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An Accelerated Genetic Algorithm for the Vehicle Routing Problem†

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Abstract

This study suggests an accelerated genetic algorithm for the vehicle routing problem (AGAVRP). This algorithm treats both the single-visit and the multiple-visit models. AGAVRP is accelerated by the OR techniques at the various stages of the algorithm. In order to improve the convergence of AGAVRP, a robust set of parameters is determined by the experimental design approach. The relative performance of AGAVRP is comparable to the other known algorithms. The advantage of the proposed algorithm is its flexibility and better convergence.

1. Introduction

Vehicle routing problem(VRP) is an optimization problem of assigning vehicles and determining their routes to distribute services and materials. In general, the objective is to minimize the routing cost with the constraints of the number of available vehicles, the load capacity, and the various types of restrictions on the routing conditions.

Traditionally, VRP model is restricted to single-visit assumption. In the single-visit model, only one vehicle is allowed to visit a demand point, which makes split deliveries impossible. However if we relax the single-visit restriction, we can get a multiple-visit model, in which more than one vehicle can visit a demand point. This concept of constraint relaxation allows further optimization which can be achieved at the price of increased

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computational costs due to the expansion of the solution space.

This research proposes an accelerated genetic algorithm for the vehicle routing problem (AGAVRP) to solve both types of VRP, the single visit and the multiple visit models. AGAVRP is accelerated by various OR techniques including the nearest neighborhood search to generate initial solutions, fixed cost transportation and convex-hull with 3-optimal heuristic to evaluate optimality and feasibility of the solutions. The parameters of this algorithm are optimized with the experimental design procedure to improve the searching power. Comparative study of the performance with various other known algorithms is also illustrated.

2. The analysis and design of the models

Suppose that there is a depot($i=0$) and a set of demand points($i=1, 2, 3, \cdots, N$) on a network, where the demand quantity $D_i(i=1, 2, 3, \cdots, N)$ is known deterministically. There are available vehicles of capacity V_k ($k=1, 2, 3, \cdots M$) in the depot. C_{ijk} is the routing cost of vehicle k from demand point i to j. Every stop of a vehicle costs α, and the maximum cost of each vehicle route is W.

The traditional VRP model determines routes and load quantities of vehicles in order to minimize total costs under the single-visit restriction. The mathematical formulations are given in eq. (1)~(8). Eq. (2) is the single visit restriction. In eq. (7), the notation T_{LEGAL} means the set of legal tours for the general VRP[4].

$$\textit{Minimize} \sum_{k=1}^{M} \sum_{i=0}^{N} \sum_{j=0}^{N} C_{ijk}\, x_{ijk} + \alpha \sum_{k=1}^{M} \left(\sum_{i=0}^{N} \sum_{j=0}^{N} x_{ijk} - 1 \right) \quad \cdots\cdots (1)$$

Subject to

$$\sum_{k=1}^{M} \sum_{i=0}^{N} x_{ijk} = 1 \qquad \forall j \quad \cdots\cdots (2)$$

$$\sum_{j=1}^{N} x_{ojk} \le 1 \qquad \forall k \quad \cdots\cdots (3)$$

$$\sum_{j=0}^{N} x_{ijk} - \sum_{j=0}^{N} x_{jik} = 0 \qquad \forall i, k \quad \cdots\cdots (4)$$

$$\sum_{i=0}^{N} \sum_{j=0}^{N} C_{ijk}\, x_{ijk} \le W \qquad \forall k \quad \cdots\cdots (5)$$

$$\sum_{i=1}^{N} D_i \sum_{j=0}^{N} x_{ijk} \le V_k \qquad \forall k \quad \cdots\cdots (6)$$

$$X \in T_{LEGAL} \qquad \forall X \quad \cdots\cdots (7)$$

$$x_{ijk} \in \{0, 1\} \qquad \forall i, j, k \quad \cdots\cdots (8)$$

The relaxed model for multiple visits and split delivery is given in eq. (9)~(18). The constant $\rho_i (= \lceil D_i / \max_k \{V_k\} \rceil)$ in the objective function is the lower bound of the required number visits to deliver the demand quantitly of customer i, and the constant β is the penalty cost for the excessive split delivery. In this model, the delivery quantitly of each vehicle must be explicitly indicated because split deliveries to a customer are allowed. The decision variable y_{ik} indicates the delivery quantity of vehicle k for customer i, and the eq. (15) is the restriction on the consistency between the routes and the delivery quantities[2, 3].

