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A Fuzzy Set Theoretic Approach for the Multigoal Plant Layout Planning

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ABSTRACT

This paper presents a fuzzy set theory based formulation and heuristic which will allow plant layout problems to be solved for more than two input goals with different weights. The heuristic combines a construction and improvement procedure to generate a good layout from the formulation. To test the effectiveness of the heuristic, a set of examples previously used by various authors is solved and the results are compared to those from other known heuristics. The comparison indicates that the proposed method performs well for each of seventeen classical test problems.

1. Introduction

The plant layout problem has the goal of locating departments in a plant to achieve the greatest efficiency in producing a product or service. This problem is usually formulated as the quadratic assignment problem (QAP). Burkard and Stratmann [2], Domschke and Drexl [3], and Kusiak and Heragu [16] have provided an extensive literature review of algorithms for the QAP.

Traditionally, two basic approaches have been used to solve plant layout problems. One is a quantitative approach which tries to minimize material handling costs between departments. Armour and Buffa [1], among others, have developed heuristics for this approach. The other is a qualitative approach which considers subjective closeness ratings. Muther [17] and many others have developed methods for this approach.

The quantitative approach does not consider qualitative factors such as the need of two

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departments to be far apart because of noise or dust problems. On the other hand, the major problem of the qualitative approach is its scoring method, which is based on preassigned numerical values for the different closeness ratings and does not incorporate the work flow cost between the various departments [21].

Each approach has its own set of advantages and disadvantages. Recently, work has been done to integrate the two approaches. Rosenblatt [21], Dutta and Sahu [4], Fortenberry and Cox [8], Rosenblatt and Sinuany-Stern [23], and Urban [25] have presented models and heuristics which minimize the material handling costs and maximize the closeness rating scores.

However, Evans et al. [6] discussed several difficulties associated with the QAP formulation of the layout design problem. One of the real difficulties in developing and using QAP formulations is the natural vagueness associated with the inputs to the models. According to Zadeh [26], fuzziness is a major source of inexactness in many humanistic systems, thus the application of conventional quantitative techniques to the study of humanistic systems is often unsuccessful. Conventional techniques are not equipped to handle fuzziness and call for a high level of precision that is difficult to obtain.

Karwowski and Evans [15] discussed the potential application of the fuzzy set theory concept to various areas of production management including layout design. The authors notice that many variables or relationships relevant to the models existing in the above problems are initially specified in an imprecise and vague manner and are later simplified for ease of analysis in an attempt to eliminate or reduce fuzziness. The fuzzy set theory based approach provides a framework for modelling problems which are inherently vague, allows for the systematic treatment of uncertainty due to fuzziness in a quantitative manner and increases understanding of complex systems, while simultaneously decreasing the cost associated with unnecessary precision [19].

Grobelny [11,12] presented the linguistic pattern approach based on Zadeh's possibility theory and Lukasiewicz's multivalued implication formula [26,27]. Raoot and Rakshit [20] presented, on the basis of Grobelny's work, a linguistic pattern based heuristic for the quadratic assignment formulation of the layout problem with single/multiple objectives. However, they consider only two goals and do not consider weights representing the relative importance of each goal.

When considering a layout problem, it is not necessary to limit the number of goals considered to two. Several different qualitative relationship charts may be used to represent different kinds of relationships. For example, the qualitative relationships desired by different managers may be considered separately. The use of common utilities by departments may be considered separately from the environmental effects between departments. It is also possible to consider several quantitative aspects. For example, flow may be separated by material size, product line, or material flow times [13]. In addition, to reflect the relative importance of each goal, weights may be applied to each goal. These goals and weights are subjectively decided by expert who is knowledgeable about the layout planning.

This paper is based on the linguistic pattern approach presented by Grobelny [11,12]. Its objectives are to develop a fuzzy set theory based formulation which will handle any number of goals and their weights, and to present a heuristic which will develop a good layout quickly. The fuzzy set theory based heuristic involves deriving an initial assignment and then, possibly, improving the initial solution through exchange between pairs of departments. To test the effectiveness of the heuristic, we solve a set of examples previously used by various authors and compare the results.

In the following section, the linguistic pattern approach suggested by Grobelny is discussed. Then a formulation and heuristic for solving this problem are presented, followed by a numerical example. Finally, a set of examples is solved and the results are compared with the solution of other known heuristics. The effectiveness and the rationale of the heuristic are then discussed.

