Journal of the Korean
OR/ MS Society
Vol. 21, No. 2, August 1996

Branch and Bound Algorithm for the Facility Layout Problem Without Shape Distortion

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Abstract

Given the flow matrix, plant size (rectangle shape) and department sizes, the algorithm in this paper provides the plant layout with rectilinear distance measure. To construct automated facility design, eigenvector approach is employed. A branch and bound computer code developed by Tillinghast is modified to find the feasible fits of departments without shape distortion (see [1]) in the plant rectangle. The computational results compared with CRAFT are shown.

I. Introduction

Facility layout problems have been analyzed by researchers for many years. In production management and operations research areas facility layout problems have received great attention. Facility layout is the problem of choosing department location in the given plant size. For the history of solution approach to generate a layout, see Raoot and Rakshit [6].

Normally, there are two criteria in the facility layout models. One is minimizing the total distance traveled (Minisum), and the other is minimizing the maximum distance traveled (Minimax). Two distance measures commonly used in facility layout models are rectilinear and Euclidean distances (see details in Francis et al. [4]).

Since the measure of effectiveness for layout problems in the plant is related to the flow of materials, it is important to have correct information about the flow of material in the plant.

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Therefore, we attack the plant layout problem of Minisum when rectilinear distance measure is used. Assuming an increasing linear function of the distances between department, the minimization of the flow of material is one of the most popular criteria for quantitative layout models. In this case the objective function can be expressed in

$$\textit{Minimize} \sum_{i=1}^{n}\sum_{i=1}^{n} c_{ij}f_{ij}d_{ij} \tag{1}$$

where $n \equiv$ the number of departments

$c_{ij} \equiv$ the unit cost per flow between department i and j

$f_{ij} \equiv$ the number of flows between departments i and j

$d_{ij} \equiv$ the distance between departments i and j

II. Branch and bound technique

The problem statement is described as follows. Given rectangle plant, n rectangle departments whose width and length are fixed and flow matrix between department, how can we generate the layout which minimizes total sum of material flows between departments when rectilinear distance metric is used. Given a flow matrix, we can obtain the scattered diagram using the eigenvector approach of Drezner [3]. The idea of eigenvector approach is as follows. If there is no area requirement of each department, then the optimal solution of problem (1) is obtained very easily. The solution will be a point that all facilities coincide. That means $d_{ij}=0$. Therefore, in the eigenvector approach, problem (1) is modified as

$$\textit{Minimize} \sum_{i=1}^{n}\sum_{i=1}^{n} c_{ij}f_{ij}d_{ij} \tag{2}$$

$$\textit{Subject to} : \textstyle\sum_{i=1}^{n}\sum_{i=1}^{n} d_{ij}^{2}=1$$

In order to compute eigenvectors, define matrix $S=\{s_{ij}\}$, where $s_{ij}=-c_{ij}f_{ij}$ for $i \neq j$ and $s_{ij}=\sum_{j} c_{ij}f_{ij}$ for all i. Then, calculate the eigenvalues of S and corresponding eigenvectors. Two eigenvectors related to the second and third least eigenvalues are x and y coordinates of each facility (department), respectively.

The generation of the final layout (with space requirement) is addressed by using the point layout obtained by eigenvector approach. The quadratic assignment approach with the branch and bound technique was developed for the above question in [7]. The idea of his method is as follows. First, select the candidates (departments) for four corner grids. If a department is chosen for a corner grid, then find other departments which can be adjacent to this department. The relationship between departments are measured by the Euclidean distance in the point layout. Of course, the departments strongly related to the preassigned department will be the adjacent department.

Since he considered only square shaped departments, his method is extended for rectangular shaped departments. The grid assignment and tree manipulation routines (see details in Tillinghast [7]) are revised for this purpose. For better illustration, the solution procedure (branch and bound technique) for layout are explained in grid assignment, tree manipulation (branching, fathoming, bachtracking) and lower bound calculation. The computational results obtained by the proposed branch and bound technique are compared with CRAFT since CRAFT has been a standard layout tool for many years [2].

