Theoretical Description of All-Optical Switching Phenomena Involving Coupled Gap Solitons

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We study the propagation of two pulses with orthogonal linear polarizations in a nonlinear periodic dielectric structure with $\chi^{(3)}$ nonlinearity. Using an envelope- function approach, we derive the coupled nonlinear Schrodinger equations governing the spatio-temporal evolutions of the two orthogonally polarized modes in a nonlinear periodic structure. We then find their solitary-wave solutions referred to as coupled gap solitons. We show that two orthogonally polarized pulses can co-propagate as a coupled gap soliton through a nonlinear periodic structure while each pulse alone will be strongly reflected due to the Bragg reflection. Based on the results, we present an all-optical switching scheme which has a novel architecture and principle. We also study the stability of coupled gap solitons to find the dragging phenomena in a nonlinear birefringent periodic medium.

I. Introduction

A linear periodic dielectric structure would reflect strongly any incoming light waves with a frequency that falls within a certain frequency band centered at the Bragg frequency. Such a frequency band is referred to as the stop gap[1,2]. However, if a periodic dielectric medium has $\chi^{(3)}$ nonlinearity, it will show entirely different transmission characteristics depending upon the field intensity. The intensity-dependent refractive index due to the third-order nonlinearity will modify the effective refractive index of the periodic dielectric structure, thereby shifting the stop gap of the structure, allowing the frequency of the input beam to tune out of the stop gap. As a result, a nonlinear periodic dielectric medium (NPDM) can transmit an input laser beam with a frequency falling within the stop gap, provided that the laser beam intensity is high enough. Such optical fields transmitted in the NPDM is referred to as gap solitons since they have solitary-propagation properties and their slowly-varying envelopes have hyperbolic secant shapes.

Recently, the theoretical interests in the gap soliton have grown owing to their unusual properties. For example, it was found[3,4] that gap solitons can propagate in a NPDM with a group velocity much slower than the usual group velocity determined by the medium refractive index dispersion. In addition, gap solitons also exhibit bistable behaviors[5-7] due to $\chi^{(3)}$ nonlinearity. The gap solitons discovered by Chen and Mills[5] were stationary solitons. A continuous-wave (CW) input beam with a certain intensity will excite such a time-independent solitary wave with a slowly-varying envelope of hyperbolic secant shape, which occupies spatially a NPDM at a standstill. Later, de Sterke *et al.*[3] and Christodulides *et al.*[4] showed that it is possible to propagate moving solitons in a NPDM with an appropriate input pulse applied to a NPDM. Based on techniques from solid-state physics, de Sterke-Sipe[8,9] developed an envelope-function approach as an analytical tool for deriving a nonlinear Schrodinger equation (NLSE) in a NPDM. On the associated dipersion curve obtained form Kronig-Penny model, a stationary gap soliton will be positioned at the Brillouin zone edge where the group velocity vanishes. However, a moving gap soliton has somehow higher peak intensity, leading to larger shift of the stop gap than a stationary gap soliton does. Consequently, the associated dispersion relation will be governed by a nonzero-slope point out of the Brillouin zone, thereby allowing the pulse to attain wave-momentum, giving rise to a traveling wave.

Meanwhile, S. Lee and Ho[10,11] studied the propagation of two orthogonally polarized solitary modes in a NPDM with $\chi^{(3)}$ nonlinearity. Specifically, we showed that two orthogonally polarized pulses can copropagate as a coupled gap soliton through a NPDM, while each pulse alone will be strongly reflected because its amplitude is less than that needed to propagate a gap soliton in a single polarization. Based on the results, we presented a new all-optical switching scheme. Compared with the other optical switch, the proposed switch has some potential advantages. For example, the control pulse in the proposed switch will switch directly the signal pulse, which means it can be exploited as a true *logic gate*. Moreover, it is very stable for the external conditions such as sound and heat waves, etc. Furthermore, the proposed switch with an extremely small switching element can be realized on a semiconductor waveguide, and thus it can be easily integrated into optical circuits. In the present paper we provide the full description and derivation of coupled gap solitons-the two orthogonally polarized $\sqrt{3/5}\,A$ sech pulse. In the previous paper[10,11] the coupled NLSE's were derived under the assumption that the nonlinear periodic waveguide has an appropriate birefringence. Thus we here study the dragging phenomena of coupled-gap solitons for the case that such birefringence exists in the periodic waveguide.

The body of the paper is comprised of the following sections. In Sec. II, we find the basic equations describing the spatio-temporal evolutions of two orthogonally polarized modes in a NPDM with $\chi^{(3)}$ nonlinearity. We show that such equations, referred to as the coupled NLSEs, have coupled-gap solitary-wave solutions in a simple case. The comprehensive demonstration for the physical properties of coupled gap solitons will be also given in this section. In Sec. III, we then present a new all-optical switching scheme based on the transmission of coupled gap solitons in a NPDM. We also search for the relations between the pulse parameters of the coupled-gap solitons and the structure parameters of the NPDM. In Sec. IV, we study the stability of the coupled-gap soiltons for the case where the two orthogonal modes propagate at different group velocities in a birefringent nonlinear periodic medium. Using an average Lagrangian formalism[12,13], we find soliton dragging phenomena in a NPDM. Finally, we discuss the results of the present paper in Sec.V.

II. Coupled Nonlinear Schrodinger Equation and Coupled Gap Soliton

The Maxwell equation in a NPDM can be written in terms of the electric field propagating along the z-axis as follows:

$$c^2\frac{\partial^2}{\partial z^2}\,\boldsymbol{E}(z,\ t)-\varepsilon(z)\frac{\partial^2}{\partial t^2}\,\boldsymbol{E}(z,\ t)=4\pi\chi^{(3)}(z)\frac{\partial^2}{\partial t^2}\,[\boldsymbol{E}(z,\ t)]^3 \tag{1}$$

where the linear dielectric tensor $\varepsilon(z)$ is periodic along the z-axis and the transverse profile of the total electric field is assumed to remain constant during its propagation in a lossless medium. Before proceeding to the main derivations, we make several assumptions; (i) Slowly varying envelope approximation and plane wave approximation; (ii) The cross-coupling coefficients $\chi^{(3)}_{abcd}$ are related to the self-coupling coefficient $\chi^{(3)}_{aaaa}$ by $\chi^{(3)}_{xxyy}=\chi^{(3)}_{xyxy}==\chi^{(3)}_{xyyx}=1/3\,\chi^{(3)}_{xxxx}$; (iii) The NPDM has an appropriate birefringence for the polarization preservation of the two orthogonal modes, and for the reason mentioned later; (iv) There is no linear coupling between two orthogonally polarized modes so that the off-diagonal elements of linear susceptibility tensor can be neglected. The nonlinear polarization in the Maxwell equation can be written as[14]

