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An Optimal Operating Policy for Two-stage Flow Lines with Machine Failures

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Abstract

Automatic transfer lines, defined as an integrated system with a number of workstations, interstation storage buffers, automatic transfer device and a control system, play a major role in mass production systems. Due to high capital investment needed for an automatic transfer line, greater care should be taken in its design so as to maximize the system performance. One way to control the system performance is to control buffer storage between successive stations, and so we give attention to the control policy of the buffer storage. To control the interstation work-in-process inventory, we propose *dual limit switches* which control the buffer storage with two parameters, R and r. Under the policy, preceding station is forced down when the inventory level in the buffer reaches R until the level falls to r. For the model developed, we analyze the system characteristics and find the optimal control parameters with a search procedure.

I. Introduction

Automatic transfer lines which can be defined as a number of automated machines and storage buffers, in series, integrated into one system by an automatic transfer mechanism and a control system are key parts of most mass production systems. Workpieces pass through successive stations with specific operations being performed at each station. A major cause of line inefficiency is breakdowns in each station. Suppose that there are no storage buffers.

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Then, when a station breaks down, the stations following it may be forced down, or starved, since the broken down station is unable to feed the downstream stations. Similarly, preceding stations may be forced down, or blocked, since the broken down station is unable to remove the semiprocessed items from the upstream stations. Interstation storage buffers are used to reduce the effect of strong interference between stations.

In this system, the operation of each station is controlled by a switch control system, what we call limit switch, with the information of storage buffer. For example, the maximum permitted inventory level, say R, of work-in-process between the two stations is predetermined by the buffer capacity. If the number of workpieces in the buffer is less than R units, preceding station continues its operation and produces its output until R units are placed in the buffer. When the buffer is full the limit switch automatically stops the operation of preceding station. Similarly, if there is no workpiece in the buffer, succeeding station cannot continue its operation. Monden[9] reported that intermittent operation by this system, he called it the full work system, is adopted in all Toyota production lines.

Such two-station production systems have been extensively studied. Buzacott and Hanifin[1] proposed a review and comparison of related topics. Buzacott and Kostelski[2] analyzed the system with discrete workpieces, finite buffers and random processing time, but without failure. Seidmann[11] discussed the system of multi-buffer. Okamura and Yamashina[10] assumed deterministic processing times with random failures. Analysis of some extension of this model using efficient interpolation approximation is discussed by Ignall and Silver[7]. The system with random processing times and random failures was studied by Gershwin and Berman[3]. Meyer et al. [8] and Wijngaard[13] analyzed the systems in which parts are not treated as discrete items but as a continuous fluid. But all these researches were concerned about buffer sizes only.

Some studies deal with buffer control methods as well as buffer sizes. Hopp et al. [5] presented an optimal control policy in case of continuous material flow. The case of random processing times and random failure was studied by Hwang and Koh[6].

In this paper we consider a two station automatic transfer line where workpiece transfers at all stations are synchronized to occur at the same time epoch. Each station can fail and be repaired. The buffer storage capacity between the two stations is finite and we control this system with (R, r) policy, or equivalently, dual limit switch system, in which the preceding station is forced down when the inventory level rises to R and restarts when it falls to r, $R \geq r+1$. In this system the maximum inventory level R is controlled by a device called upper limit switch and the restarting point r by lower limit switch.

The purpose of this study is to gain insight into the policy to control the buffer storage in two-station transfer lines by the following study of the problem. First, a Markov model of the problem is proposed to derive the steady state probability of each system state. Next, based on the model developed, an optimal buffer control policy is found by numerical search.

II. Model Development

2. 1 Model Description and Assumptions

The transfer line produces one kind of commodity, and consists of two stations. At each station an operation is carried out on a workpiece. The stations are arranged serially so that each workpiece enters the line at the first station, and begins to transfer from the station to the next at the same instant. The interval between successive transfers is called the cycle time. There is always a supply of workpieces available to the first station of the line. The final station will deposit the completed workpiece into a storage area which has an infinite capacity. The inter-station buffer storage has, however, a fixed capacity R.

Without loss of generality we define the time units so that the cycle time is one. This is accomplished by dividing all the time parameters used in the model by original cycle time. The transformation maps many equivalent problems to one that is easier to manipulate. The transporting time between the stations is assumed to be negligible or subsumed by the unit production time (cycle time).

The performance of a particular station is defined by a set of four states :

(1) Operating : A station is in working order and carrying out its function [abbrev. : U].

(2) Broken down and under repair : A station is subject to breakdowns, which are random in both occurrence and duration. These breakdowns may be a result of a malfunction, or time required to change or adjust tools, settings, and so forth [abbrev. : D].

(3) Starving : The subsequent station is in working order, but unable to operate because it has no workpiece to process [abbrev. : S].

(4) Blocking : Once the inventory level reaches R, the preceding station cannot produce until the inventory level falls to r [abbrev. : B].

The role that a buffer plays is to diminish or eliminate the possibility of starving and blocking by means of its storing and replenishing functions.

The following fundamental assumptions are made :

(1) The probability that station i (i=1, 2) breaks down in a cycle, given that it was oper-

ative at the end of the previous cycle, is f_i which is called failure rate. The failure rate of a forced down (starving or blocking) station is assumed to be zero. In effect we are considering only operation dependent failures.