$$\textit{Minimize} \ \sum_{k=1}^{M}\sum_{i=0}^{N}\sum_{j=0}^{N}C_{ijk}\ x_{ijk}$$
$$+\alpha \ \sum_{k=1}^{M}\left(\sum_{i=0}^{N}\sum_{j=0}^{N}x_{ijk}-1\right)+\beta \ \sum_{i=1}^{N}\left(\sum_{k=1}^{M}\sum_{j=0}^{N}x_{ijk}-1\right) \quad \cdots (9)$$

Subject to

$$\sum_{j=1}^{N} x_{ojk} \le 1 \qquad \forall k \quad \cdots (10)$$
$$\sum_{j=0}^{N} x_{ijk} - \sum_{j=0}^{N} x_{jik}=0 \qquad \forall i,\ k \quad \cdots (11)$$
$$\sum_{i=0}^{N}\sum_{j=0}^{N} C_{ijk}\ x_{ijk} \le W \qquad \forall k \quad \cdots (12)$$
$$\sum_{i=1}^{N} y_{ik} \le V_k \qquad \forall k \quad \cdots (13)$$
$$\sum_{k=1}^{M} y_{ik} = D_i \qquad \forall i \quad \cdots (14)$$
$$y_{ik}-\min\ \{D_i,\ V_k\} \sum_{j=0}^{N} x_{jik} \le 0 \qquad \forall i,\ k \quad \cdots (15)$$
$$X \in T_{LEGAL} \qquad \forall X \quad \cdots (16)$$
$$x_{ijk} \in \{0,\ 1\} \qquad \forall i,\ j,\ k \quad \cdots (17)$$
$$y_{ik} \ge 0 \qquad \forall i,\ k \quad \cdots (18)$$

The best scenario for the effect of allowing split delivery is as follows; The demand of each customer is $V/2+\delta$ where V is the load capacity of all the vehicles and δ is a very small positive number, so that we must assign a vehicle for every customer. If split deliveries are allowed, the number of vehicles needed is reduced by approximately half, and the usage rate of the load capacity is doubled. As a result, we can save nearly half of the costs[1-3, 9].

3. Accelerated genetic algorithm

The key ideas of the genetic search algorithm are the principle of the survival of the fittest, exchange of the genetic information, and evolution of the population. To implement the principle of the fittest, the fitness of each solution is evaluated based on the objective function value. To simulate genetic information exchange process, the offsprings are created by combining the genetic code.

The genetic algorithm harnesses implicit parallelism[15]. Its search is performed utilizing the total information from a number of points in the feasible space. Intentional use of the probabilistic transition rule effectively prevents the algorithm from premature convergence into a local optimum. These distinguishing features make it popular in various NP-hard class problems[7, 12].

This section describes the ideas of the AGAVRP. The genetic algorithm's development process consists of the following stages: design and synthesis of each component procedure including the solution coding scheme, the initial population generating method, the evaluation technique of individual solution, the parent selection rule, the genetic operators, the generation replacement policy, and the termination condition.

3.1 The design of AGAVRP components

(1) Solution coding and initial population generating

To represent a solution with the binary code, the vehicle-customer assignment pattern is formulated by the binary matrix Z_p of the eq. (19) and (20). Population $P(t)$ at time t consists of a set of Z_p where L is the population size and T is the maximum number of the generation replacement.

$$Z_p = \{Z_p(i,\ k) : Z_p(i,\ k) = \begin{cases} 1, & \textit{if vehicle } k \textit{ visits node } i \\ 0, & \text{otherwise} \end{cases} \quad \forall i,\ k\} \quad \cdots (19)$$

$$P(t) = \{Z_p : p = 1,\ 2,\ \cdots,\ L\} \quad t = 0,\ 1,\ 2,\ \cdots,\ T \quad \cdots (20)$$

In general, the initial population $P(0)$ is generated randomly to get an unbiased sampling effect from the whole solution space. In addition to this randomized method, this research adopts a heuristic method based on an OR technique, in which routes are constructed by the random seed selection rule and the nearest neighborhood search method.