$$\boldsymbol{P}^{NL}=\chi^{(3)}[(1-B)(\boldsymbol{E}\cdot\boldsymbol{E}^*)\,\boldsymbol{E}+B(\boldsymbol{E}\cdot\boldsymbol{E})\,\boldsymbol{E}^*] \tag{2}$$

where $\chi^{(3)}=3\chi^{(3)}_{xxxx}$ and $B=\chi^{(3)}_{xyyx}/\chi^{(3)}_{xxxx}$. Using the assumption (ii), we find

$$\boldsymbol{P}^{NL}=\chi^{(3)}\left[\frac{2}{3}(\boldsymbol{E}\cdot\boldsymbol{E}^*)\,\boldsymbol{E}+\frac{1}{3}(\boldsymbol{E}\cdot\boldsymbol{E})\,\boldsymbol{E}^*\right]. \tag{3}$$

The electric field for the two orthogonally polarized modes in a linear periodic structure ($\boldsymbol{P}^{NL}=0$) can be written in terms of Bloch functions as follows:

$$\boldsymbol{E}(z,\ t)=\begin{bmatrix}\varphi_{mx}(z)\,e^{-i\omega_{mx}t}+C.C\\ \varphi_{my}(z)\,e^{-i\omega_{my}t}+C.C\end{bmatrix}\begin{bmatrix}\boldsymbol{i}_x\\ \boldsymbol{i}_y\end{bmatrix} \tag{4}$$

where $\boldsymbol{i}_x$ and $\boldsymbol{i}_y$ are the unit vectors along the x and y axis, respectively, and φ_{mx} and φ_{my} are the corresponding Bloch functions satisfying the orthogonality relations:

$$\langle mi|\varepsilon_i(z)|mi'\rangle\equiv\int_0^L\varphi^*_{mi}(z)\,\varepsilon_i(z)\,\varphi_{mi'}(z)\,dz=\delta_{mi,mi'} \tag{5}$$

where $i=x$ or y.

In order to study the dynamical behavior of the slowly-varying envelopes in the two modes, we use an envelope-function approach to derive the coupled NLSEs in a NPDM. Following de Sterke *et al.*[8,9], the electric field $\boldsymbol{E}(z, t)$ can be expressed in terms of different length scales, $z_n=\mu^n z$ and time scales, $t_n=\mu^n t$:

$$\boldsymbol{E}(z,\ t)=\mu(e_{1x}\boldsymbol{i}_x+e_{1y}\boldsymbol{i}_y)+\mu^2(e_{2x}\boldsymbol{i}_x+e_{2y}\boldsymbol{i}_y)+\cdots, \tag{6}$$

where $\mu(\ll 1)$ is a scalar constant, e_{nx} and e_{ny} show the variations of the electric fields in each polarization on the different space and time scales, and $n=0,\ 1,\ 2,\ \cdots$. We now substitute Eq. (6) into Eq. (1) and then calculate order by order for μ^n. First, collecting all terms proportional to $\mu(n=1)$, we find

$$-\left[c^2\frac{\partial^2}{\partial z_0^2}-\varepsilon_x(z_0)\frac{\partial^2}{\partial t_0^2}\right]e_{1x}=0$$

$$-\left[c^2\frac{\partial^2}{\partial z_0^2}-\varepsilon_y(z_0)\frac{\partial^2}{\partial t_0^2}\right]e_{1y}=0 \tag{7}$$

Since the above equations are just a linear form of the Maxwell equation ($\boldsymbol{P}^{NL}=0$), we have

$$e_{1x}=a_x(z_1,\ z_2,\ \cdots,;\ t_1,\ t_2,\ \cdots)\,\varphi_{mx}(z_0)\,e^{-\omega_{mx}t_0}+C.C$$

$$e_{1y}=a_y(z_1,\ z_2,\ \cdots,;\ t_1,\ t_2,\ \cdots)\,\varphi_{my}(z_0)\,e^{-\omega_{my}t_0}+C.C \tag{8}$$

where the slowly-varying envelope functions in the x and y-polarization, a_x and a_y are determined by subsequent stages. Next, collecting all terms proportional to μ^2, we find

$$-\left[c^2\frac{\partial^2}{\partial z_0^2}-\varepsilon_x(z_0)\frac{\partial^2}{\partial t_0^2}\right]e_{2x}=2\left[c^2\frac{\partial}{\partial z_0}\frac{\partial}{\partial z_1}-\varepsilon_x(z_0)\frac{\partial}{\partial t_0}\frac{\partial}{\partial t_1}\right]e_{1x}$$

$$-\left[c^2\frac{\partial^2}{\partial z_0^2}-\varepsilon_y(z_0)\frac{\partial^2}{\partial t_0^2}\right]e_{2y}=2\left[c^2\frac{\partial}{\partial z_0}\frac{\partial}{\partial z_1}-\varepsilon_y(z_0)\frac{\partial}{\partial t_0}\frac{\partial}{\partial t_1}\right]e_{1y} \tag{9}$$

Using the orthogonality relations given in Eq. (5), we find the following relations for the subspace spanned by φ_{mi}:

$$\frac{\partial a_x}{\partial t_1}+\omega'_{mx}\frac{\partial a_x}{\partial z_1}=0$$

$$\frac{\partial a_y}{\partial t_1}+\omega'_{my}\frac{\partial a_y}{\partial z_1}=0 \tag{10}$$

which means the envelopes, a_x and a_y travel with the group velocity, ω_{mx}' and ω_{my}', respectively. By the remaining states φ_{lx} and φ_{ly} ($l\neq m$), the additional coupled terms appear, for example $\partial a_x/\partial z_1(\Lambda_{l,m})_x$ where $\Lambda_{l,m}=2ic/d\ \langle l|\Omega|m\rangle_x/(\omega_{lx}^2-\omega_{mx}^2)$.