2.1 Grid assignment

If a grid, say *g*, is occupied by a department, say *i*, we have to search the grid sets neighboring the grid *g* suitable for the size of the department *i*. First, suppose department *i* requires square size, say 3 by 3. In this case, four possible considerations will be needed to assign the department *i* in the plant site. Figure 1 shows four possible combinations of 3 by 3 department shape.

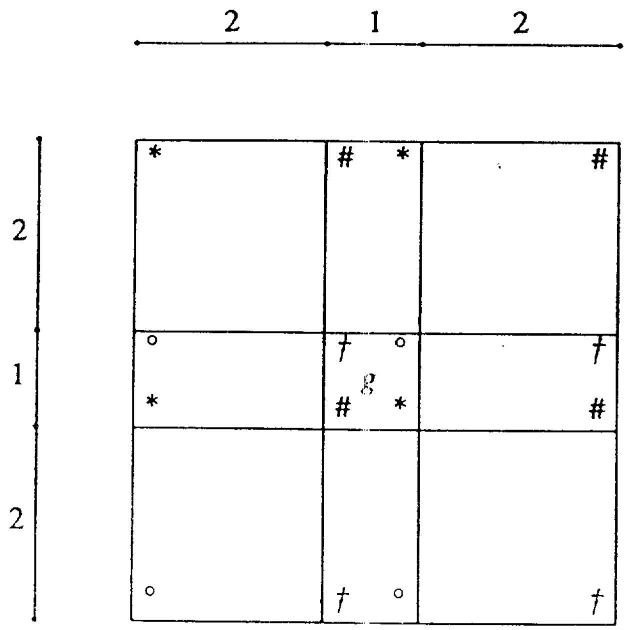

Figure 1. Possible grid location combinations for a square

But, if department i requires a rectangular shape, say 2 by 3, then the grid assignment will be more complex. Because of the orientation of department i, it is necessary to consider two cases : x-axis orientation and y-axis orientation. There are eight possible combinations for rectangle, four possible combinations with the x-axis orientation and four possible combinations with the y-axis combinations. Figure 2 and Figure 3 show all combinations with the x-axis and y-axis orientations, respectively.

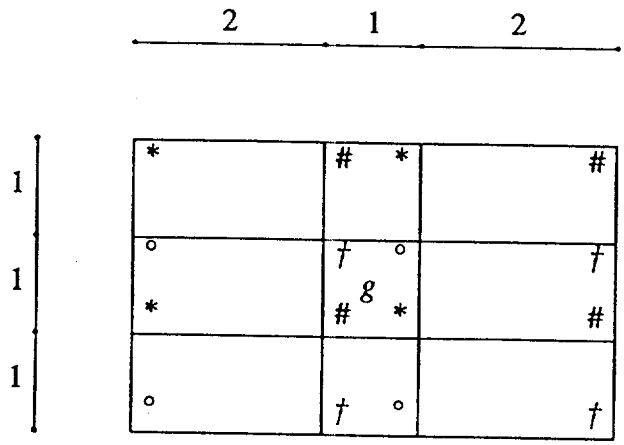

Figure 2. Possible grid location combinations for a rectangle (x-axis)

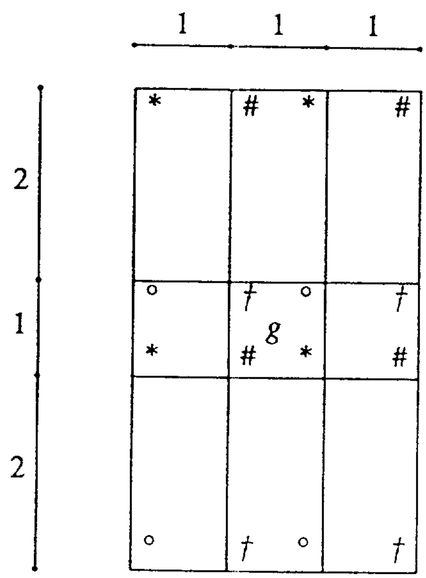

Figure 3. Possible grid location combinations for a rectangle (y-axis)