(2) Feasibility and optimality test

For the feasibility test, the possibility of perfect delivery is checked. This test can be modeled by the feasibility test of the transportation problem(eq. (21)~(24)), in which the supply and the demand corresponds to the load capacity of the vehicles and the order quantities of the customers. The decision variable f_{ki} is the quantity delivered to a customer i by a vehicle k and the constant F_{ki} is the cost coefficient.

$$\textit{Minimize} \quad \sum_{k=1}^{M} \sum_{i=1}^{N} F_{ki} f_{ki} \quad \cdots (21)$$

subject to

$$\sum_{i=1}^{N} f_{ki} \le V_k \qquad \forall k \quad \cdots (22)$$

$$\sum_{k=1}^{M} f_{ki} = D_i \qquad \forall k \quad \cdots (23)$$

$$f_{ki} \ge 0 \text{ (only)if } Z_p(i, k)=1 \qquad \forall i, k \quad \cdots (24)$$

To accelerate the efficient construction of low-cost routes, the cost coefficient is set to an approximate marginal cost[2, 11] of eq. (25) where $C(i, j, k)$ means C_{ijk}. The seed node $Seed(k)$ of a vehicle k is determined by the farthest selection rule.

$$\begin{aligned} F_{ki} &\approx [\{C(0, Seed(k), k)+C(Seed(k), i, k)+C(i, 0, k)\} \\ &\quad - \{C(0, Seed(k), k)+C(Seed(k), 0, k)\}]/D_i \\ &\approx [C(Seed(k), i, k)+C(i, 0, k)-C(Seed(k), 0, k)]/D_i \qquad \cdots (25) \end{aligned}$$

The result of the feasibility test can be used to correct the solution $Z_p(i, k)$. The redundancy in the vehicle-customer assignment causes degeneracy of the transportation problem. Correcting this redundancy makes the optimality evaluation more accurate and faster.

The corrected solution is used to evaluate optimality, but the original solution is used to recombine genetic codes. This temporary correction is similiar to an interaction of the genotype and phenotype based on the presence of a dominant and a recessive genes.

The minimum cost route covering all the assigned customers of a vehicle is determined to evaluate the optimality. This corresponds to the traveling salesman problem. The convex hull method[17] and the 3-optimal procedure[16] are applied.

(3) Fitness evaluation

Total cost $C(p)$ of a solution p is evaluated based on the results of the feasibility and the optimality test. In eq. (26), C_{route} is the sum of the routing cost, C_{stop} is the stopping cost, C_{split} is the penalty cost of the excessive split, C_{unload} is the penalty cost or the maximum additional expenses for the possible undelivered quantity, C_{long} is the penalty cost of exceeding upper limit on the routing cost of the vehicle, and γ is the cost coefficient for the penalty C_{long}. In eq. (26d), $(D_i-\sum_{k=1}^{M} y_{ik})$ is the sum of the undelivered demand quantity, $\lceil(D_i-\sum_{k=1}^{M} y_{ik})/max_k\{V_k\}\rceil$ is the minimum necessary number of vehicles, and $max_k\{2C_{0ik}\}$ is the maximum round trip cost, so the meaning of the eq. (26d) is the upper bound of the routing cost for the undelivered demand.