Finally, we calculate terms proportional to μ^3 where a_x and a_y are nonlinearly coupled due to the third-order nonlinearity. By the expression of the electric field in terms of the multiple scales, we rewrite the nonlinear polarization $\boldsymbol{P}^{NL}$ as

$$\boldsymbol{P}^{NL}=\chi^{(3)}[\mu^3(e_{1x}\boldsymbol{i}_x+e_{1y}\boldsymbol{i}_y)^3+3\mu^4(e_{1x}\boldsymbol{i}_x+e_{1y}\boldsymbol{i}_y)^2(e_{2x}\boldsymbol{i}_x+e_{2y}\boldsymbol{i}_y)+\cdots]. \tag{11}$$

Taking here only terms proportional to μ^3, and then substituting Eq. (8) into those terms, we find the expressions for the nonlinear polarizations in x and y-axis. Using the results obtained from the previous stages, we obtain

$$2i\omega_{mx}\left[\frac{\partial a_x}{\partial t_2}+\omega'_{mx}\frac{\partial a_x}{\partial z_2}\right]+c^2\langle m|m\rangle_x\frac{\partial^2 a_x}{\partial z_1^2}-\frac{\partial^2 a_x}{\partial t_1^2}+4\pi\Big[3\omega_{mx}^2\int_0^L\chi^{(3)}(z_0)|\varphi_{mx}(z_0)|^4dz_0|a_x|^2a_x+2\omega_{mx}^2\int_0^L\chi^{(3)}(z_0)|\varphi_{my}(z_0)|^2|\varphi_{mx}(z_0)|^2dz_0|a_y|^2a_x+(\omega_{mx}-2\omega_{my})^2\int_0^L\chi^{(3)}(z_0)\,\varphi_{my}^2(z_0)\,\varphi_{mx}^{*2}(z_0)\,dz_0a_y^2a_x^*e^{2i(\omega_{mx}-\omega_{my})t}\Big]=0,$$

$$2i\omega_{my}\left[\frac{\partial a_y}{\partial t_2}+\omega'_{my}\frac{\partial a_y}{\partial z_2}\right]+c^2\langle m|m\rangle_y\frac{\partial^2 a_y}{\partial z_1^2}-\frac{\partial^2 a_y}{\partial t_1^2}+4\pi\Big[3\omega_{my}^2\int_0^L\chi^{(3)}(z_0)|\varphi_{my}(z_0)|^4dz_0|a_y|^2a_y+2\omega_{my}^2\int_0^L\chi^{(3)}(z_0)|\varphi_{my}(z_0)|^2|\varphi_{mx}(z_0)|^2dz_0|a_x|^2a_y+(\omega_{my}-2\omega_{mx})^2\int_0^L\chi^{(3)}(z_0)\,\varphi_{mx}^2(z_0)\,\varphi_{my}^{*2}(z_0)\,dz_0a_x^2a_y^*e^{2i(\omega_{my}-\omega_{mx})t}\Big]=0, \tag{12}$$

where the high-order harmonic generations are not considered. If we use the relation $\omega''_{mx}=c^2/\omega_{mx}\langle m|m\rangle_x-(\omega'_{mx})^2/\omega_{mx}$ and then replace t_2, ξ_1, and $a(x_i;\ t_i)$ with t, z, and $\sqrt{L}a\,(x_i;\ t_i)$ at the limit of $\mu\to 1$, respectively, we can obtain the coupled nonlinear differential equations governing temporal evolutions of two envelopes as follows:

$$i\frac{\partial a_x}{\partial t}+\frac{1}{2}\omega''_{mx}\frac{\partial^2 a_x}{\partial z^2}+(\alpha_{mxx}|a_x|^2+\alpha_{mxy}|a_y|^2)a_x+\beta_{mxy}a_y^2a_x^*e^{2i(\omega_{mx}-\omega_{my})t}=0$$

$$i\frac{\partial a_y}{\partial t}+\frac{1}{2}\omega''_{my}\frac{\partial^2 a_y}{\partial z^2}+(\alpha_{myy}|a_y|^2+\alpha_{myx}|a_x|^2)a_y+\beta_{myx}a_x^2a_y^*e^{2i(\omega_{my}-\omega_{mx})t}=0 \quad (13)$$

where $\alpha_{mxx}=6\pi\omega_{mx}L\int_0^L\chi^{(3)}(z_0)|\varphi_{mx}(z_0)|^4dz_0$, $\alpha_{mxy}=4\pi\omega_{mx}L\int_0^L\chi^{(3)}(z_0)|\varphi_{mx}(z_0)|^2|\varphi_{my}(z_0)|^2dz_0$, $\alpha_{myx}=4\pi\omega_{my}L\int_0^L\chi^{(3)}(z_0)|\varphi_{mx}(z_0)|^2|\varphi_{my}(z_0)|^2dz_0$, $\beta_{mxy}=[2\pi(\omega_{mx}-2\omega_{my})^2/\omega_{mx}]L\int_0^L\chi^{(3)}(z_0)\varphi_{mx}^2(z_0)\varphi_{my}^{*2}(z_0)dz_0$. In this paper, we are interested in the case in which the last terms (called the analogous four-wave-mixing term) in Eq. (13) can be neglected. It is valid if x and y-polarized fields have different center frequencies (or if the fields are in a birefringent medium). In the latter case, uncoupled x and y-polarized solitons will propagate with different group velocities. As will be seen in Sec. IV, the coupled x and y-polarized solitons, however, will exhibit dragging phenomena. For the both cases, Eq. (13) will be reduced to the coupled NLSE's. As discussed below, in the case where the terms of concern is not negligible, simple solutions can be found if $\omega_{mx}=\omega_{my}$.

The coupled NLSEs can be solved in the special case of stationary solitons by introducing the detuning factors δ_x and δ_y, where $a_x=\psi_x(z)e^{-i\delta_x t}$ and $a_y=\psi_y(z)e^{-i\delta_y t}$. In terms of these detuning factors, Eq. (13) can be rewritten as

$$\frac{d^2\psi_x}{dz^2}-B_x^2\psi_x+2\frac{B_x^2}{A_{xx}^2}|\psi_x|^2\psi_x+2\frac{B_x^2}{A_{xy}^2}|\psi_y|^2\psi_x=0,$$

$$\frac{d^2\psi_y}{dz^2}-B_y^2\psi_y+2\frac{B_y^2}{A_{yy}^2}|\psi_y|^2\psi_y+2\frac{B_y^2}{A_{yx}^2}|\psi_x|^2\psi_y=0, \quad (14)$$

where $A_{xx}=\sqrt{-2\delta_x/\alpha_{mxx}}$, $A_{xy}=\sqrt{-2\delta_x/\alpha_{mxy}}$, $A_{yy}=\sqrt{-2\delta_y/\alpha_{myy}}$, $A_{yx}=\sqrt{-2\delta_y/\alpha_{myx}}$, $B_x=\sqrt{-2\delta_x/\omega''_{mx}}$, and $B_y=\sqrt{-2\delta_y/\omega''_{my}}$. First let us consider a special case where the two orthogonal fields have the same shape but not necessarily the same amplitude so that $a_x=\psi_x e^{-i\delta t}\equiv\psi e^{-i\delta t}$ and $a_y=K\psi e^{-i\delta t}$. We also assume that $B_x=B_y$. In this case the coupled NLSEs are simplified to two NLSEs of the same form. These two NLSEs can then be solved by following similar methods used in Refs. [8] and [9]. The pulse envelope solutions are given by the following solitary wave forms.