2.2 Tree manipulation

In order to search the feasible solution, a branch and bound technique is used. Suppose department i, j, k, l and m requires a square shape. The tree structure in branch and bound is constructed as Figure 4. If department j needs a rectangular shape, the tree structure must be altered because of the grid assignment. Figure 5 shows the adjustment of the tree structure when the size of department j is changed from square to rectangle. The notations of j^x and j^y in Figure 5 represent the x-axis oriented grid assignment and the y-axis oriented grid assignment of department j, respectively.

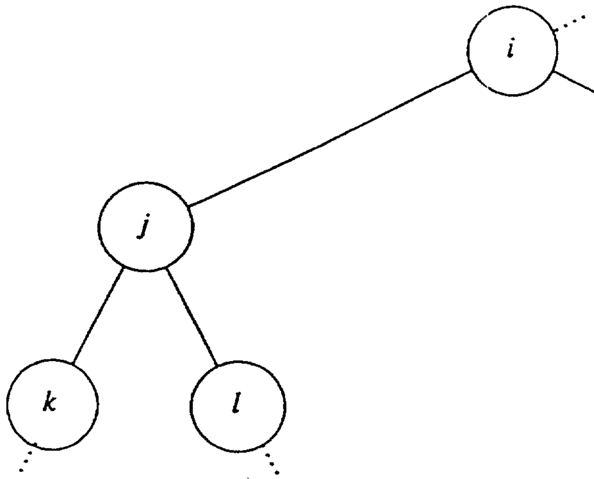

Figure 4. Tree structure when the shape of department j is square

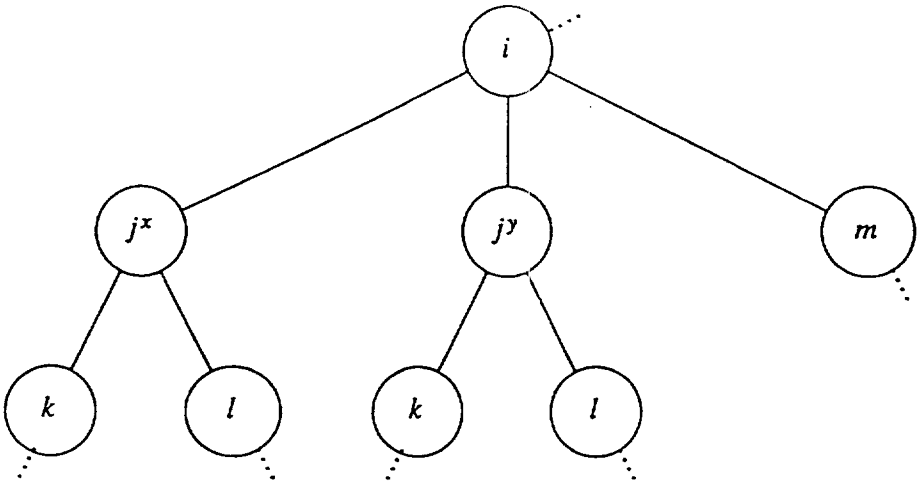

Figure 5. Tree structure when the shape of department j is rectangle

2.3 Constructing a new lower bound

Definition (rectilinearly adjacent)

If the distance between the centroids of two departments is equal to the sum of the minimum length or width of both departments, then both departments are called rectilinearly adjacent. (see Figure 6)