This research adopts the rank-based, shared fitness function to overcome the problem of premature convergence[8, 13]. The rank-based fitness $f(p)$ is defined by eq. (27) where $r(p)$ is the rank of solution p and $F = (F_{min}, F_{max})$ is the scaling factor. The shared fitness function $f_s(p)$ defined by eq. (28) evaluates the m solutions of the same rank. Finally the relative fitness $f_r(p)$ is defined by eq. (29).

$$C(p)=C_{route}+C_{stop}+C_{split}+C_{unload}+C_{long} \quad \cdots (26)$$

$$C_{route}=\sum_{k=1}^{M}\sum_{j=0}^{N}\sum_{i=0}^{N} C_{ijk}\ x_{ijk} \quad \cdots (26a)$$

$$C_{stop}=\alpha \sum_{k=1}^{M}(\sum_{j=0}^{N}\sum_{i=0}^{N} x_{ijk}-1) \quad \cdots (26b)$$

$$C_{split}=\beta \sum_{k=1}^{M}(\sum_{j=0}^{N}\sum_{i=0}^{N} x_{ijk}-\rho_i) \quad \cdots (26c)$$

$$C_{unload}=2\sum_{i=1}^{N} \max_k\{C_{0ik}\}\lceil(D_i-\sum_{k=1}^{M} y_{ik})/\max_k\{V_k\}\rceil \quad \cdots (26d)$$

$$C_{long}=\gamma \sum_{k=1}^{M} \max(0,\ (\sum_{i=0}^{N}\sum_{j=0}^{N} C_{ijk}\ x_{ijk}-W)) \quad \cdots (26e)$$

$$f(p)=F_{max}-(F_{max}-F_{min})(r(p)-1)/(L-1)$$
$$\text{where } r(p)=1,\ 2,\ \cdots,\ L \quad \cdots (27)$$

$$f_s(p)=f(p_i)/m$$
$$\text{where } f(p_i)=const,\quad i=1,\ 2,\ \cdots,\ m \quad \cdots (28)$$

$$f_r(p)=f_s(p)/\sum_{p=1}^{L} f_s(p) \qquad p=1,\ 2,\ \cdots,\ L \quad \cdots (29)$$

(4) Genetic operations

In order to create the next population, the parent solutions which will take part in the genetic recombination process are selected by the statistically biased sampling based on the relative fitness of current population. In this study, the stochastic remainder sampling without replacement is applied[5].

The genetic reproduction and recombination process follows to create offsprings from the selected parents. In addition to crossover and mutation, this research adopts inversion. Crossover creates offspring Z_3 and Z_4 from parents Z_1 and Z_2 at the rate of P_c by the eq. (30) and (31) where l_c is the point of crossover. Mutation manipulates genetic codes by eq. (32) at the rate of P_m. Inversion operator defined by eq. (33) creates a symmetrically reversed genetic code with applying rate of P_r.

$$Z_3(i,\ k)=\begin{cases} Z_1(i,\ k), & N\cdot(k-1)+i\le l_c \\ Z_2(i,\ k), & N\cdot(k-1)+i > l_c \end{cases} \quad \cdots (30)$$

$$Z_4(i,\ k)=\begin{cases} Z_2(i,\ k), & N\cdot(k-1)+i\le l_c \\ Z_1(i,\ k), & N\cdot(k-1)+i > l_c \end{cases} \quad \cdots (31)$$

$$Z_p(i,\ k)=\begin{cases} 1, & Z_p(i,\ k)=0 \\ 0, & Z_p(i,\ k)=1 \end{cases} \quad \cdots (32)$$

$$Z_p(i,\ k)=Z_p(N-(i-1),\ M-(k-1)) \quad \cdots (33)$$

(5) Replacement with elitism

In this research, replacement scheme with elitism is used. The generation replacement technique has some potential drawback such that the best member of the population may fail to survive in the next generation. The elitist strategy fixes this loss of potential sources by copying the best member of each generation into the succeeding generation[7].

The termination condition is set to the maximum number of generation replacements or to the state in which no more significant improvement occurs during a certain period.

3.2 The procedure of AGAVRP

The procedure of AGAVRP is as follows. $S^*(t)$ indicates the best solution ever existed until time t. $BS(t)$ and $WS(t)$ indicate the best and the worst solution at time t, respectively.