$$\psi_x(z)=\psi(z)=\frac{1}{\sqrt{1/A_{yx}^2+K^2/A_{yy}^2}}\mathrm{sech}(Bz), \quad (15)$$

$$\psi_y(z)=K\psi(z)=\frac{K}{\sqrt{1/A_{xx}^2+K^2/A_{xy}^2}}\mathrm{sech}(Bz), \quad (16)$$

where K is given by $K^2=(1/A_{xx}^2-1/A_{yx}^2)/(1/A_{yy}^2-1/A_{xy}^2)$. If we assume that the medium and pulse characteristics are identical for TE and TM mode with a slightly different frequency, then $\varphi_{mx}\simeq\varphi_{my}$, $\omega_m\equiv\omega_{mx}\simeq\omega_{my}$, $\omega''_{mx}\simeq\omega''_{my}$, and thus $\alpha_m\equiv\alpha_{mxx}\simeq\alpha_{myy}\simeq 3/2\,\alpha_{mxy}$. In this case the envelope functions of the gap solitons can be further simplified, giving:

$$a=a_x=a_y=\sqrt{3/5}\,A\ \mathrm{sech}(Bz)e^{-i\delta t}, \quad (17)$$

where $A=\sqrt{-2\delta/\alpha_m}$ and $B=\sqrt{-2\delta/\omega''_m}$. For the general case of moving solitons, the solutions give:

$$a=a_x=a_y=\sqrt{3/5}Ae^{iB_2z}e^{-i(\delta+\Delta)t}\,\mathrm{sech}\,B_1(z-v_gt), \quad (18)$$

where $A=\sqrt{-2\delta/\alpha_m}$, $B_1=\sqrt{-2\delta/\omega''_m}$, $B_2=\sqrt{2\Delta/\omega''_m}$, $v_g=\sqrt{2\Delta\omega''_m}$, and the center frequency of the soliton is $\omega_c=\omega_m+\delta+\Delta$. The factor $\delta+\Delta$ corresponds to the frequency detuning of the pulse center frequency from the edge frequency of the stop band. We note that our solution can be straight-forwardly extended to the case where the analogous four-wave-mixing (AFWM) term is not negligible provided $\omega_{mx}=\omega_{my}$. In this case there is also a coupled soliton solution where a_x is proportional to a_y. In fact, with a_x proportional to a_y, the AFWM term would simply give rise to an effective change in the coefficients of the bracketed terms in Eq. (13). For example, the case where $a_x=a_y$ discussed above, would have a solution of $a=a_x=a_y=(1/\sqrt{2})A\,\mathrm{sech}(Bz)e^{-i\delta t}$ instead.

In order to understand the physics behind the coupled gap solitons, we here consider the case in which the two polarized modes $a=a_x=a_y=\sqrt{3/5}Ae^{iB_2z}e^{-i(\delta+\Delta)t}\,\mathrm{sech}\,B_1(z-v_gt)$ are temporally synchronized. The conceptual reasons for the transmissivity and the existence of gap soliton can be described as follows. The field intensity in a nonlinear medium locally modifies the refractive index $n=n^{(0)}(z)+n^{(2)}I$ so that the medium is locally tuned out of the stop gap as illustrated in Fig. 1. To be specific, the photonic band (; the dispersion property of periodic structures) shifts down by δ due to the positive nonlinearity. Recall $\omega_c=\omega_m$

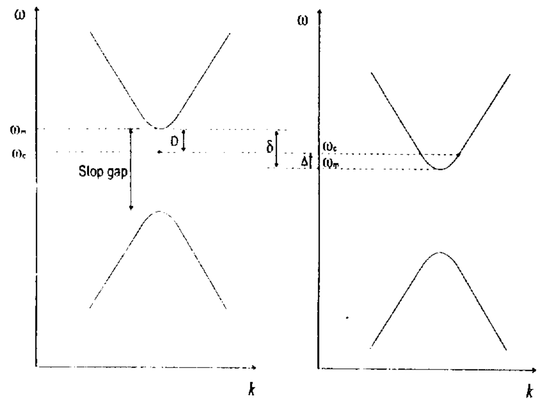

Fig. 1. Illustrative diagram for a shift of the stop gap due to the positive nonlinearity.

$+\delta+\Delta$ where δ and Δ have the opposite signs. Consequently the center frequency of the field ω_r is located right on the edge of the photonic band (for $\Delta=0$) or within the allowed band (for $\Delta>0$). The latter case corresponds to the moving-soliton solution while the former case is responsible for the stationary-soliton solution. In general, the existence of a soliton should be supported by the two mechanisms-dispersion and nonlinearity. For the case of NPDM, the curvature of photonic bands ($\partial^2\omega/\partial k^2$) provides strong dispersive properties, thereby balancing $\chi^{(3)}$ nonlinearity. Therefore a pulse with an appropriate power can propagate as a gap soliton through NPDM even if its center frequency initially locates within the stop gap. Meanwhile, in the case of coupled-gap solitons, the pulse intensity in the self-polarization as well as that in the other polarization will modify the periodic index owing to the self- and cross-coupling process of $\chi^{(3)}$ nonlinearity. Thus the refractive index in the x-polarization for the case of coupled-gap solitons are written as

$$n_x(z)=n^{(0)}(z)+n^{(2)}|E_x(z)|^2+\frac{2}{3}n^{(2)}|E_y(z)|^2 \qquad (19)$$

where $E_x(z)=E_y(z)=\sqrt{3/5}\,A\,\mathrm{sech}\,(Bz)$, while the refractive index in a gap soliton is given by

$$n(z)=n^{(0)}(z)+n^{(2)}|E(z)|^2 \qquad (20)$$

where $E(z)=A\,\mathrm{sech}\,(Bz)$. Comparing these two equations, we find that the amount of the refractive-index change in coupled gap solitons is identical to that in a gap soliton.

