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Reflection of a Gaussian Beam from a Planar Dielectric Interface

Yeon H. Lee
Electronics Department, Sung Kyun Kwan University, Suwon, 440-746, Korea

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When a Gaussian beam is incident to a planar dielectric interface at an angle other than Brewster angle or the critical angle of total reflection, we derive the six nonspecular effects of rotation, lateral shift, focal shift, Rayleigh length change, magnitude and phase changes in the complex amplitude of the reflected beam simultaneously by taking account of the boundary condition. In the derivation we assume a Gaussian beam of fundamental mode to emerge from the interface and then match at the interface the constant, linear, and quadratic variations of the amplitude and phase of the reflected beam with those of the incident beam multiplied by the reflection coefficient. Our calculation shows that the six nonspecular effects can result from a linear variation of the natural logarithm of the reflection coefficient at the interface.

I. INTRODUCTION

When a Gaussian beam with curved phase front is incident to a planar dielectric interface, the incident angle of the phase front may vary as a function of position at the interface. Since Fresnel formula of reflection varies rapidly near the critical angle of total reflection,[1] the reflection coefficient at the interface may vary rapidly as a function of position in this case. Since, in general, the reflection coefficient is given by a complex number, it may cause changes in the amplitude and phase of the beam reflected from the interface.

It has been shown by many researchers that the changes can make the reflected beam appear not only to be shifted in the lateral direction as in Goos-Hänchen shift,[2] but also to be rotated,[3,4] shifted in the longitudinal direction,[5,6] modified in the beam waist,[7] changed in the magnitude and phase of the complex amplitude.[8] In most of the previous studies angular spectrum analysis has been widely employed in predicting consequences of the changes in the amplitude and phase of the reflected beam. In this case, an incident Gaussian beam is represented by a superposition integral of angular spectra. Then the reflected beam is reconstructed from the angular spectra of the incident beam multiplied by the complex reflection coefficient. Improvements in the predictions have been made in the limit where a ratio of beam waist to wavelength is not large.[9] However, in this case, the reflected beam is reconstructed from the angular spectra by ignoring the longitudinal amplitude and phase changes of the incident and reflected Gaussian beams.

Since the nonspecular effects are very small in general, there are many difficulties in experiments. Several experiments however have been performed in microwave region where the effects appear relatively large.[10] And despite the difficulties some experiments have been done in optical wavelength region.[11,12] However, in these experiments, the lateral shift and the rotation of the reflected beam have been observed as a single event due to the experimental difficulties.

In this paper, we employ a new method other than the angular spectrum analysis. We first take the natural logarithm of Fresnel formula of reflection and then expand it in Taylor series at the interface assuming the beam incident angle to be other than Brewster and the critical angles. Next, we obtain the amplitude and phase distributions of the electric field at the interface from the electric field of the incident beam and the complex reflection coefficient. To solve this boundary value problem, we assume, emerging from the interface is a Gaussian beam of fundamental mode that is rotated relative to the reflection direction predicted by geometrical optics, shifted in the transverse and longitudinal directions, changed in its Rayleigh length, and changed in the magnitude and phase of its complex amplitude. By matching the constant, linear, and quadratic variations of both amplitude and phase of the reflected beam at the interface with the given boundary values we obtain the six nonspecular effects simultaneously.

II. THE ELECTRIC FIELD DISTRIBUTION AT THE DIELECTRIC INTERFACE

Fig. 1 shows three coordinate systems (x, z), (ξ, η), and (ξ', η'). Here (x, z) coordinates are for a planar dielectric interface, which is represented by xy plane. To the interface a Gaussian beam of fundamental mode