Step 0. [initialization]

Set time $t=0$. Generate initial population using eq. (19) and (20). The number of heuristic solutions is L_h, and the number of randomly generated solutions is $L_r(L_h+L_r=L)$.

Step 1. [generation replacement]

Repeat the following steps until the termination condition is satisfied.

1.1 [feasibility test]

Test the feasibility of delivery by solving the transportation problem of eq. (21)~(25).

1.2 [optimality test]

Correct the genetic code temporarily according to the result of the feasibility test. To solve the traveling salesman problem for each vehicle, apply the convex-hull insertion method and the 3-optimal route improvement procedure.

1.3 [fitness evaluation]

Determine the rank of each solution member based on the total cost in eq. (26). Evaluate rank-based, shared, relative fitness applying eq. (27)~(29). If $BS(t)$ is inferior to $S^*(t)$, replace $WS(t)$ with $S^*(t)$.

1.4 [reproduction and recombination]

Repeat the following steps to construct a succeeding generation.

1.4.1 [parents selection]

Select parents Z_1 and Z_2, using the stochastic remainder sampling without replacement.

1.4.2 [reproduction]

Reproduce offsprings Z_3 and Z_4 from the selected parents Z_1 and Z_2.

1.4.3 [crossover]

Apply crossover operator with rate P_c according to eq. (30) and (31).

1.4.4 [inversion]

Apply inversion operator with rate P_r according to eq. (33).

1.4.5 [mutation]

Apply mutation operator with P_m according to eq. (32).

1.5 [replacement and termination test]

Set time $t=t+1$. Replace current population $P(t)$ with succeeding one. If the termination condition is satisfied, then terminate the algorithm. Otherwise repeat step 1.

4. The experiments and the analysis

Extensive experiments are performed to analyze the searching power and to optimize the parameters of the AGAVRP on a number of 486(50 MHz) personal computers with the algorithm programmed by Lahey FORTRAN compiler(ver. 5.1). Ten test problems of the single-visit model are taken from Eilon et al.[10]. A number of good solutions are known from the previous research including the branch-and-bound algorithm, the savings heuristic, or the 3-optimal algorithm[6, 10, 18].

4.1 Parameter tuning

Some researches on parameter tuning were reported in literature[8, 14, 19]. But because AGAVRP has many different characteristic variants compared to the standard genetic algorithm, a new parameter set is to be found.

Table 1. The experimental level of the parameters

Scaling factors (F)	Crossover rate (P_c)	Mutation rate (P_m)	Inversion rate (P_r)	Heuristic sol. rate (P_h)
(83, 117)	0.8	0.002	0.02	0.02
(50, 150)	0.9	0.005	0.05	0.05
(17, 183)	1.0	0.008	0.08	0.08

The parameters of AGAVRP include the population size L, the maximum number of generations T, the scaling factors $F=(F_{min}, F_{max})$, the applying rate P_c, P_m, P_r of each operator, and the ratio P_h of heuristic solutions over the population size. In this study, the population size and the maximum number of generations are fixed to $L=100$ and $T=1000$ according to the preliminary experimental study. The experimental levels of the parameters are set to the values shown in table 1.

According to a 5-way factorial design with 5 replication, experiments are conducted for the 7 test problems(No. 1~No. 7) on all the combinations of the test levels, so that the total number of experiments is 8,505(=5 · 7 · 3^5). Some problems (No. 8, 9 and 10) are excluded from these experiments because their sizes are too large.

The results of the experimental study are summarized in table 2, including the best parameter levels and the average cost for each test problem. This result coincides with the conclusion of the Schaffer et al.[19] that the best parameter levels of the genetic algorithm are varying from problem to problem.