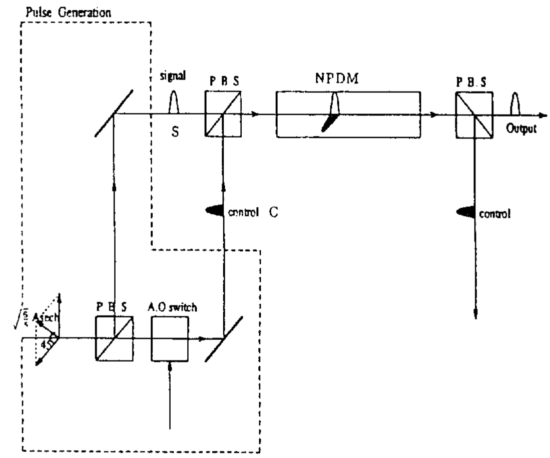

Fig. 2. Schematic of light-by-light switching based on the transmission of coupled gap solitons. A. O, acousto-optical.

III. All-optical switch based on the transmission of coupled-gap solitons

In this section, we present a new all-optical switching scheme based on the previous results. The scheme and principle are different from the intensity-dependent switch studied in the literature[15]. In our proposed switching scheme, we makes use of the fact from Eq. (18) that two orthogonal $\sqrt{3/5}\,A$ sech pulses can copropagate as coupled gap solitons due to cross-coupling $\chi^{(3)}$ nonlinearity although one $\sqrt{3/5}\,A$ sech pulse will be strongly reflected. A single $\sqrt{3/5}\,A$ sech pulse will not propagate through the medium because in the formalism here, the condition required to propagate a gap-soliton pulse in a single polarization is given by a pulse with amplitude A, i.e. a A sech pulse. Thus a $\sqrt{3/5}\,A$ sech pulse with an amplitude smaller than A will be strongly reflected. This proposed light-by-light switch is shown in Fig. 2. The main part of the device consists simply of a very small switching element-GaAs waveguide, and two Polarization Beam Splitters (PBS). In the pulse generation part, a $\sqrt{6/5}$ A sech pulse is divided by a PBS into two $\sqrt{3/5}\,A$ sech pulses which are orthogonally polarized with each other. The two synchronized pulses-the signal S and control C are then combined at another PBS, and coupled into the GaAs waveguide. The S will be transmitted through the waveguide only with the presence of the C due to the characteristic of coupled gap solitons. The S is then separated from the coupled state by another PBS at the waveguide output.

There are several kinds of pulse and structure parameters concerning a switching operation in the proposed scheme. A key relation that connects the pulse parameters and the structure parameters is $\omega_c = \omega_m + \delta + \Delta$. First of all, we need ω_c (center frequency of pulse) to be located around ω_m (band-edge frequency) inside the stop gap. Since ω_o (midgap frequency or Bragg frequency) and $\Delta\omega$ (bandwidth of stop gap) are determined by Λ (spatial period of structure) and Δn (index-modulation depth), respectively, one can adjust $\omega_m = \omega_o + \Delta\omega/2$ to a tuning range of the source laser by an appropriate design of Λ and Δn. Let us define a total detuning by $D \equiv |\delta + \Delta| = |\omega_m - \omega_c|$, which indicates a separation of ω_c from the band-edge. D will be, of course, controlled by tuning of ω_c in a tunable laser. As D decreases with the fixed peak intensity, τ_p (temporal width) should be shortened and the gap solitons will propagate in NPDM with a higher group velocity. For good switching performance, we require that ω_c be far enough from the edge so that the entire pulse spectrum is within the stop band. Let us call this requirement the detuning condition. From the above relations, we find the detuning condition given as follows:

$$D = |\delta + \Delta| \geq 2\pi(\text{pulse} - \text{spectrum})/2 = 2\pi\sqrt{\delta(D+\delta)}$$
$$\Rightarrow D \geq -0.976\delta \quad (21)$$

As will be seen later, such a detuning condition will restrict significantly the allowed range of parameters.

Meanwhile, $\sqrt{3/5}\,A$ (peak amplitude) influences τ_p (temporal width) and also σ_p (spatial width). First, σ_p is inversely proportional to A (; $\sigma_p = \sqrt{\omega''_m/\alpha_m}/A$ from the relations of the soliton parameters given in the previous section). Basically, L (the length of NPDM) should be designed such that it exceeds at least σ_p. In addition, for a good "off-condition" in the switch, we require enough length of NPDM for a sole $\sqrt{3/5}$ A sech pulse to decay out sufficiently at the output end of NPDM. The waveguide length needed for a coverage of the spatial width, however, will provide a good off-state. Second, the dependence of τ_p on

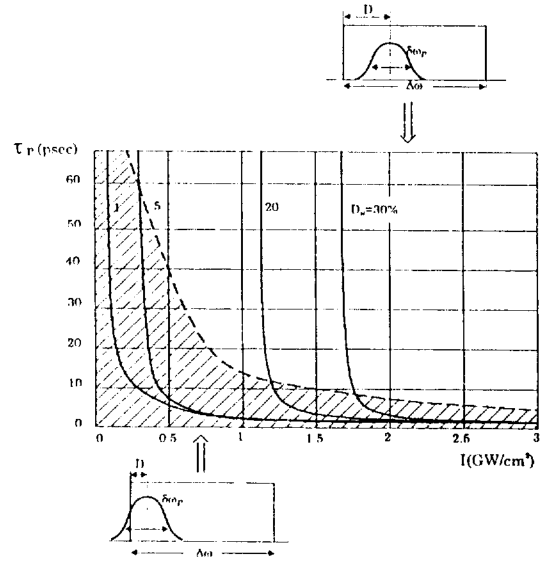

Fig. 3. Dependence of a temporal width on the intensity for each fixed normalized detuning; the dashed line indicates a limitation of pulse bandwidth in conjunction with the center frequency and intensity as illustrated in the miniature figures.