In order to construct a lower bound in a partial layout, Tillinghast [7] separated departments into two groups : assigned departments and unassigned departments. Assuming all departments unassigned in a partial layout are rectilinearly adjacent to each other and all departments assigned, he constructed a lower bound as

$$L_b=\sum_{i\in A}\sum_{j\in A}c_{ij}d_{ij}+\sum_{i\in A}\sum_{j\in A(\cup U)}c_{ij}(m_i+m_j)$$

where A≡set of assigned departments

U≡set of unassigned departments

m_i≡minimum length or width of department i

d_i≡ distance between department i and j

Since distance measure is rectilinear, we can limit the number of departments rectilinearly adjacent to a department, say x. Even though departments i and j are adjacent to department x in Figure 6, only department i is rectilinearly adjacent to department x. Therefore, at most, four departments can be rectilinearly adjacent to a single department x. Since the shapes of departments are square or rectangle, four departments occupy four edges of department x. After four departments are assigned to be rectilinearly adjacent, we compute the minimum length or width of these four departments and make a minimum size of rectangle (ring) for an integrated department. Figure 7 represents how we can make a ring to define an integrated department. A rectangle with a dotted line indicates the integrated department (note that the integrated department has the minimum size of rectangle which includes 4 additional departments). Next, find the candidates for departments which can be rectilinearly adjacent to the integrated department. That procedure is repeated until all unassigned departments are located for computing lower bound in order of the sequence determined by flow values from department x.

If the shape of department x is a very thin rectangle, the two departments rectilinearly adjacent to department x do not provide a real bound. In order to avoid this problem, we reduce all rectangles to squares whose side length is equal to the minimum of its length and width for computation of lower bound. For example, department x, 2 by 3 rectangle, is changed to 2 by 2 square, so as to calculate the lower bound. This will provide an approximate lower bound which is very likely to be a true lower bound.