2. Linguistic Pattern Approach

The linguistic pattern approach [11,12] is based on possibility theory, estimation of truth and the procedure of inference [26,27].

If $X=\{x\}$ is a set of objects, then the fuzzy set A in X is said to have a membership function in which the range of the values is [0, 1], i.e.

$$A: X \rightarrow [0,\ 1] \tag{1}$$

whereas

$$A = \{A(x), x\} \qquad \text{for all } x \in X, \tag{2}$$

so eventually fuzzy set A is a set of ordered pairs of the form in equation (2). Function $A(x)$ determines the grade of the membership of an element x in the set A.

The truth value of a given statement p with respect to the criterion r is defined as 'consistency'. Let p ='A is F', r='A is G', where A is the name of the variable and F and G denote fuzzy sets of the membership functions $F(x)$ and $G(x)$, respectively, then

$$\begin{aligned} \text{Cons } (A \text{ is } F,\ A \text{ is } G) &= \text{POSS } (A \text{ is } F \mid A \text{ is } G) \\ &= \sup_{x \in X} \{F(x)\ G(x)\} \end{aligned} \tag{3}$$

where $\wedge$ denotes a minimum operator and the possibility measure is defined as supremum (sup).

POSS-possibility-is defined from the concept of possibility distribution introduced by Zadeh [24]. The consistency, which is a numerical value calculated using equation (3) denotes the grade of closeness of sets F and G and the possibility of the fact that variable A equals G and at the same time is F.

The truth value of implication can be estimated according to the infinite valued logic by Lukasiewicz [27]. The truthfulness of the implication $A \Rightarrow B$, denoted by $|A \Rightarrow B|$ (where $|A|$ means the grade of truth of the expression A, $|B|$ means the grade of truth of the expression B) is calculated from

$$|A \Rightarrow B| = \min(1, 1-|A|+|B|) \tag{4}$$

In solving a layout problem, linguistic patterns are statements which can be treated as both recommendation and the criteria of evaluating a given layout. As an illustration, consider the following goal concerning the proper layout: "A department pair with heavy flow should be allowed to be as close to each other as possible." A small modification of the above statement will allow us to obtain the following sentence called a linguistic pattern.

If FLOW_{ik} for a given pair of departments is very HEAVY Then the DISTANCE_{jl} between them is VERY SMALL.

Or in a shorter form:

If FLOW_{ik} = VERY HEAVY Then DISTANCE_{jl} = VERY SMALL.

In the above statement, FLOW and DISTANCE are linguistic variables and VERY HEAVY and VERY SMALL are values of the linguistic variables. The linguistic pattern enables us to evaluate any layout numerically by comparing this layout with the pattern. Using linguistic patterns means evaluating a truth value which shows how a given layout fulfills those patterns. It can be done by using equations (3) and (4).

3. Mathematical Model

A quadratic assignment formulation using the linguistic pattern is developed to represent the problem of assigning departments to locations. The formulation presented here allows us to consider more than two goals with different weights and to combine individual goals into a single composite goal. The objective function in this study is to minimize the difference value between the desirable and actual truth distance. The rationale for the objective function is to find the final layout which satisfies the linguistic patterns to the maximum extent. In the formulation, we assume that the departments are of equal area. The distances between various locations are measured by a rectangular distance. The proposed model is shown in equations (5)-(8).

$$\text{Minimize } Z = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} \sum_{l=1}^{n} A_{ijkl} X_{ij} X_{kl} \tag{5}$$

Subject to:

$$\sum_{i=1}^{n} X_{ij}=1 \qquad j=1,\dots,n \tag{6}$$

$$\sum_{j=1}^{n} X_{ij}=1 \qquad i=1,\dots,n \tag{7}$$

$$X_{ij}=0 \text{ or } 1 \qquad \text{for all } i, j \tag{8}$$

where

$$X_{ij}=\begin{cases} 1 & \text{if department } i \text{ is assigned to location } j \\ 0 & \text{otherwise} \end{cases}$$

A_{ijkl}='cost' of having department i at location j and department k at location l

In order to consider more than two input goals by comparable units of measure, all goals are combined into one composite goal. Several steps are used for developing the composite goal. First, the relationships between department pairs for each goal are converted to the desirable distance using the appropriate linguistic pattern, as suggested by Raoot and Rakshit [20]. Second, all desirable distances are multiplied by weights representing the relative importance of each goal m (α_m). Next, the total of weighted desirable distances for each pair of departments is calculated. The composite desirable distance resulting from this conversion is shown in equation (9).