(2) The probability that the repair on station i ($i=1, 2$) is completed in a cycle, given that the station broken down or was under repair during the previous cycle, is r_i which is called repair rate.

(3) When a station is broken down, its unit is scrapped.

2. 2 Markov Chain Model Representation

Suppose we observe the state of the automatic transfer line just after each transfer of units. Let X_j be the system state at the jth observation epoch. The state is described by a triple (n, α, β) in which n means the number of work-in-process in the buffer ($0 \le n \le R$) while α and β are the performances of the preceding and succeeding stations, respectively (α and β can be U(up), D(down), S(starving), or B(blocking)). Therefore, the state space E and the total number of states NS are as follows :

$$
\begin{aligned}
E=\{ & (0, D, S), (0, U, S), \\
& (n, U, U), (n, U, D), (n, D, U), (n, D, D) : n=0, 1, \ldots, R-1, \\
& (R, B, D), \\
& (k, B, U), (k, B, D) ; k=R-1, R-2, \ldots, r+2, r+1 \}
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
NS &= 2+4R+1+2(R-r-1) \\
&= 6R-2r+1
\end{aligned} \tag{2}
$$

It can be verified that $\{ X_j \}$ is an irreducible Markov chain with transition probability matrix P in which the states are arranged by the order of the state space E.

$$
P=\begin{bmatrix}
A_1 & A_0 & & & & & & & & & & & & & \\
A_2 & B_1 & B_0 & & & & & & & & & & & & \\
 & B_2 & B_1 & B_0 & & & & & & & & & & & \\
 & & B_2 & B_1 & B_0 & & & & & & & & & & \\
 & & & & & \ddots & & & & & & & & & \\
 & & & & & & B_2 & B_1 & B_0 & & & & & & \\
 & & & & & & & B_2 & B_1 & D_0 & & & & & \\
 & & & & & & & & & D_1 & D_2 & & & & \\
 & & & & & & & & & & E_1 & E_0 & & & \\
 & & & & & & & & & & & & \ddots & & \\
 & & & & & & & & & & & & & E_1 & E_0 \\
 & & & & & & E_2 & & & & & & & & E_1
\end{bmatrix} \tag{3}
$$

or for the special case of $r=R-1$,

$$
\mathrm{P}=\begin{bmatrix}
A_1 & A_0 & & & & & & & \\
A_2 & B_1 & B_0 & & & & & & \\
 & B_2 & B_1 & B_0 & & & & & \\
 & & B_2 & B_1 & B_0 & & & & \\
 & & & & & \ddots & & & \\
 & & & & & B_2 & B_1 & B_0 & \\
 & & & & & & B_2 & B_1 & D_0 \\
 & & & & & & & D_3 & D_1 & D_2
\end{bmatrix} \tag{4}
$$

where

$$
A_0=\begin{bmatrix} 0 & 0 & 0 & 0 \\ \bar{f}_1 & 0 & 0 & 0 \end{bmatrix},
$$

$$
A_1=\begin{bmatrix} \bar{r}_1 & r_1 \\ f_1 & 0 \end{bmatrix},
$$

$$
A_2=\begin{bmatrix} f_1\bar{f}_2 & 0 \\ f_1 r_2 & 0 \\ \bar{r}_1\bar{f}_2 & r_1\bar{f}_2 \\ \bar{r}_1 r_2 & r_1 r_2 \end{bmatrix},
$$

$$
B_0=\begin{bmatrix} 0 & \bar{f}_1 f_2 & 0 & 0 \\ 0 & \bar{f}_1\bar{r}_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},
$$

$$
B_1=\begin{bmatrix} \bar{f}_1\bar{f}_2 & 0 & 0 & f_1 f_2 \\ \bar{f}_1 r_2 & 0 & 0 & f_2\bar{r}_2 \\ 0 & r_1 f_2 & 0 & \bar{r}_1 f_2 \\ 0 & r_1\bar{r}_2 & 0 & \bar{r}_1\bar{r}_2 \end{bmatrix},
$$

$$
B_2=\begin{bmatrix} 0 & 0 & f_1\bar{f}_2 & 0 \\ 0 & 0 & f_1 r_2 & 0 \\ r_1\bar{f}_2 & 0 & \bar{r}_1\bar{f}_2 & 0 \\ r_1 r_2 & 0 & \bar{r}_1 r_2 & 0 \end{bmatrix},
$$

$$
D_0=\begin{bmatrix} \bar{f}_1 f_2 \\ \bar{f}_1\bar{r}_2 \\ 0 \\ 0 \end{bmatrix},
$$

$D_1=\bar{r}_2$

$D_2=(r_2\ 0)$,

$D_3=(r_2\ 0\ 0\ 0)$,

$$E_0=\begin{bmatrix} \bar{f}_2 & 0 \\ r_2 & 0 \end{bmatrix},$$

$$E_1=\begin{bmatrix} 0 & f_2 \\ 0 & \bar{r}_2 \end{bmatrix},$$

$$E_2=\begin{bmatrix} \bar{f}_2 & 0 & 0 & 0 \\ r_2 & 0 & 0 & 0 \end{bmatrix},$$

and $\bar{f}_i=1-f_i$, $\bar{r}_i=1-r_i$; $i=1, 2$.