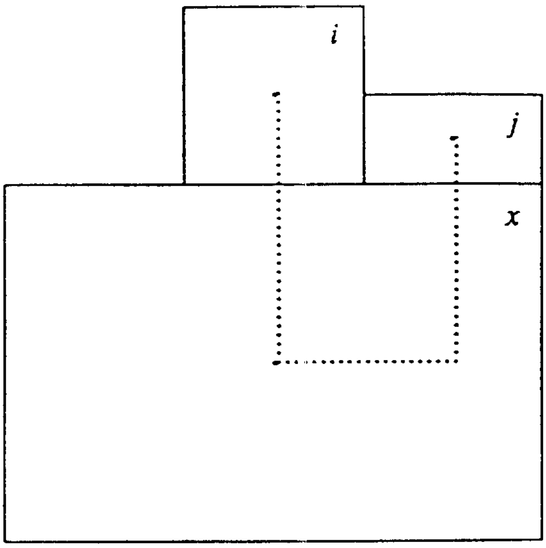

Figure 6. Illustration of rectilinearly adjacent

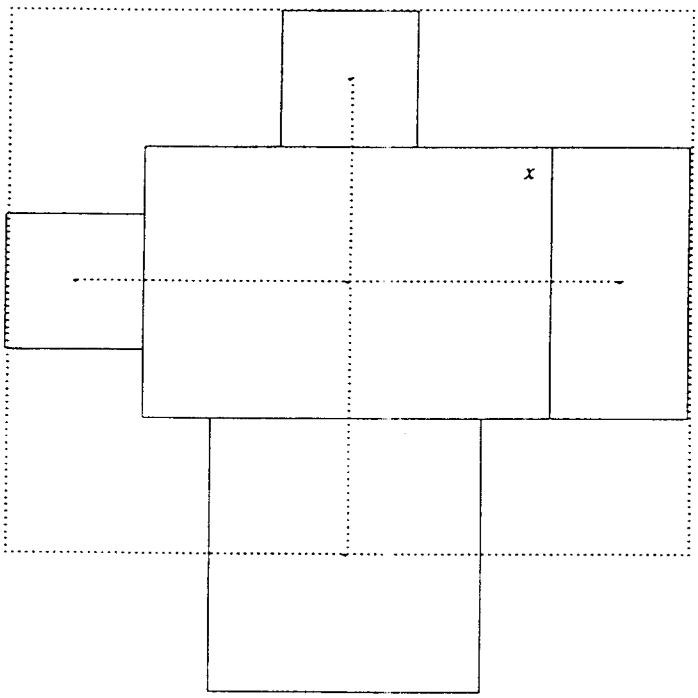

Figure 7. Illustration of integrated department

2.4 Rotation of point layout

The departments whose point locations are closest in terms of Euclidean distance to each corner are candidates for the corner location (grid) of the plant. The candidates for corner locations will vary with the point layout on the grid representation on the plant. That means we can generate a different layout by rotating the orientation of point layout.

For computational analysis, the point layout is rotated 10 times, with possibly different candidates for each corner. Of course, if we consider all possible cases, where the number of departments for a corner location is the same as the total number of departments, rotation of the point layout is not necessary. Because of CPU time (actually it is not important even though run time is fairly long), we limit the number of departments considered for location at a corner. Since rotation presents consideration of much more possible cases, it generally produces a better layout.

Ⅲ. Computational results

We introduced a new facility layout technique using the brach and bound technique with eigenvector concept. Even though Drezner presented a scattered diagram (point layout) which provides good insight for the design analyst, he generated the final layout only when all facility sizes were equal. When the sizes and shapes of departments are fixed, the extensions we develop can be used to find final layout.

We generated 25 test problems (see Table 1 for the characteristics of test problems) and tested them on an IBM RISC 6000. The flow matrix (flow matrix with 70% zero and 30% nonzero elements in the range [10, 100], flow matrix with 50% zero and 50% nonzero elements in the range [30, 80], flow matrix with 30% zero and 70% nonzero elements in the range [50, 60]), number of departments, plant size and department size of each problem are in Kim [5]. We obtained solutions in less than two hours when we confined the number of candidates for a corner location and the node limits on the proposed technique. Of course, in practice, layout design times for a factory are several months duration and node limits or time limits are not necessary. CRAFT accepts the constraint of department size but it has no function to restrict the shape of department, it is difficult to directly compare CRAFT with the branch and bound technique in this paper. For example, the branch and bound technique obtained an optimal solution in test problem 25, but CRAFT generated a final layout with lower material handling cost by using shape distortion. In order to directly compare both methods, we have to obtain the layout without shape distortion by CRAFT. One way to do that is to assign department size to 1 unit (1x1) or 2 units (1x2) as much as possible. We generated the test problems considering shape distortion.

In this situation, the solution quality (material handling cost as well as shape distortion) by CRAFT is very high because of the following two reasons. First, CRAFT is an improvement algorithm. Therefore, it requires an initial layout which affects the final layout. The space filling curve technique for initial layout performs very well. Second, CRAFT reduces the material handling cost by exchanging departments. Since there are so many combinations of interchanging departments because of department size, CRAFT can provide a good layout. According to the computational results, shape ratio (number of departments with areas less than three divided by total number of depertment) is an important factor to analyze Table 1. We can divide the test problems into three groups.

(1) shape ratio $\leq$ 80%

CRAFT generated a final layout with shape distortion. It is difficult to say which method is better.

(2) 80% $<$ shape ratio $\leq$ 90%

CRAFT generated final layout with shape distortion. The branch and bound technique is better than CRAFT.

(3) 90% $<$ shape ratio $\leq$ 100%

CRAFT generated final layout without shape distortion. Generally, the branch and bound technique is better than CRAFT if the shape ratio is less than 95%.

Even though shape is an important factor, CRAFT is recommendable when the shape ratio is greater than or equal to 90 percent. Usually CRAFT cannot present a layout without shape distortion when shape ratio is less than or equal to 90 percent. In this case, it is wise to avoid the CRAFT approach if the shape of the department is an important factor. The branch and bound technique has a strong point especially when the shape ratio is between 90 percent and 95 percent. We confined our test runs when number of nodes explored exceed 500,000. If there is no limitation of nodes explored, we can expect that the branch and bound technique will provide better solutions.