$$S_{ik} = \sum_{m=1}^{t} \alpha_m S_{ikm} \tag{9}$$

where

α_m = weight for goal m

t = number of goals

S_{ikm} = desirable distance between department i and k for goal m

Finally, the truth value of the composite desirable distance for each pair of departments is calculated according to equation (3). The desirable truth values indicate the desired 'degree of closeness' between the department pairs. The problem is formulated by substituting equation (10) for A_{ijkl} in equation (5).

$$A_{ijkl} = d_{ik} - a_{jl} \tag{10}$$

where

d_{ik} = desirable truth distance between departments i and k with respect to the appropriate linguistic variable using equation (3)

a_{jl} = actual truth distance between locations j and l with respect to the appropriate linguistic variable using equation (3)

It is possible to relax constraint (6), allowing the number of locations to be more than the number of departments. In this case, some locations will be left after the layout is complete, and equation (6) is replaced with equation (11), where p is the number of possible locations [10].

$$\sum_{i=1}^{n} X_{iu} \leq 1 \qquad u = 1, \ldots, p \tag{11}$$

4. The heuristic algorithm

The proposed heuristic algorithm for solving the plant layout quadratic assignment problem consists of two phases: Ⅰ. Construction Ⅱ. Improvement. The construction heuristic is employed to obtain a good starting initial layout for the improvement method. In the improvement procedure, the initial solution is improved through exchange between pairs of departments.

Ⅰ. Construction

Step 1. Establish a linguistic description of the relationship between department pairs using the values of the appropriate linguistic variable.

Step 2. Using equation (3), find the truth value of the individual relationship with respect to the value of the appropriate linguistic pattern.

Step 3. Based on the truth value of the individual relationship, derive the desirable distance between all department pairs. In order to assure the maximum degree of satisfaction with the linguistic pattern, the derived distance value should be such that its truth value with respect to the value of DISTANCE used in the appropriate pattern is greater than or equal to the truth value of the linguistic variable value [20]. Next, all desirable distances are multiplied by weights representing the relative importance of each goal.

Step 4. Calculate the total of weighted desirable distances for each pair of departments and, using equation (3), find the truth value of the composite desirable distance with respect to the value of the appropriate linguistic variable.

Step 5. Determine the sum of desirable truth distances from each department to all other departments, D_i, where

$$D_i = \sum_{k=1}^{n} (d_{ik} + d_{ki}) \quad (12)$$

Rank the departments according to their decreasing values of D_i.

Step 6. According to equation (3), calculate the truth values of actual distances with respect to the value of the appropriate linguistic variable. Based on the actual truth distance, determine the sum of actual truth distances from each location to all other locations, A_j, where

$$A_j = \sum_{l=1}^{n} (a_{jl} + a_{lj}) \quad (13)$$

Step 7. Divide the locations into subsets L_k, $k=1,\ldots K$, of equal values. Rank the subsets in decreasing order of A_j.

Step 8. Choose a next candidate department and location. The department ranked first is the candidate department and will be placed into the layout next. If the current subset of locations L_k is empty, select the next subset. If the current subset of locations L_k has only one location, assign the candidate department to the location. If the current subset of locations L_k has more than one location, calculate the difference value of placing the candidate department in each candidate location and place the candidate department in the candidate location that results in the lowest additional difference value. The difference value has already been defined in section 3.

Step 9. Increase the total difference value by the additional difference value incurred by placing the department in the location in step 8 and eliminate the department and location from the lists. If no more candidate departments are available, go to step 10. Otherwise, go to step 8.

Ⅱ. Improvement

Step 10. Given the current layout, calculate the difference value for each department pair. Divide the department pairs into two groups, Q_1 and Q_2. If $d_{ik} > a_{jl}$(i.e., desirable truth distance > actual truth distance), the department pair is assigned to Q_1 and needs to be kept closer. If $d_{ik} < a_{jl}$(i.e., desirable truth distance < actual truth distance), the department pair is assigned to Q_2 and needs to be kept further apart. If $d_{ik} = a_{jl}$, however, the department pair does not need to be adjusted and is ignored.

Step 11. Calculate the averages of Q_1 and Q_2. The department pairs above the average of Q_1

or below the average of Q_2 are candidates for improvement. If no such department pair exists, stop. Otherwise, go to step 12.