$I(=|A|^2)$ described by $\tau_p = 1/2\sqrt{\alpha_m/2}\,I\,((\alpha_m/2)I - D)$ is very strong especially around the stationary soliton condition. Note that the spectrum of the gap soliton will be restricted by $\Delta\omega$ and furthermore the detuning condition. Peculiarly, temporal width as well as the spatial width will contract with increasing I (peak intensity) because of a dependence of v_g (group velocity) on I. The v_g is governed by the relation $v_g = \sqrt{\omega''_m(\alpha_m I - 2D)}$ and it will mainly influence a switching latency. The v_g will be usually far below the nondispersive group velocity where the soliton characteristics vanish. Nevertheless, the increase of the switching latency due to the slow group velocity is not crucial in an extremely short waveguide used in the proposed scheme. Fig. 3 shows the dependence of temporal width on the peak intensity for a various detunings where the characteristic parameters for the calculation are chosen from the specific GaAs waveguide[16], and the detuning (%) is normalized by the size of the stop gap $\Delta\omega$. The region where the detuning condition is satisfied is the unshaded region in the upper right-hand corner of the dashed line. As discussed the above, the detuning condition is satisfied when the pulse spectrum $\delta\omega_p$ is within the stop band $\Delta\omega$ as illustrated in the insert at the upper right corner of Fig. 3. We can observe a drastic variation of the temporal width τ_p around the condition of the stationary coupled-gap solitons in contrast to a slow variation of τ_p in the other region.

IV. Stability of coupled-gap solitons in a nonlinear birefringent periodic waveguide

In Sec. II, we have assumed that the NPDM has an appropriate birefringence. We now study the stability of coupled gap solitons in a nonlinear birefringent periodic medium. Using a variational approach[12,13], we will show that there exists an interaction force which can lock the two orthogonally polarized modes together in a NPDM. Suppose the refractive indices for TE and TM modes have slightly different average values but the same modulation depth. The different average indices will lead to different slopes of the asymptotic lines in ω-k plane, and thus to different Bragg frequencies and edge frequencies. For such a case, we need to reform the propagation equations given in Eq. (13) since the AFWM terms can be neglected. Let us consider the evolutions of the two orthogonally polarized modes in a new framework moving at $z=z_1-\langle v_g\rangle t$ where $\langle v_g\rangle=(\omega_x'+\omega_y')/2$ is the average group velocity. Accordingly, the propagation equations become

$$i\frac{\partial a_x}{\partial t}+i\Delta V\frac{\partial a_x}{\partial z}+\frac{1}{2}\omega''\frac{\partial^2 a_x}{\partial z^2}+R(|a_x|^2+\frac{2}{3}|a_y|^2)a_x=0$$
$$i\frac{\partial a_y}{\partial t}-i\Delta V\frac{\partial a_y}{\partial z}+\frac{1}{2}\omega''\frac{\partial^2 a_y}{\partial z^2}+R(|a_y|^2+\frac{2}{3}|a_x|^2)a_y=0 \quad (22)$$

where $\Delta V=\omega_x'-\langle v_g\rangle$, $\omega''=\omega_{mx}''=\omega_{my}''$, and $R=\alpha_{mxx}\simeq\alpha_{myy}\simeq 3/2\,\alpha_{mxy}$. The above equations describe the temporal evolutions of the two orthogonally polarized solitons with constant spatial widths. In this case, the temporal widths of solitons will be changed by small variations in the group velocities. Suppose that Eq. (22) have the following input conditions given by

$$a_x=A_x\,\mathrm{sech}\,\varepsilon A_x(z-\xi_x)\,e^{iC_x(z-\xi_x)+iD_x}$$
$$a_y=A_y\,\mathrm{sech}\,\varepsilon A_y(z-\xi_y)\,e^{iC_y(z-\xi_y)+iD_y} \quad (23)$$

where $\varepsilon=\sqrt{5\alpha_m/3\omega_m''}$ is constant. One can observe the temporal evolutions of four parameters, A_j (amplitude), ξ_j (center), C_j (momentum), D_j (Phase) by using a variational method. Note that the parameters given above are virtual quantities for a mathematical treatment, and the conversion into the physical quantities will be made at the end of this section. Using the Euler equation given by

$$\frac{\partial}{\partial z}[\partial L/\partial(\partial a_j^*/\partial z)]+\frac{\partial}{\partial t}[\partial L/\partial(\partial a_j^*/\partial t)]=\frac{\partial L}{\partial a_j^*}, \quad (24)$$

we find that Eq. (22) can be derived from the Lagrangian given as follows:

$$L=L_x+L_y+L_{xy}$$
$$L_j=\frac{i}{2}\left(a_j^*\frac{\partial a_j}{\partial t}-a_j\frac{\partial a_j^*}{\partial t}\right)+(-1)^j\frac{i}{2}\Delta V\left(a_j^*\frac{\partial a_j}{\partial z}-a_j\frac{\partial a_j^*}{\partial z}\right)-\frac{1}{2}\omega''\left|\frac{\partial a_j}{\partial z}\right|^2+\frac{1}{2}R|a_j|^4,\ j=x,\ y$$
$$L_{xy}=\frac{2}{3}R\,|a_x|^2|a_y|^2. \quad (25)$$

In order to observe the evolutions of the soliton parameters, we introduce the averaged Lagrangian defined as

$$\langle L\rangle\equiv\int_{-\infty}^{\infty}L dz. \quad (26)$$

We thus find the averaged Lagrangian of the propagation equation are given by

$$\langle L_j\rangle=\frac{2A_j}{\varepsilon}\left(C_j\frac{d\xi_j}{dt}-\frac{dD_j}{dt}-\Delta VC_j-\frac{\omega''}{2}C_j^2\right)+A_j^3\left(\frac{2R}{3\varepsilon}-\frac{\varepsilon\omega''}{3}\right) \quad (27)$$

and

$$\langle L_{xy}\rangle=\frac{2R}{3\varepsilon}A_xA_y^2\int_{-\infty}^{\infty}\frac{dz_x}{\cosh^2 z_x\cosh^2(z_x+\zeta)}=\frac{8R}{3\varepsilon}A_xA_y^2\frac{\zeta\coth\zeta-1}{\sinh^2\zeta_r} \quad (28)$$

where $z_x=\varepsilon A_x(z-\zeta_x)$, $z_y=\varepsilon A_y(z-\xi_y)$, and $\zeta=z_x-z_y$ is nothing but a normalized distance between the centers of the two solitons in case of $A_x=A_y$. Using the Euler equation again for four parameters, A_j, C_j, D_j, ξ_j, we find the variational equations involving the soliton parameters as follows:

$$\frac{\delta\langle L\rangle}{\delta D_j}=0 \Rightarrow \frac{dA_j}{dt}=0 \tag{29}$$

$$\frac{\delta\langle L\rangle}{\delta \xi_j}=0 \Rightarrow -\frac{2}{\varepsilon}\frac{dA_jC_j}{dt}+\frac{\partial\langle L_{xy}\rangle}{\partial \xi_j}=0 \tag{30}$$

$$\frac{\delta\langle L\rangle}{\delta C_j}=0 \Rightarrow \frac{d\xi_j}{dt}+(-1)^j\Delta V-\omega'' C_j=0 \tag{31}$$

$$\frac{\delta\langle L\rangle}{\delta A_j}=0 \Rightarrow \frac{dD_j}{dt}=\frac{\omega''}{2}C_j^2+\left(\frac{2R}{\varepsilon}-\varepsilon\omega''\right)A_j^2 + \frac{\varepsilon}{2}\frac{\partial\langle L_{xy}\rangle}{\partial A_j}=0 \tag{32}$$

where $\langle L\rangle=\langle L_x\rangle+\langle L_y\rangle+\langle L_{xy}\rangle$.