Let $\mathbf{x}$ be the steady state probability vector of the Markov chain { X_j }. Then $\mathbf{x}$ is the unique nonnegative solution of

$$\mathbf{x}\mathrm{P}=\mathbf{x} \text{ and } \mathbf{xe}=1, \tag{5}$$

where $\mathbf{e}=(1, 1,\ldots,1)^T$ is an NS-component column vector. This system of equations is redundant since it has NS unknowns and $NS+1$ equations. And we set a transformed system of equations as follows :

$$\mathbf{x}\mathrm{Q}=\mathbf{b}, \tag{6}$$

where $\mathbf{b}=(1, 0, 0,\ldots, 0)$ is an NS-component row vector and Q is the $NS\times NS$ matrix which is derived by converting the first column of (I−P) to $\mathbf{e}$ where I is an identity matrix of the same dimension as P. It is well known that Q is a nonsingular matrix and

$$\mathbf{x}=\mathbf{b}\mathrm{Q}^{-1}. \tag{7}$$

The right hand side of the above equation is the first row vector of Q^{-1}. Therefore the steady state probability vector of the Markov chain { X_j } is the first row vector of Q^{-1} and all the thing we have to do is to calculate the inverse matrix of Q.

In calculation of the inverse matrix we have a limitation of the matrix size which depends on the size of the buffer storage. The number of system states may be very large when the buffer size is big. Shanthikumar and Tien[12] developed a method to break the limitation when the single limit switch is used. In our dual limit switch system, however, their method cannot be applied and we have a limitation of buffer size which depends on the computer memory size used. But, in spite of the increase of the number of system states in our dual limit switch system, this limit is almost same as the single switch system by matrix partition method in the calculation of inverse matrix.

Let

$s=2+4R$

and $t=NS-s$. (8)

We partition Q as follows:

$$Q=\begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}, \tag{9}$$

where α is an $s \times s$ submatrix, β an $s \times t$ submatrix, γ a $t \times s$ submatrix, and δ a $t \times t$ submatrix. Then Q^{-1} is partitioned in the same way as Q, that is,

$$Q=\begin{bmatrix} A & B \\ C & D \end{bmatrix}, \tag{10}$$

where A is $s \times s$, B is $s \times t$, C is $t \times s$, and D is $t \times t$. Hadley[4] shows that

$$A=(\alpha-\beta\delta^{-1}\gamma)^{-1}, \tag{11}$$

$$B=-A\beta\delta^{-1}. \tag{12}$$

Note that we have no concern at C or D since we need only the first row of the inverse matrix.

Submatrix δ is an upper-triangular matrix and its inverse can easily be derived. And the $s \times s$ matrix $\beta\delta^{-1}\gamma$ has only four nonzero elements. Therefore we need the memory size of almost same as $s \times s$ matrix.

Once the long run probability of each state is obtained, the following performance measures can be calculated. Here, we let $p(n, \alpha, \beta)$ be the steady state probability of the chain $\{X_j\}$, that is,

$$p(n, \alpha, \beta)=\lim_{j\to\infty} \Pr\{X_j=(n, \alpha, \beta)\}. \tag{13}$$

1) Utilization ratio of a station: time devoted to producing items during a unit of time

$$e_1=p(0, U, S)+\sum_{n=0}^{R-1}\{p(n, U, U)+p(n, U, D)\} \tag{14}$$

$$e_2=\sum_{n=0}^{R-1}\{p(n, U, U)+p(n, D, U)\}+\sum_{k-r+1}^{R-1}p(k, B, U) \tag{15}$$

2) System productivity: output from the system per unit time

$$P=e_2\bar{f}_2 \tag{16}$$

3) Average work-in-process in the buffer storage

$$\bar{I}=\sum_{n=1}^{R-1} n\cdot\{p(n, U, U)+p(n, U, D)+p(n, D, U)+p(n, D, D)\}$$
$$+R\cdot p(R, B, D)+\sum_{k=r+1}^{R-1} k\cdot\{p(k, B, U)+p(k, B, D)\} \tag{17}$$

2. 3 Computational Experience

In this subsection we describe the results of a set of numerical experiments which demonstrate that the model behaves reasonably as its parameters are varied.

First, we observe the response of the system performance to failure rate and repair rate : f_i and r_i, $i=1$, 2. Figure 1 and 2 show the result. In these figures one parameter was varied over a range (0.001 through 0.75), while all others were held constant. The standard values of the parameters were : $f_1=0.1$, $f_2=$ 0.1, $r_1=0.25$, $r_2=0.25$, $R=20$, and $r=10$.

The graph of the system productivity is plotted in Figure 1. It shows that as f_1 or f_2, failure rate of each station, increases, system productivity P decreases and that as r_1 or r_2, repair rate of each station, increases, P increases. Figure 2 shows that as f_1 or r_2 increases, average work-in-process $\bar{I}$ decreases and that as f_2 or r_1 increases, $\bar{I}$ increases. All the results agree the intuition.