Table 1. Comparison between CRAFT and eigenvector approach

test problem	shape ratio*	CARFT (min,max)[a]	eigenvector** (min,max)[b](min,max)[c]	# of dept.	% of deviation from CARFT
1	0.71	(330,430)	(408,446)(348[d],362)	7	−0.052
2	0.8	(3380,4200)	(3988,4954)(3938,4736)	15	−0.142
3⁻	0.87	(4350,5450)	(4428,5900)(4302,4700)	15	+0.011
4	0.73	(4430,6110)	(4112,6052)(4784,5824)	15	−0.064
5	0.53	(5370,7360)	(7288,7468)(6152,7076)	15	−0.127
6	0.6	(4910,7510)	(6430,6996)(5468,6120)	15	−0.102
7	0.73	(4350,4790)	(4680,5484)(4644,5120)	15	−0.063
8	0.73	(3970,5230)	(5944,6996)(4076,4864)	15	−0.026
9	0.73	(4190,5130)	(5364,6612)(4528,5072)	15	−0.075
10	0.6	(5130,7330)	(6240,10072)(6080,6980)	15	−0.156
11	0.8	(3860,4660)	(4408,4524)(3916,4356)	15	−0.014
12†	0.94	(4160,4790)	(4152,4764)(4336,5064)	17	+0.002
13	0.65	(5620,8100)	(6400,8668)(5928,6892)	17	−0.052
14	0.47	(5730,7770)	(6776,12744)(6700,7128)	17	−0.145
15	0.53	(6260,8430)	(***,***)(6736,8460)	17	−0.071
16	0.76	(5420,6450)	(6482,7256)(5512,6204)	17	−0.017
17†	1.0	(4330,4660)	(4260,5016)(4270,4958)	17	+0.016
18°	0.94	(3960,4850)	(4068,4456)(4384,5292)	17	−0.027
19⁻	0.82	(4690,5390)	(5116,6240)(4632,5952)	17	+0.013
20	0.53	(5890,7290)	(9108,9108)(6580,7948)	17	−0.105
21†	0.94	(4320,5040)	(4184,4412)(4340,4788)	17	+0.033
22°	1.0	(5030,6060)	(5584,7096)(6504,7980)	20	−0.099
23	0.9	(5550,7510)	(6744,8570)(7318,8368)	20	−0.177
24°	1.0	(5060,6740)	(6682,7356)(6890,7760)	20	−243
25°	0.95	(4990,5810)	(5764,6808)(6924,8424)	20	−0.134

a : best and worst solution with 10 different initial layout
b : best and worst solution when 4 departments are cnadidates for corner
c : best and worst solution when 5 departments are candidates for corner
d : optimal solution
† CARFT : provides feasible but inferior solution
° CARFT : provides feasible and superior solution
– CARFT : provides infeasible and inferior solution
* : (# of departments with areas less than three)/(total # of departments)
** : node limit is 500,000 and 10 rotations
*** : no feasible fit found

Ⅳ. Conculsion

This paper addressed the problem of assigning *n* departments without shape disrortation for plant layout problem. To use the idea of the eigenvector generated from flow matrix, a branch and bound computer code developed by Tillinghast was modified to handle the departments with rectangle shape. Using the concept of rectilinear adjacency, we constructed a new approximate lower bound which reduced the number of nodes traveled to complete the run without affecting the objective.

If the department shape is fixed, CRAFT cannot generate a feasible solution. But, when the shape ratio is greater than 90%, CRAFT provides a good solution since there are a greater number of combinations of interchanging departments. In eigenvector approach, we rotated the point layout for a better solution because rotation changes the department candidates for corner location of the plant. Further research for the layout problem without shape distortion will be in how to find a better solution by using simulated annealing technique or tabu search method.

Rrference

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