Let us examine the physical implications of the above equations for the case of symmetric solutions, i.e $A_x=A_y$. Eq. (29) shows that the amplitudes of two solitons will be preserved during their propagation ($A_x=A_y=A=const$). From Eq. (30) and Eq. (31), we find the following relations given by $C_x=-C_y=C$ and

$$\frac{dC}{dt}=\frac{\varepsilon^2}{2}\frac{\partial\langle L_{xy}\rangle}{\partial\zeta}$$
$$\frac{d^2\zeta}{dt^2}=2\varepsilon A\,\omega''\frac{dC}{dt} \tag{33}$$

The above equations suggest the possibility of a potential well due to the interaction between two modes, $\langle L_{xy}\rangle$. We can formulate a potential energy function from the energy conservation law:

$$\frac{1}{2}\left(\frac{d\zeta}{dt}\right)^2+U(\zeta)=const \tag{34}$$

where the first term corresponds to a kinetic energy, and $U(\zeta)$ is responsible for a potential energy. From Eq. (28), Eq. (33), and Eq. (34), we know that the potential energy function is

$$U(\zeta)=2\varepsilon^3\omega'' A(\langle L_{xy}(0)\rangle-\langle L_{xy}(\zeta)\rangle) =\frac{8}{3}\varepsilon^2\omega'' RA^4\left(\frac{1}{3}-\frac{\zeta\coth\zeta-1}{\sinh^2\zeta}\right). \tag{35}$$

The configuration of $U(\zeta)$ on a normalized scale is shown in Fig. 4 where the distance between two peaks of the solitons ζ is normalized by a spatial width of the soliton. Unless the kinetic energy exceeds the limit of potential energy, the two orthogonally polarized modes can be confined by the potential well. For a small ζ,

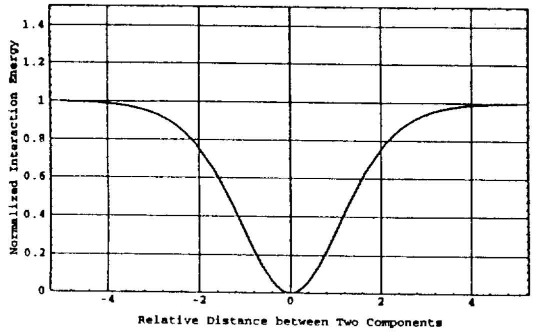

Fig 4. Configuration of a potential well due to the nonlinear interaction of two orthogonally polarized modes.

$$U(\zeta)=\frac{16}{45}\varepsilon^2\omega'' RA^4\zeta^2 \tag{36}$$

which means that the two modes confined in the potential well will oscillate and form a bound state. Eq. (31) describes how the centers of the solitons will evolve temporally in a nonlinear birefringent periodic medium. As far as the solitary-waves are preserved, they should be on the frame moving at $\overline{v_g}$. The two modes propagate with the group velocities, $\overline{v_g}+\omega'' C(t)+\Delta V$, and *vice versa*, alternating in the lead. It is noted that $\omega'' C(t)$ compensates ΔV, i.e. the group velocity mismatch due to the birefringence.

We can now find the threshold condition for the bound state of the two orthogonally polarized modes. It is equivalent to the condition that a particle be confined to the potential well given in Eq. (36). That is $[1/2\,(d\zeta/dt)^2]_{\max}\le U_{max}$. Since $U_{max}=8/9\varepsilon^2\omega'' RA^4$ and $(d\zeta/dt)_{\max}=2\varepsilon A\Delta V$ at $t=0$, we obtain the threshold condition given as follows:

$$A_{thresh}\ge\frac{3}{2}\Delta V/\sqrt{\omega'' R}. \tag{37}$$

However, the amplitude given the above is not the physical quantity in the actual waveguide. As mentioned previously, the peak amplitude of a stationary-gap soliton is the least condition for the formation of a gap soliton, and thus the threshold condition for the dragging phenomena in the nonlinear periodic waveguide is given by

$$A=\sqrt{\frac{3}{5}}A_{stat}+A_{thresh}$$
$$=\sqrt{\frac{3}{5}}\sqrt{2D/\alpha_m}+\frac{3}{2}\,\Delta V/\sqrt{\omega''R}. \qquad (38)$$

Let us find out how much birefringence in the actual waveguides is required for the neglection of AFWM terms. Then we will find a threshold intensity for the dragging phenomena of coupled gap solitons in case that such birefringence exists in the periodic waveguide. Consider two orthogonally polarized sech pulses of $\tau_p=100$ psec at the detuning of $D_N=3\%$ in the NPDM with $\Delta n/\bar{n}=0.1\%$. From some numerical calculations, we know that higher peak intensity than $I=0.175$ GW/cm^2 is required to generate the coupled gap solitons with the above conditions. In absence of birefringence, the group velocity of the synchronized gap solitons will be $v_g=4.21\times10^6$ m/sec and thus the spatial width will be $\sigma_p=0.421$ mm. We take the waveguide length as $L=8.42$ mm, which is 20 times as large as the spatial width. Consequently, the time taken by the coupled gap solitons to propagate through the NPDM will be approximately $t_l\equiv L/v_g=2\times10^{-9}$ sec. At the present time, we need to introduce a beat time in a birefringent periodic medium defined as

$$t_B\equiv 2\pi/2|\omega_{mx}-\omega_{my}|. \qquad (39)$$

Since $\omega_m=4.069\times10^{15}/\bar{n}$ in this case, one can rewrite t_B as

$$t_B=7.721\times10^{-16}/|\frac{1}{\bar{n}_x}-\frac{1}{\bar{n}_y}|. \qquad (40)$$

However, since we need $t_l\gg t_B$ for the neglection of AFWM terms, it follows from the above relations that $|\bar{n}_y-\bar{n}_x|\gg 4.042\times10^{-6}$. Consequently, we conclude that the birefringence of $|\bar{n}_y-\bar{n}_x|=6\times10^{-5}$ will be enough for the neglection of AFWM terms.