Step 12. Select the candidate pair for improvement. While one member (i) of the pair is fixed in its location, the other member (k) of the pair is exchanged with i's 'relative departments'. The relative department (m) should be such that the actual distance between i and m is shorter (for Q_2, longer) than the actual distance between i and k. If this interchange results in improvement, this potential change becomes a candidate for change. If no such candidate exists, stop. Otherwise, choose the best candidate for change and perform the interchange, and go to step 10.

5. A numerical example

A numerical example from Raoot and Rakshit [20] is used for illustrative purposes. The example has also been used by Rosenblatt [21], Dutta and Sahu [4], Fortenberry and Cox [8]. Because the quantitative and qualitative relationship information in the numerical example is available as flow volume and closeness rating score, the relationship is converted to a linguistic description using the values of linguistic variables [20].

The linguistic variables QUAN-LINK, QUAL-LINK and DISTANCE, their primary variable values, base variable values (i.e., universe of discourse), membership functions and linguistic pattern are presented in appendix. Because a final layout is sensitive to the linguistic variable information, the information used by Raoot and Rakshit [20] was adapted to compare this algorithm's results with theirs.

Step 1. Quantitative relationship expressed using values of linguistic variable QUAN-LINK:

	1	2	3	4	5	6
1	–	Low	Medium	V. low	Low	Low
2	Low	–	Low	V. low	V. low	High
3	Medium	Low	–	V. low	V. low	Medium
4	V. low	V. low	V. low	–	Medium	V. low
5	Low	V. low	V. low	Medium	–	V. high
6	Low	High	Medium	V. low	V. high	–

Qualitative relationship expressed using values of linguistic variable QUAL-LINK:

	1	2	3	4	5	6
1	–	Medium	V. low	V. low	High	Low
2	Medium	–	Medium	V. low	High	V. low
3	V. low	Medium	–	Negative	V. low	Negative
4	V. low	V. low	Negative	–	V. low	V. low
5	High	High	V. low	V. low	–	High
6	Low	V. low	Negative	V. low	High	–

Step 2. Truth value matrices with respect to VERY HIGH for the quantitative and qualitative relationship:

	1	2	3	4	5	6
1	–	0.10	0.40	0.00	0.10	0.10
2	0.10	–	0.10	0.00	0.00	0.90
3	0.40	0.10	–	0.00	0.00	0.40
4	0.00	0.00	0.00	–	0.40	0.00
5	0.10	0.00	0.00	0.40	–	1.00
6	0.10	0.90	0.40	0.00	1.00	–

	1	2	3	4	5	6
1	–	0.40	0.00	0.00	0.90	0.10
2	0.40	–	0.40	0.00	0.90	0.00
3	0.00	0.40	–	−1.00	0.00	−1.00
4	0.00	0.00	−1.00	–	0.00	0.00
5	0.90	0.90	0.00	0.00	–	0.90
6	0.10	0.00	−1.00	0.00	0.90	–

Step 3. Weighted desirable distance matrices for the quantitative and qualitative relationship:

	1	2	3	4	5	6
1	–	1.00	0.75	1.25	1.00	1.00
2	1.00	–	1.00	1.25	1.25	0.50
3	0.75	1.00	–	1.25	1.25	0.75
4	1.25	1.25	1.25	–	0.75	1.25
5	1.00	1.25	1.25	0.75	–	0.50
6	1.00	0.50	0.75	1.25	0.50	–

	1	2	3	4	5	6
1	–	0.75	1.25	1.25	0.50	1.00
2	0.75	–	0.75	1.25	0.50	1.25
3	1.25	0.75	–	1.50	1.25	1.50
4	1.25	1.25	1.50	–	1.25	1.25
5	0.50	0.50	1.25	1.25	–	0.50
6	1.00	1.25	1.50	1.25	0.50	–

In this example, an equal weight of 0.5 was applied to each quantitative and qualitative goal.

Step 4. Total of the weighted desirable distances for each pair of departments is given in the following matrix.