Table 2. The best parameter levels for each problem

Prb.	F	P_c	P_m	P_h	P_r	Avg. cost
1	–	–	–	–	–	114.0
2	(17,183)	0.7	0.001	0.7	0.001	291.6
3	(90,110)	0.9	0.001	0.9	0.001	589.2
4	(30,170)	0.5	0.005	0.5	0.005	949.0
	(10,190)	0.7	0.009	0.7	0.009	949.0
	(10,190)	0.7	0.009	0.7	0.009	949.0
5	(70,130)	0.9	0.005	0.9	0.005	877.0
6	(50,150)	0.9	0.009	0.9	0.009	1260.2
7	(70,130)	0.9	0.001	0.9	0.001	811.6
	(70,130)	0.9	0.001	0.9	0.001	811.6

The fact that the best parameter levels are varying from problem to problem requires clearer definition of the optimality for the parameter. In the genetic algorithm, the optimization of the parameter set requires robustness such that it is not sensitive to the change of characteristics of problem space and always guarantees the satisfactory solutions for various types of problems.

This study uses the average normalized search power as a measure of robust performance.

For each parameter set u and the problem v, the normalized search power $NAC(u,v)$ is defined in eq. (34), where $AC(u, v)$ means average costs of the 5 trials, $AAC(v)$ and $DAC(v)$ is the average and the standard deviation of the $AC(u, v)$'s over all parameter sets. Finally, the average normalized search power $ANAC(u)$ is defined for each parameter set u in eq. (35) and it is used as a measure of comparison.

$$NAC(u, v) = (AC(u, v) - AAC(v))/DAC(v) \quad \forall u, v \quad \cdots (34)$$

$$ANAC(u) = \sum_{v=1}^{7} NAC(u, v)/7 \quad \forall u \quad \cdots (35)$$

Table 3 shows the values of parameter sets and their average normalized search power for the top five. The best parameter set is found when the scaling factor, the rate of the operators(crossover, mutation, inversion), and the inclusion rate of heuristic solution are set to $F=(50, 150)$, $P_c=0.8$, $P_m=0.002$, $P_r=0.02$, $P_h=0.05$ respectively.

Table 3. The top five performance

Rank	*F*	P_c	P_m	P_h	P_r	$ANAC(u)$
1	(50, 150)	0.8	0.002	0.05	0.02	−1.470
2	(17, 183)	0.9	0.002	0.05	0.02	−1.443
3	(83, 117)	0.8	0.002	0.02	0.05	−1.122
4	(17, 183)	0.9	0.002	0.08	0.05	−1.096
5	(50, 150)	1.0	0.002	0.02	0.02	−1.011

4.2 The performance analysis under the best parameter set

This section describes the experimental study about the performance analysis of the AGAVRP under the best parameter set found in section 4.1. This analysis is conducted over all the 10 test problems and for both types of the models mentioned above. For each problem, the best result in three AGAVRP runs is compared to the best one in the solutions by 8 types of algorithms including branch-and-bound, 3-optimal, and savings heuristic(table 4).

For this analysis, the population size, and the maximum number of generations are fixed to 100 and 5000, respectively. The search process is also terminated when no improvement is made in 2000 generations.

In case of the single visit model, *Rel. Dev.* means a relative deviation compared with the previously known best solutions of the single visit model from 8 types of algorithms[6, 10, 18]. But in case of the multiple visit model, there is no known solution for this problem set.

Table 4. The comparative performance of the AGAVRP

Prb. No.	Single Visit Model		Multiple Visit Model	
	Tot. Cost	*Rel. Dev.*(%)	*Tot. Cost*	*Rel. Dev.*(%)
1	114	0.00	114	0.00
2	290	0.00	282	−2.76
3	585	0.00	585	0.00
4	949	0.00	949	0.00
5	877	0.23	877	0.00
6	1265	−10.54	1231	−2.69
7	810	0.00	810	0.00
8	533	1.72	527	−1.13
9	895	4.80	876	−2.12
10	886	7.13	861	−2.82

he *Rel. Dev.* of this case means a relative deviation compared with the best AGAVRP solutions of the single visit model, so it means cost saving ratio of the multiple visit model compared to the single visit model.