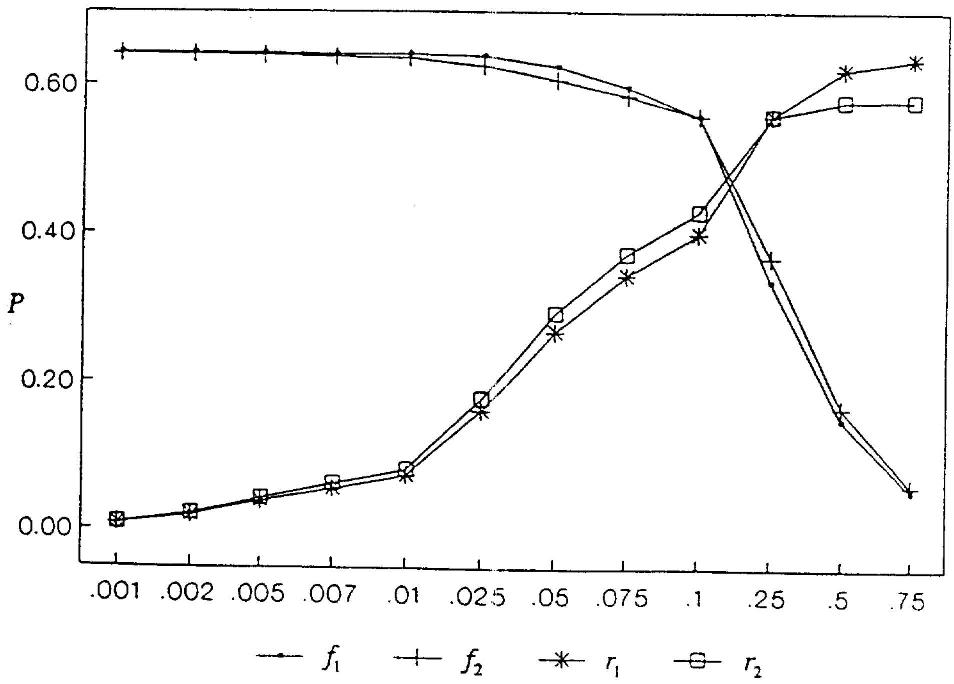

Figure 1. Effects of f_1, f_2, r_1 and r_2 to P

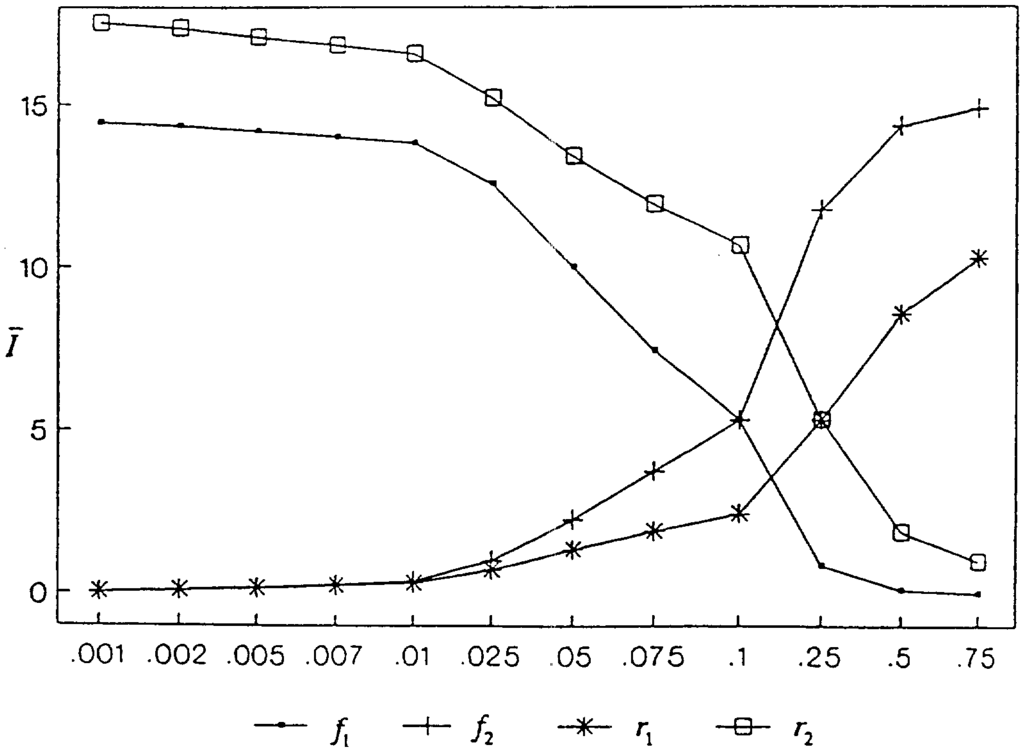

Figure 2. Effects of f_1, f_2, r_1 and r_2 to $\bar{I}$

Next we observe the effects of the buffer size on the system performances. The changes of system performance due to the change of buffer storage capacity in six cases, which is summarized in Table 1, were observed.

In the table we define *isolated efficiency of station i* which means time devoted to producing items during a unit of time without considering force down effect as follows :

$$\rho_i = r_i / (f_i + r_i),\ i=1,\ 2. \qquad (18)$$

Table 1. Summary of numerical experiments

Case	f_1	f_2	r_1	r_2	ρ_1	ρ_2	ρ_1/ρ_2
1	0.1	0.1	0.1	0.5	0.500	0.833	0.6
2	0.05	0.1	0.1	0.5	0.667	0.833	0.8
3	0.1	0.1	0.5	0.5	0.833	0.833	1.0
4	0.1	0.05	0.5	0.1	0.833	0.667	1.25
5	0.1	0.1	0.5	0.1	0.833	0.500	1.67
6	0.05	0.1	0.5	0.1	0.909	0.500	1.82

Based on the table, we plotted two graphs, Figure 3 and 4. In these figures we assumed that the restarting point of the preceding station r has a half value of buffer size R, *i.e.*, $r=R/2$. From Figure 3, the following observations can be made :

1) As buffer capacity increases, P increases with an upper bound.