Suppose that $\bar{n}_x=3.23630$ and $\bar{n}_y=3.23636$ for the above birefringence. We note that a band-edge frequency in each polarization, ω_m will change in inverse proportion to its average index, $\bar{n}$. Thus, in this case, the detunings will be given as $D_{Nx}=(\omega_{mx}-\omega_c)/\Delta\omega=0.03618$ (i.e. 3.618%), and $D_{Ny}=(\omega_{my}-\omega_c)/\Delta\omega=0.01763$(i.e. 1.763%). Thus we know $V_{Nx}=0.04158$ and $V_{Ny}=0.05852$, giving $v_{gx}=3.8544\times10^6$ m/sec and $v_{gy}=5.4243\times10^6$ m/sec, respectively. Here we found another unusual property of gap soliton that if the identical gap solitons in orthogonally polarized modes are launched into a birefringent periodic medium, the gap soliton in the slow axis will travel faster than that in the fast axis. The fundamental reason is as follows. Due to the birefringence in a waveguide, the stop gap in the slow-axis occupies a lower location than that in the fast-axis does, which means the gap soliton in the slow-axis has the smaller detuning than that in the fast-axis for the same center frequency. As a result, the group velocity of a gap soliton in the slow-axis is larger than that in the fast-axis if both gap solitons are identical. Substituting $\Delta V=(v_{gx}-v_{gy})/2$ and the associated parameters into Eq. (38), one can obtain the threshold condition for the dragging phenomena given by $I>0.1883$ GW/cm^2. However, if $0.175<I<0.1883$ GW/cm^2, then the gap solitons in two orthogonally polarized modes will propagate with different average group velocities, giving rise to the break-up of coupled-gap solitons. Note that any kinds of coupled-gap solitons can not be formed if $I<0.175$ GW/cm^2 for the detuning given

Table 1. The threshold conditions for dragging phenomena in some specific birefringent periodic waveguides.

	$\Delta n/\bar{n}=0.1\%$		$\Delta n/\bar{n}=0.2\%$	
	$\tau_p=100$ psec	$\tau_p=10$ psec	$\tau_p=100$ psec	$\tau_p=10$ psec
Normalized Detuning(%)	3	30	2	20
Center Angular Frequency ($\times10^{15}$ 1/sec)	1.25723	1.25689	1.25785	1.25739
Spatial Width(mm)	0.421	0.134	0.26	0.082
Waveguide Length(mm)	8.44	4	5.2	2.463
Required Birefringence	4×10^{-5}	3×10^{-4}	4×10^{-5}	3×10^{-4}
Peak Intensity(GW/cm^2)	0.19	1.79	0.25	2.34
Average Group Velocity(m/sec)	4.639×10^6	1.346×10^7	2.619×10^6	8.251×10^6

above.

Following a similar procedure, we can also calculate for the case of $\tau_p=10$ psec in the waveguide with a modulation depth of $\Delta n/\bar{n}=0.1\%$, and for the cases of $\tau_p=100$ psec and 10 psec in the waveguide with a modulation depth of $\Delta n/\bar{n}=0.2\%$. The results are summarized in Table 1 where the waveguide lengths L's for $\tau_p=100$ psec and for $\tau_p=10$ psec are taken as 20 times and 30 times as large as their spatial widths, respectively. From Tab. 1, we know that a relatively strong birefringence is required for the neglection of the AFWM terms. Comparing the waveguide of $\Delta n/\bar{n}=0.1\%$ with that of $\Delta n/\bar{n}=0.2\%$, we find that, in case of $\Delta n/\bar{n}=0.2\%$, the smaller detuning and the higher peak intensity is needed to generate the coupled gap solitons with the same temporal width. On the whole, the threshold intensities for the dragging phenomena are quite near the boundaries of the detuning conditions such that, at the utmost, only a little portion of the pulse spectrum penetrates into the allowed band. The locities of decoupled gap solitons in the birefringent periodic waveguides. As mentioned in Sec. IV, the group velocities in the each polarized mode of coupled gap solitons will change periodically due to the potential well formed by the cross-coupling process of $\chi^{(3)}$ nonlinearity. However, the whole unit of coupled gap solitons will move with the group velocities given in Table 1.

V. Conclusion and discussion

Our studies for a novel switching phenomena have proceeded by the following steps; (i) Coupled NLSEs and coupled gap soliton as a principle of opto-optical switching phenomena; (ii) A novel all-optical switching scheme as a application; (iii) Threshold conditions for the dragging phenomena as a stability analysis of coupled gap solitons in NPDM with birefringence. By means of an envelope-function approach, we found the coupled NLSE governing the spatio-temporal evolutions of two orthogonally polarized modes in NPDM. If we employ a coupled mode formalism[4,17], we need four coupled equations to describe the propagations of two counterpropagating modes for each polarized mode. In this case, a pair of propagation equations for each polarized mode will be linearly coupled owing to a periodic perturbation in the refractive index. Moreover, each propagation equation will be nonlinearly coupled with itself and three other equations by self-phase modulation (SPM), cross-phase modulation (XPM), and various AFWM terms. Depending upon whether the polarizations of interacting modes are in parallel or orthogonal, the nonlinear coefficients of XPM terms will be 2 or 2/3 times as big as that of SPM term, respectively. We found that it would be extremely difficult to obtain four solitary-wave solutions by means of a coupled mode formalism even if all AFWM terms can be neglected under strict assumptions.

In conclusion, we have proposed a compact all-optical switch and also investigated its stability. Its implementation is simple because the main part is consisted simply of two PBSs and a switching element-GaAs waveguide, which is compact in size. Moreover the proposed device can potentially exhibit good switching performances, including good switching speed with reasonable pulse width, and good on-off intensity contrast ratio. From a practical viewpoint, however, we note that much higher input power than I would be required for the generation of gap solitons because of the reflections at the interfaces between the Bragg grating region and the surrounding region. Provided that the above technical problem can be solved, a gap soliton as well as coupled gap solitons will have a great prospect for a wide variety of exploitations.

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