	1	2	3	4	5	6
1	–	1.75	2.00	2.50	1.50	2.00
2	1.75	–	1.75	2.50	1.75	1.75
3	2.00	1.75	–	2.75	2.50	2.25
4	2.50	2.50	2.75	–	2.00	2.50
5	1.50	1.75	2.50	2.00	–	1.00
6	2.00	1.75	2.25	2.50	1.00	–

Desirable truth distance matrix:

	1	2	3	4	5	6
1	–	0.50	0.50	0.00	0.80	0.50
2	0.50	–	0.50	0.00	0.50	0.50
3	0.50	0.50	–	0.00	0.00	0.00
4	0.00	0.00	0.00	–	0.50	0.00
5	0.80	0.50	0.00	0.50	–	1.00
6	0.50	0.50	0.00	0.00	1.00	–

Step 5. Total of the desirable truth distances from each department to all other departments in decreasing order.

$D_5=5.6$, $D_1=4.6$, $D_2=4.0$, $D_6=4.0$, $D_3=2.0$, $D_4=1.0$.

Step 6. Actual truth distance matrix:

	1	2	3	4	5	6
1	–	1.00	0.50	1.00	0.50	0.00
2	1.00	–	1.00	0.50	1.00	0.50
3	0.50	1.00	–	0.00	0.50	1.00
4	1.00	0.50	0.00	–	1.00	0.50
5	0.50	1.00	0.50	1.00	–	1.00
6	0.00	0.50	1.00	0.50	1.00	–

Sum of the actual truth distances from each location to all other locations in decreasing order.

$A_2 = A_5 = 8$, $A_1 = A_3 = A_4 = A_6 = 6$.

Step 7. Subsets according to their decreasing values of A_j.

$L_1 = \{2,5\}$, $L_2 = \{1,3,4,6\}$

Step 8 and 9. The current subset of locations is L_1 with two members. Assign department 5 to location 2. The current subset has only one location. Thus department 1 is assigned to location 5. Because L_1 is empty, the current subset of locations is L_2 with four members. Departments 5 and 1 and locations 2 and 5 are eliminated from the lists.

At this stage the candidate department is 2 and the candidate locations are 1, 3, 4 and 6. If candidate department 2 is placed in each candidate location, the total difference values are all 1.0. As we have ties, select the first location in the subset. Assign department 2 to location 1. L_2 is reduced to three members. Current difference value is 1.0.

If the next candidate department 6 is assigned to each candidate location (3, 4 and 6), the additional difference values are 0.6, 1.8 and 1.8, respectively. Assign department 6 to location 3 which resulted in the lowest additional difference value. The current difference value is changed to 1.6. The next candidate department is 3 and the candidate locations are 4 and 6.

If department 3 is assigned to each candidate location, the additional difference values are 1.2 and 2.2, respectively. Assign department 3 to location 4. Finally, department 4 is assigned to location 6. At this stage we have an initial layout:

2	5	6
3	1	4

Step 10. The difference value for each pair of departments is given in the following matrix. Due to symmetry of the distance matrices ($d_{ik} = d_{ki}$ and $a_{jl} = a_{lj}$) in this example, only the upper triangular part of the matrix is necessary.

	1	2	3	4	5	6
1	–	0.00	−0.50	−1.00	−0.20	0.00
2		–	−0.50	0.00	−0.50	0.00
3			–	−0.50	−0.50	0.00
4				–	0.00	−1.00
5					–	0.00
6						–

Step 11. The averages of Q1 and Q2 are 0.00 and −0.587, respectively. There is no department pair in Q1. The department pairs below the average of Q_2 are 1−4 and 4−6. These pairs need to be kept further apart and selected as candidates for possible improvement.

Step 12. Department 1 is fixed and department 4 is moved. The candidate locations to which department 4 moves are 1 and 3, because the actual distance between department pairs (1−2 and 1−6) is longer than the actual distance between 1 and 4. Consequently, two department exchanges (2−4 and 6−4) are necessary.

Here, the 'flow * distance * closeness rating' measure is used as the improvement criterion to compare with the final values obtained by other algorithms. The current initial assignment value is 368. Department exchange (2−4) yields the same value as the initial assignment. Department exchange (6−4), on the other hand, does not result in improvement.

Next, department 4 is fixed and department 1 is moved. The candidate locations to which department 1 moves are 1, 2 and 4. Department exchanges (2−1, 5−1 and 3−1) only worsen the value. The same result is obtained if departments 4 and 6 are used. Thus the heuristic can stop. If department exchange (2−4) resulting in 368 is performed, we have the following alternative layout:

1	5	6
3	2	4

If the same approach as above is applied to the alternative layout, we have no department pair resulting in an improvement. Hence, the heuristic stops.