2) For a given buffer control policy (R and r), P value is ranked by the order of the following factors :

① min $\{\rho_1, \rho_2\}$,

② max $\{\rho_1, \rho_2\}$,

③ ρ_1,

④ ρ_2.

From the second observation, we can say that balancing the isolated efficiency of each station is most important to the increase of system productivity.

Figure 4 shows that as buffer capacity increases, work-in-process increases and that for a given R and r the rank of average inventory level depends upon the order of the ratio, ρ_1/ρ_2.

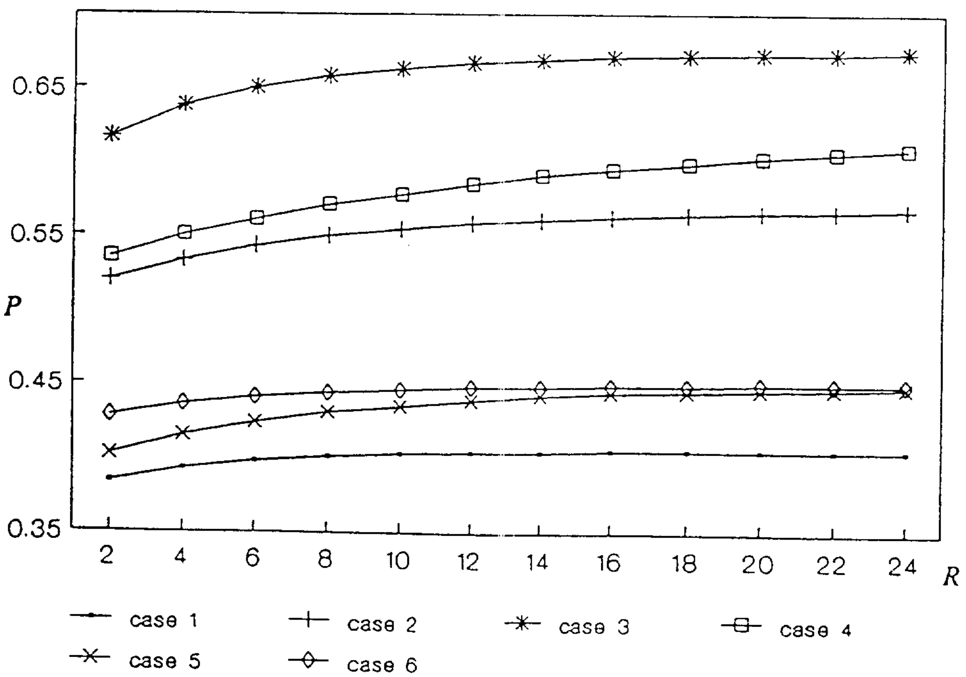

Figure 3. Effects of buffer size to P

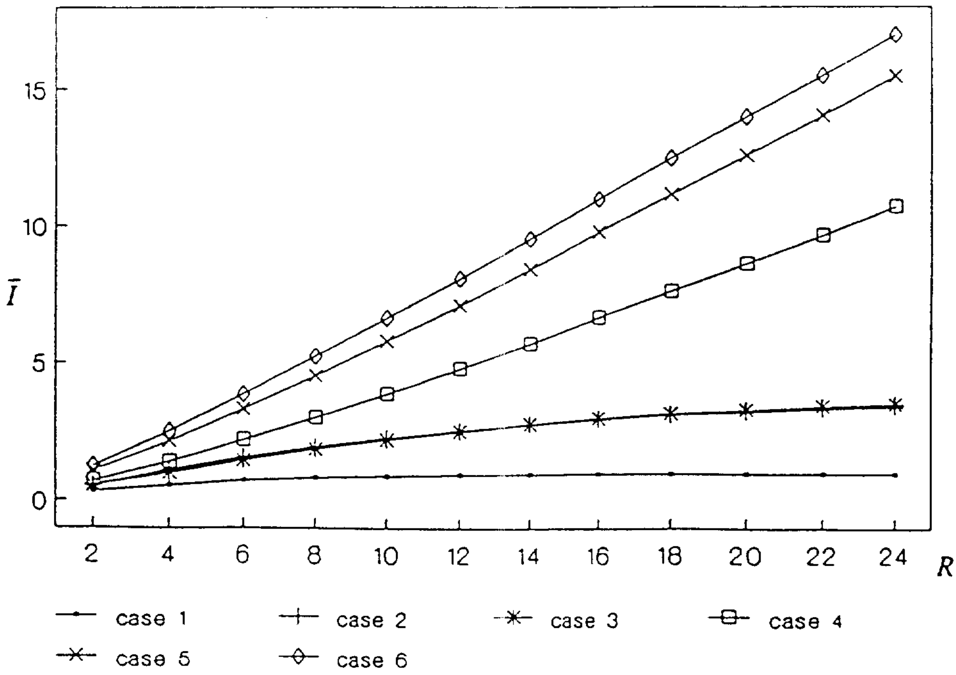

Figure 4. Effects of buffer size to $\bar{I}$

Ⅲ. Optimal Control Policy

3. 1 Objective Function and Solution Procedure

In this section we consider an optimal control policy (R and r) which minimize an objective function. Before setting the objective function, we define two terms, $FREQ_1$ and $FREQ_2$, which are blocking frequency of station 1 (preceding station) and starving frequency of station 2 (succeeding station), respectively. Then it can be easily shown that :

$$FREQ_1 = p(r+1,\ B,^*)/T_1 \qquad (19)$$

and

$$FREQ_2 = p(0,\ U,\ S)/T_2, \qquad (20)$$

where $p(r+1, B,*)=p(r+1, B, U)+p(r+1, B, D)$,

T_1=expected time that the system remains at state $(r+1,B,*)$ once it enters the state

