than the flows of the GPMC due to the kinematic effect.

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〈접수 : 1996년 2월 18일〉