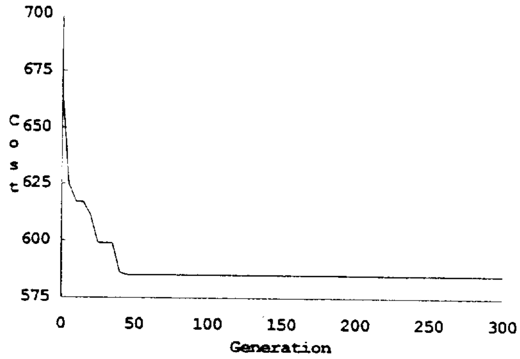
In case of the single visit model, the AGAVRP solutions of problem 1~7 are superior or approximately equal to the previously known best solutions. Especially, the AGAVRP finds 10.54 % better solution for problem 6. The reason for the lower performances for some problems is presumed that these problems are excluded from the parameter tuning stage.

In case of the multiple visit model, general tendency of cost reduction is observed. In problem 2 of single visit model, the AGAVRP solution is equal to the optimum found by the branch-and-bound algorithm. Thus, the net cost saving effect is 2.76 %, which is solely due to the permission of the split delivery.

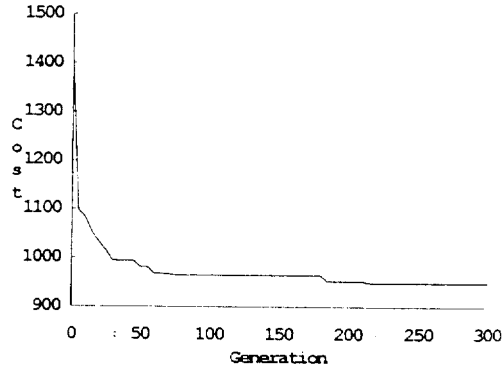
Figure 1 is the convergence chart of AGAVRP for problem 3 and 4. Similar convergence patterns are observed for the other problems. The distinctive feature of AGAVRP is the fact that the convergence is fast especially in early stage.

4.3 Side product of parameter tuning

In the process of parameter tuning of this research, some solutions observed are even better than previously known solutions. These new best solutions presented in table 5 can be used new bench mark.



(a) Problem No. 3



(b) Problem No. 4

Figure 1. Convergence of the AGAVRP

Table 5. New best solutions found

Problem No.	6	8
previous best solution new best solution	1414 1227	524 522
improvement ratio(%)	13.22	0.38

5. Conclusions

This research proposes AGAVRP(accelerated genetic algorithm for the vehicle routing problem) of single and multiple visit model. The AGAVRP is accelerated by the OR techniques at the stage of generating initial solution, evaluating optimality and feasibility of the solutions. It adopts many ideas which are different from traditional genetic algorithms such as simplified coding, encoding by OR techniques, distinction of the genotype and the phenotype, rank-based shared fitness, elitism, and inversion operator.

Extensive experimental study has been conducted by the 5-way factorial design with replication in order to set the parameters of the AGAVRP based on the concept of robustness. The best parameter set is found such that the scaling factor, the rate of the operator crossover, mutation, inversion, and the inclusion rate of heuristic solution is set to $F=(50,$

150), P_c=0.8, P_m=0.002, P_r=0.02, P_h=0.05 respectively.

With the best parameter set, the performance of the AGAVRP is compared with that of previously known algorithms. In the single visit model, the solutions found with the AGAVRP are superior or approximately equal to the best known solutions for the 7 problems out of 10 problems. Especially for the problem 6, 10.54% better solution is found. In the multiple visit model, cost saving effect is observed. As a side product of this research, new bench mark solutions for some problems are found.

The advantage of the proposed algorithm is its flexibility and better convergence. The AGAVRP is applicable to both the single visit and multiple visit model without any algorithmic modification. Convergence is quite fast especially in early stages of the algorithmic process, and many alternative good solutions are available in the entire algorithmic stages due to the population evolution nature of the genetic algorithm.

Future research should be directed to the development of an effective method to test convergence and the termination condition. Also more work on the two-stage search algorithm is expected, that is the global search at the first stage utilizing the early and fast convergence character, and the local search at the second stage.

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