T_2=expected time that the system remains at state $(0, U, S)$ once it enters the state, and

state$(r+1, B,*)$=an artificial state which contain states $(r+1, B, U)$ and $(r+1, B, D)$

Using conditional expectation, one can show that:

$$T_1=1+f_2/r_2 \tag{21}$$

$$T_2=(r_1+f_1)/(r_1\bar{f}_1) \tag{22}$$

In the special case of $r=R-1$,

$$FREQ_1=p(R, B, D)/T_1, \tag{23}$$

where $T_1=1/r_2$.

Now we propose an objective function with two decision variables, R and r, as follows:

$$\begin{aligned} COST(R, r)=&C_s\{p(0, D, S)+p(0, U, S)\} \\ &+C_h\bar{I} \\ &+C_{f1}FREQ_1+C_{f2}FREQ_2, \end{aligned} \tag{24}$$

where C_s=shortage (starving) cost of station 2, [\$/time unit],

C_h=inventory holding cost, [\$/time unit],

C_{fi}=restarting cost of station i after blocking or starving, i=1, 2, [\$/restart].

Then one can find optimal control policy (R and r) which minimize the above equation using the solution procedure as follows:

procedure Opt_Control

read f_i, r_i, C_s, C_{f_1} and C_{f_2}

$R \leftarrow 0$

repeat

  $R \leftarrow R+1$, $r \leftarrow -1$, $COST(R, -1) \leftarrow \infty$

  repeat

    $r \leftarrow r+1$

    calculate $p(n, \alpha, \beta)$ for all $(n, \alpha, \beta) \in E$

    calculate $FREQ_1$ and $FREQ_2$

    calculate $COST(R, r)$ by equation (24)

  until $r=R-1$ or $COST(R, r) > COST(R, r-1)$

if $r=R-1$ then

$COST_R \leftarrow COST(R, r)$, $r_R^* \leftarrow r$

else

$COST_R \leftarrow COST(R, r-1)$, $r_R^* \leftarrow r-1$

until $COST_R > COST_{R-1}$

$R^* \leftarrow R-1$, $r^* \leftarrow r_{R-1}^*$

end procedure

3. 2 Numerical Examples and Observations

In this subsection, we observe the behavior of the system in accordance with the changes of the cost parameters. Since the problem structure of equation (24) has a form which does not permit analytic analysis, sample problems are solved to do this.

Setting $C_{f_1} = C_{f_2} = 100$, $C_h = 10$ and $C_s = 200$ as the standard values, for each cost parameter, nine different levels are chosen by multiplying the standard value by 1/5, 1/4, 1/3, 1/2, 1, 2, 3, 4 and 5, respectively.

Assuming that $f_1 = 0.1$, $f_2 = 0.2$, $r_1 = 0.25$ and $r_2 = 0.25$, our solution procedure determines R^* and r^* for each level of a cost parameters while the others are fixed to the standard values.

The results depicted in Figure 5, 6 and 7 show the changes of R^*, r^* and $\bar{I}$ in accordance with the perturbations of C_s, C_h and $C_f(=C_{f_1} = C_{f_2})$, respectively. In these figures we can observe that R^* and r^* increase as C_s increases or C_h decreases and that the distance between R^* and r^* becomes larger as C_f increases.

Figure 8 and 9 show the effects of each cost parameter to the system productivity P and cost ratio $RATIO=COST(R^*, r^*)/\hat{C}$ in which $\hat{C}$ is the optimal cost of the single limit switch system (i. e., $r^*=R^*-1$), respectively. In these figures the horizontal axis indicates the value multiplicated to the standard value of each cost parameter while the others have the standard values. In Figure 8 we can find that the system productivity increases as C_s or C_f increases or as C_h decreases. This shows that the system productivity is proportional to average work-in-process (See Figure 5, 6 and 7.). Figure 9 shows that the smaller C_h or the larger C_f (According to Figure 6 and 7, this results in larger distance between R^* and r^*.) makes the smaller cost ratio which means the dual limit switch system is better than single limit switch system.