6. Comparison with other procedures

Raoot and Rakshit [20] presented a set of problems which can serve as a benchmark for comparing the effectiveness of other algorithms with the QAP. These problems have also been used by Golany and Rosenblatt (G+R) [10], Rosenblatt (R) [22,23], Dutta and Sahu (D+S) [4, 5], Fortenberry and Cox (F+C) [8] and Urban (U) [25]. The proposed heuristic solved the same problems. The summaries of results obtained for single and multiple goal problems are presented in Table 1 and 2, respectively.

In the case of multiple goal problems the heuristics of Rosenblatt (R) [22,23], Dutta and Sahu (D+S) [4,5], Fortenberry and Cox (F+C) [8] generated different solutions for different initial solutions and for different weights. Only the 'best' solutions obtained by these heuristics are included in Table 2. In Table 1 and 2, 'I' and 'F' indicate the initial and final solutions, respectively.

These tables show that the improvement procedure after the construction procedure was necessary to achieve a good layout. Also, no other procedure resulted in any better layouts than the proposed construction-improvement heuristic. Out of 14 single-objective problems, for problems 1, 3, 5 and 7 the initial assignment values are optimal, for problems 2, 4, 6, 8, 9, 10 and 11 the final assignments are optimal, for problem 12 the final assignment value is equal to the 'best' value obtained by other heuristics, and for problems 6, 13, and 14 the final values are better than the 'best' solutions achieved so far.

Out of the three multiple goal problems, in two cases the initial or final assignment value is equal to the 'best' value achieved by other heuristics, and for the third case the final value is better than the 'best' value obtained by other heuristics.

In a majority of the cases, improvement is obtained by applying pairwise exchange. The minimum, maximum and average numbers of the improvement procedure iteration are 1, 5 and 2.17, respectively. This indicates that good layouts may be produced in a reasonable amount of computational time. The proposed heuristic is more effective than other heuristics when the number of departments is large (see problems 13, 14 and 17 in tables).

The main differences between this heuristic algorithm and the one proposed by Raoot and Rakshit [20] are as follows. In Raoot and Rakshit's, only two goals are considered and weights can not be applied to each goal. In this algorithm, on the other hand, any number of goals may be handled and weights can be applied to each goal so that the final layout appropriately reflects the relative importance of each goal.

Their method requires more computational efforts to combine all goals into one composite goal than our method does. They use two steps. First, for each department pair, they find the sum of the truth values of the desirable distances with each of the values of the linguistic variable DISTANCE. Next, they select the distance value for which the sum of truth values is maximum. In this approach, however, all goals are combined into one composite goal by just summing the weighted desirable distances.

They select the department pairs ((desired degree of closeness > actual degree of closeness) or (desired degree of closeness=0 and actual degree of closeness=1)) as candidates for possible improvement. This algorithm, however, selects the department pairs above the average of Q_1 or below the average of Q_2. Thus, this approach identifies the pairs both for keeping closer and for keeping further apart and it can reduce the number of pairs investigated for possible improvement. In the example in section 5, their approach selects 6 pairs (2−4, 4−6, 1−2, 1−6, 2−6 and 4−5), while our approach selects only 2 pairs (2-4 and 4-6).

Table 1. Procedure Comparison for Single Goal Layout Problems.