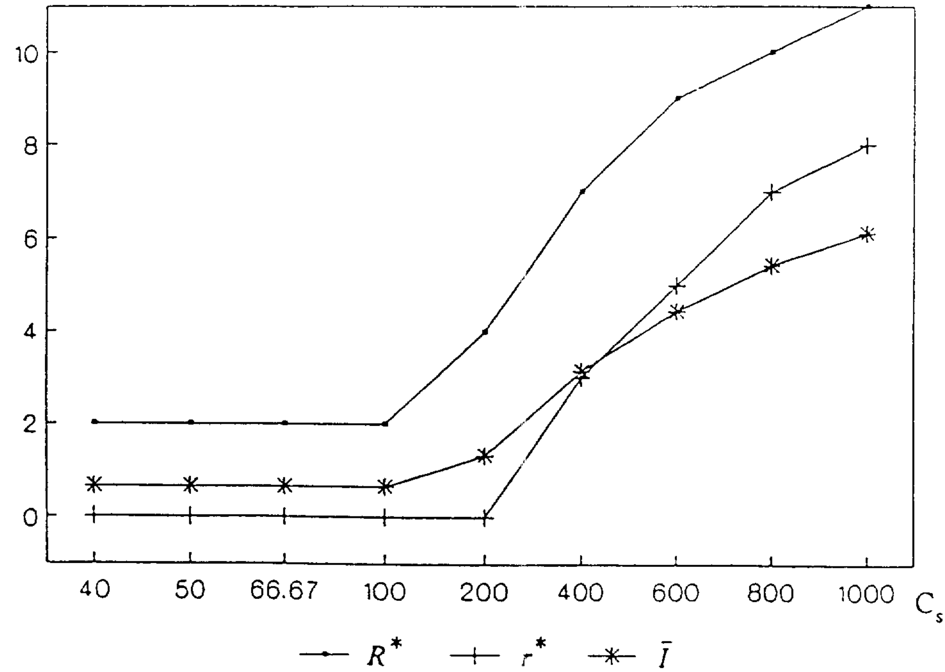

Figure 5. Effects of C_s to R^*, r^* and $\bar{I}$

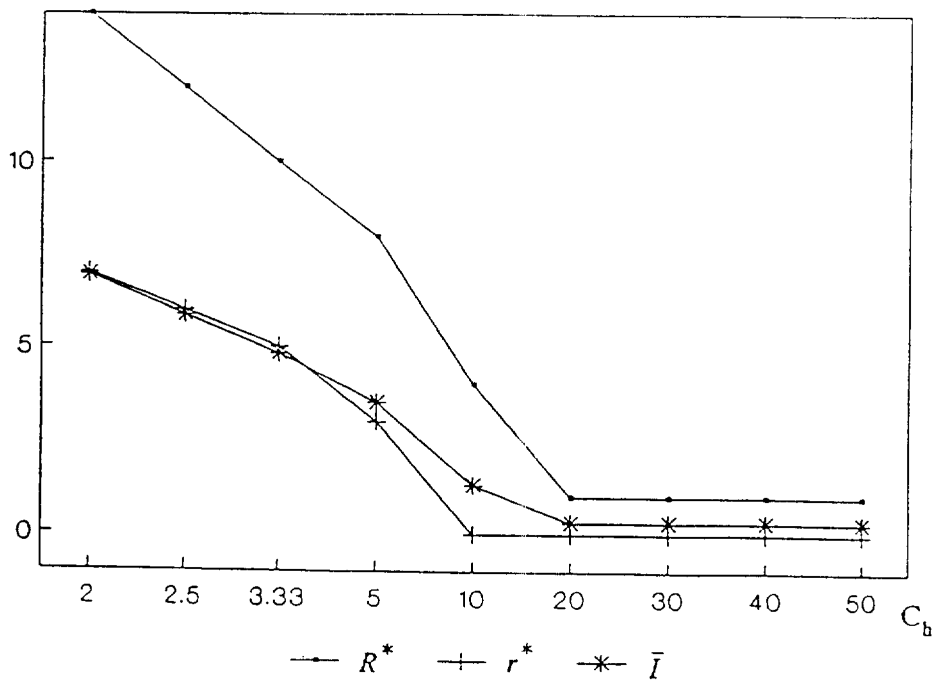

Figure 6. Effects of C_h to R^*, r^* and $\bar{I}$

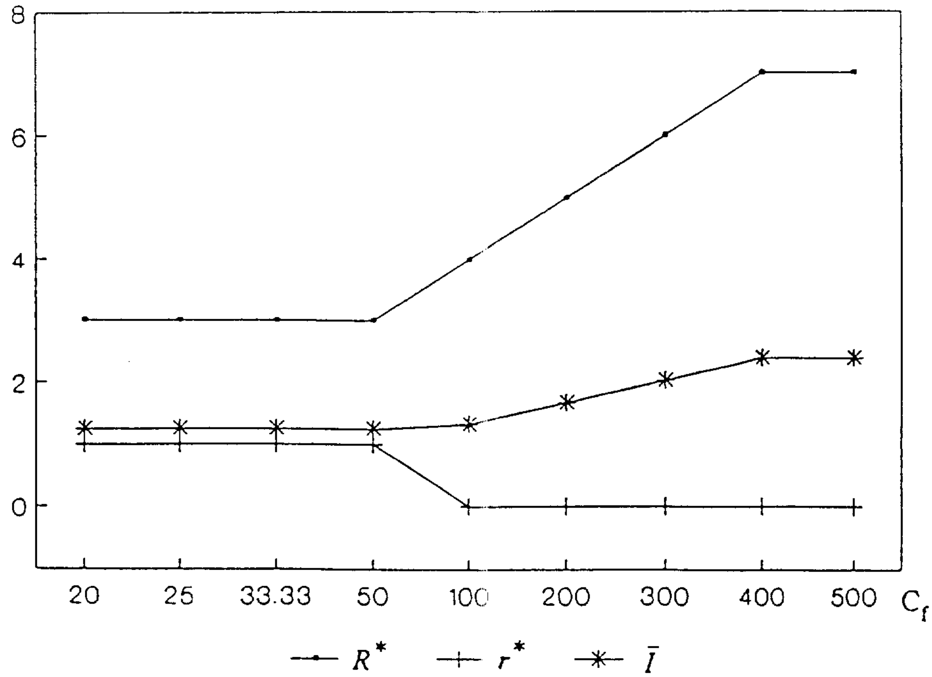

Figure 7. Effects of $C_f(=C_{f_1}=C_{f_2})$ to R^*, r^* and $\bar{I}$

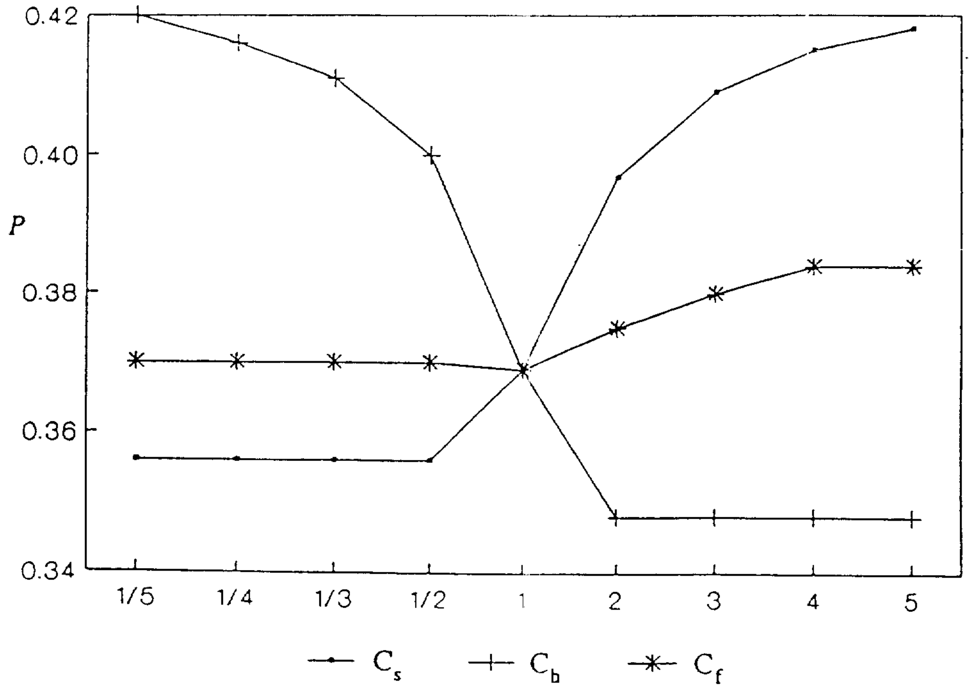

Figure 8. Effects of cost parameters to P

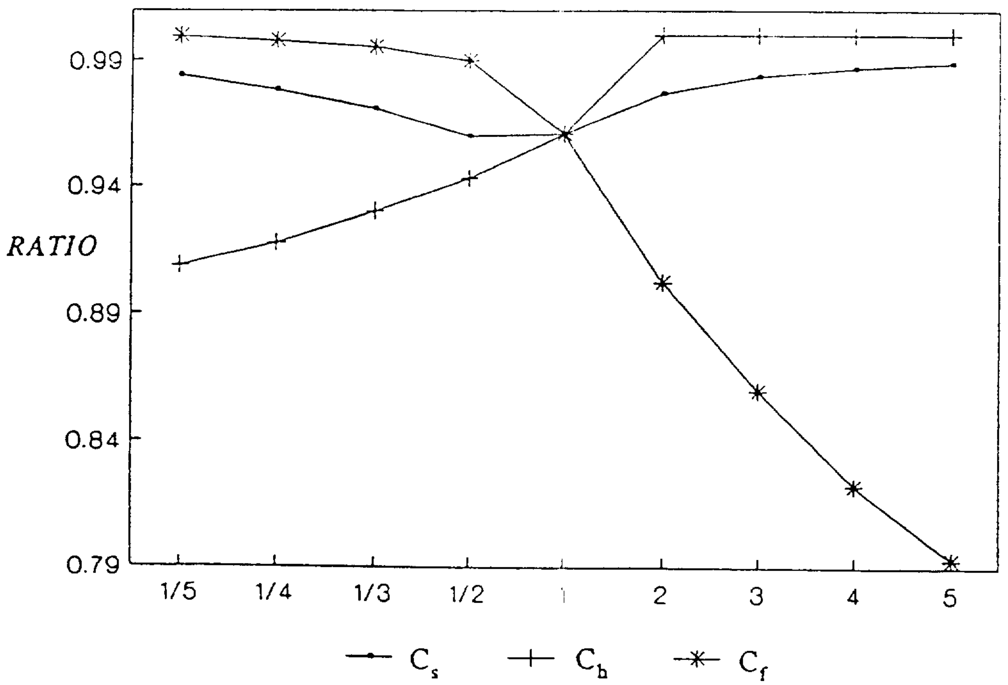

Figure 9. Effects of cost Parameters to *RATIO*

Ⅳ. Conclusions

In this paper we analyzed a transfer line consisting of two unreliable stations with the same deterministic processing time and a finite storage buffer controlled by dual limit switches. First of all, we identified the system states whose steady state probabilities are determined by the Markov process model. Then, using a search procedure, we found the optimal control parameters which minimize the system cost per unit of time.

Once the cost and control parameters are known, the system performance including the utilization ratio of each station, the production rate of the system and the average work-in-process inventory, can be determined.

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