Problem No	Problem Size	Problem Reference	Solution	Locations 1	2	3	4	5	6	7	8	9	10	11	12	Cost Obtained	Cost Optimal
1	4	Gavett (1968)	G+R(I)	2	4	3	1									403	403
			R+R(I)	2	4	3	1									403	
			fuzzy(I)	2	4	3	1									403	
2	4	Zimmermann and Sovereign (1974)	G+R(I)	2	4	3	1									437	389
			(F)	4	2	3	1									389	
			R+R(I)	4	2	3	1									389	
			fuzzy(I)	1	2	3	4									494	
			(F)	4	2	3	1	(1)								389	
3	4	Tompkins and White (1984)	G+R(I)	2	3	1	4									164	164
			R+R(I)	3	1	2	4									224*	
			fuzzy(I)	2	3	1	4									164	
4	5	Nugent *et al.* (1968)	G+R(I)	5	2	4	1	3								26	25
			R+R(I)	5	4	2	1	3								26	
			(F)	4	5	1	2	3								25	
			fuzzy(I)	4	2	5	1	3								31	
			(F)	4	1	5	2	3	(1)							25	
5	6	Francis and White (1974)	F+W(I)	5	1	4	6	3	2							98	92
			G+R(I)	3	6	2	1	5	4							92	
			R+R(I)	2	6	3	4	5	1							92	
			fuzzy(I)	5	6	2	4	1	3							92	
6	6	Nugent *et al.* (1968)	G+R(I)	2	5	4	1	6	3							57	43
			(F)	4	5	2	1	6	3							53	
			R+R(I)	1	5	4	2	6	3							51	
			(F)	1	4	5	2	3	6							47	
			fuzzy(I)	6	5	4	2	1	3							47	
			(F)	6	5	4	3	2	1	(2)						43	
7	6	Rosenblatt (1986)	G+R(I)	1	4	2	5	3	6							12964	12822
			(F)	6	4	2	5	3	1							12894	
			R+R(I)	2	4	6	5	3	1							12822	
			fuzzy(I)	1	3	5	6	4	2							12822	
8	6	Rosenblatt (1986)	G+R(I)	1	3	6	5	4	2							14883	14853
			(F)	5	3	6	1	4	2							14853	
			R+R(I)	1	3	5	6	4	2							15328	
			(F)	5	3	6	1	4	2							14853	
			fuzzy(I)	1	3	6	5	4	2							14883	
			(F)	5	3	6	1	4	2	(1)						14853	
9	6	Rosenblatt (1986)	G+R(I)	1	5	3	2	4	6							13172	13172
			R+R(I)	1	5	3	2	4	6							13172	
			fuzzy(I)	1	5	2	6	4	3							13866	
			(F)	3	5	1	6	4	2	(2)						13172	
10	6	Rosenblatt (1986)	G+R(I)	1	5	2	3	6	4							13545	13032
			R+R(I)	2	6	4	3	5	1							13349	
			(F)	4	6	1	3	5	2							13032	
			fuzzy(I)	2	5	1	4	6	3							13365	
			(F)	3	5	2	4	6	1	(2)						13032	
11	6	Rosenblatt (1986)	G+R(I)	4	1	3	5	2	6							13172	12819
			(F)	4	1	5	3	2	6							12819	
			R+R(I)	5	1	3	6	2	4							13110	
			(F)	5	1	4	6	2	3							12819	
			fuzzy(I)	3	1	5	6	2	4							13350	
			(F)	4	1	5	3	2	6	(2)						12819	
12	7	Nugent *et al.* (1968)	G+R(I)	1	7	6	2	4	5	3						78	73
			R+R(I)	4	6	5	3	1	7	2						96	
			(F)	4	5	6	7	1	2	3						74	
			fuzzy(I)	4	6	5	3	1	7	2						96	
			(F)	4	5	6	7	1	2	3	(3)					74	
13	8	Nugent *et al.* (1968)	G+R(I)	1	8	7	3	2	4	6	5					120	107
			R+R(I)	7	8	4	1	3	6	5	2					118	
			(F)	6	8	7	3	5	4	1	2					112	
			fuzzy(I)	7	8	4	3	2	1	5	6					126	
			(F)	3	8	4	7	2	1	5	6	(1)				109	
14	12	Hiller (1963) Nugent *et al.* (1968)	G+R(I)	1	4	6	3	10	8	11	5	2	7	9	12	351	297**
			(F)	1	4	6	5	10	8	11	3	2	7	9	12	327	
			R+R(I)	10	7	9	12	6	8	11	2	5	4	10	1	314	
			fuzzy(I)	3	7	9	12	6	8	11	2	5	4	10	1	330	
			(F)	12	7	9	3	4	8	11	1	5	6	10	2 (3)	289	

* R+R [20] present the value of 164 which is not correct.

** G+R [10] and R+R [20] present the value of 297 which is not correct.

( ) value indicates the number of the improvement procedure iteration.

Table 2. Procedure Comparison for Multiple Goal Layout Problems.

Problem			Solution	Locations												Cost	
No	Size	Reference		1	2	3	4	5	6	7	8	9	10	11	12	Obtained	Optimal
15	6	Rosenblatt (1986) Dutta and Sahu (1983) Fortenberry (1985)	R	1	5	6	3	2	4							368	not known
			D+S	1	5	6	3	2	4							368	
			F+C	3	1	4	2	5	6							368	
			R+R(I)	1	5	6	3	2	4							368	
			fuzzy(I)	2	5	6	3	1	4							368	
16	8	Dutta and Sahu (1982, 1983) Rosenblatt (1983) Fortenberry and Cox (1985)	D+S	4	8	5	1	3	6	7	2					1106	not known
			R	3	1	5	8	4	2	7	6					1028	
			F+C	8	5	1	3	6	7	2	4					1028	
			R+R(I)	3	1	5	8	4	2	7	6					1028	
			fuzzy(I)	8	5	1	4	3	7	6	2					1422	
			(F)	8	5	1	3	6	7	2	4 (3)					1028	
17	12	Fortenberry and Cox(1985) Urban(1987)	F+C	7	6	4	1	5	12	10	2	9	11	8	3	4540	not known
			U	7	6	4	3	11	5	8	2	9	12	10	1	4598	
			R+R(I)	7	6	4	1	10	5	8	2	9	11	12	3	4734	
			(F)	7	6	4	1	5	12	10	2	9	11	8	3	4540	
			fuzzy(I)	9	7	10	1	11	6	5	2	4	12	8	3	4866	
			(F)	9	2	10	2	11	5	8	3	4	7	6	1 (5)	4464	

( ) value indicates the number of the improvement procedure iteration.

In this method, instead of all possible departments, only departments (with a shorter or longer actual distance) are exchanged with one member of the candidate pair.

These considerations result in a reduction in the time required to design the layout.

7. Conclusions

When considering department layout problems, it is often desirable to consider multiple goals and their weights in the model. However, one of the real difficulties in developing and using QAP formulations is the natural vagueness associated with the inputs to the models. The proposed method presented in this paper may consider any number of goals and allow for the systematic treatment of uncertainty due to fuzziness in a quantitative manner. By combining a construction and improvement procedure, good layouts may be developed for a very reasonable amount of computational efforts for single or multiple goal layout problems. Also, the heuristic performs well for both symmetric and non-symmetric layout configurations.

The procedure may be used alone or the output layout could be used as a starting layout input for other methods, which should lead to better results than a randomly generated starting point.

In several steps of the algorithm presented here, ties are broken by simply selecting the first department or location in the list. Thus, it is desirable to apply the approach several times whenever ties occur and to choose the best layout. Rules for tie breaking in this study are quite simple. However, additional work should address this issue. In addition, actual industrial problems which have more than two goals with different weights should be tested to evaluate the effectiveness in terms of computational time and quality of solution.

Appendix. Linguistic Variable Information.

Variable / Values	Universe of discourse										
QUAN-LINK	0	1	2	3	4	5	6	7	8	9	10
Very Low	1.0	1.0	0.9	0.5	0.2	0.0	0.0	0.0	0.0	0.0	0.0
Low	0.0	0.1	0.8	1.0	0.9	0.6	0.1	0.0	0.0	0.0	0.0
Medium	0.0	0.0	0.2	0.6	0.8	1.0	0.8	0.6	0.1	0.0	0.0
High	0.0	0.0	0.0	0.0	0.0	0.1	0.7	0.9	1.0	0.9	0.7
Very High	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.4	0.8	1.0	1.0
QUAL-LINK	0	1	2	3	4	5	6	7	8	9	10
Very Low	1.0	1.0	0.5	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Low	0.0	0.8	1.0	0.7	0.1	0.0	0.0	0.0	0.0	0.0	0.0
Medium	0.0	0.0	0.7	1.0	0.8	0.4	0.0	0.0	0.0	0.0	0.0
High	0.0	0.0	0.0	0.5	1.0	0.9	0.3	0.0	0.0	0.0	0.0
Very High	0.0	0.0	0.0	0.0	0.3	1.0	1.0	0.8	0.3	0.0	0.0
Negative	0.0	0.0	0.0	0.0	0.0	0.2	0.7	1.0	1.0	1.0	1.0
DISTANCE	1.0	1.5	2.0	2.5	3.0	4.0	5.0	6.0	7.0	8.0	9.0
Very Close	1.0	0.5	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Close	0.8	1.0	0.7	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Near by	0.2	0.8	1.0	0.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Far	0.0	0.0	0.0	0.8	1.0	1.0	1.0	1.0	1.0	1.0	1.0

Linguistic patterns for placement and evaluation of facility locations:

(1) If QUAN-LINK = Very high, then DISTANCE = Very close.

(2) If QUAL-LINK = Very high, then DISTANCE = Very close.

(3) If QUAL-LINK = Negative, then DISTANCE = Far.

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