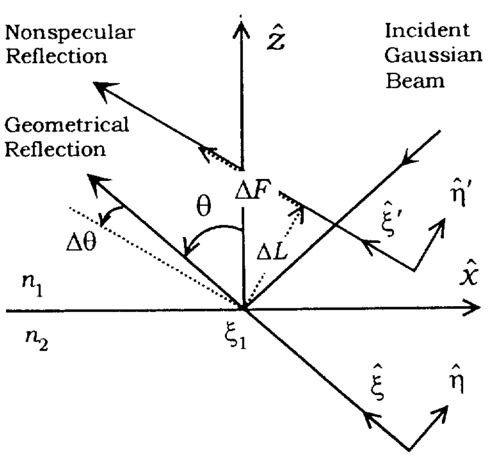

Fig. 1. A Gaussian beam is incident on a dielectric interface, which is represented by xy plane. A unit vector $\hat{\xi}$ represents the reflection direction predicted by geometrical optics. A unit vector $\hat{\xi}'$ represents the propagation direction of the beam reflected from the interface. (ξ', η') coordinates can be obtained from (ξ, η) coordinates by rotating it by $\Delta\theta$ at $\xi=\xi_1$ and then by displacing it in the transverse and longitudinal directions by ΔL and ΔF, respectively. The origin of (ξ, η) coordinates corresponds to the location of the mirror image of the incident beam waist.

is assumed to incident at an angle of θ relative to the surface normal, or z-axis. (ξ, η) coordinates are to represent a simple geometrical reflection of the incident beam. Since there is nothing changed in a beam after the geometrical reflection, except for its propagation direction, from the incident beam, we assume at the interface that the scalar-electric-field distribution of a beam propagating along $\hat{\xi}$ axis can be obtained from that of the incident beam multiplied by the reflection coefficient of unity. It is assumed to be unity for simplicity in calculation. (ξ', η') coordinates are for nonspecular reflection of the incident beam in which the reflection coefficient is assumed to vary as a function of position at the interface.

The incident Gaussian beam is assumed to have propagated a distance ξ_1 from its beam waist before it reaches the interface. If the reflection coefficient, for the curved phase front of the incident beam, is assumed to be uniform at the interface (unity for simplicity), the incident beam should reflect in the direction predicted by geometrical optics and should have its beam waist at the distance of ξ_1 from the interface in $-\hat{\xi}$ direction. In this case the electric field of the beam is given in (ξ, η) coordinates as

$$\varepsilon_{gr}(\xi, \eta)=\frac{1}{2}E_o\exp\left[i\frac{k\eta^2}{2q}-\ln\left(1+i\frac{\xi}{\xi_o}\right)+ik\xi\right]$$

$$\exp(-i\omega t)+c.c. \quad (1)$$

Here gr represents that the beam propagates in the direction predicted by geometrical optics. E_o is complex amplitude. k is wavenumber given by $2\pi n_1/\lambda$ with refractive index n_1 and wavelength λ. q is Gaussian beam parameter given by $\xi-i\xi_o$ with Rayleigh length ξ_o. ω is angular frequency. c.c represents complex conjugate.

Fig. 2 shows a Gaussian beam propagating along $\hat{\xi}$ axis after reflecting from the dielectric interface, where the reflection coefficient is assumed for a moment to be unity, and its phase front at a position $\xi=\xi_2$. As can be seen from the figure the interface plane is represented in (ξ, η) coordinates by the equation $\eta+(\xi-\xi_1)\cot\theta=0$. Therefore, the electric field distribution of the beam at the interface can be obtained from (1) by replacing η with $-(\xi-\xi_1)\cot\theta$. Since the reflection coefficient is assumed to be unity in the calculation, the derived electric field should be the same with the electric field distribution of the incident beam at the

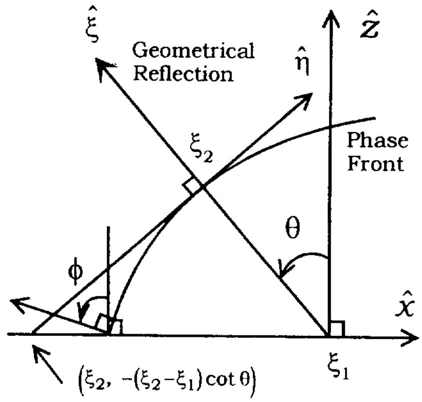

Fig. 2. In (ξ, η) coordinates the dielectric interface is represented by the equation $\eta=-(\xi-\xi_1)\cot\theta$. When the incident Gaussian beam is assumed to follow the path of the geometrical reflection, the propagation direction of its phase front at a point of the dielectric interface will make an angle of ϕ relative to the surface normal.

interface. Therefore,

$$\varepsilon_i(x)=\frac{1}{2}E_o\exp\left[i\frac{k}{2q}(\xi-\xi_1)^2\cot^2\theta - \ln\left(1+i\frac{\xi}{\xi_o}\right)+ik\xi\right]\exp(-i\omega t)+c.c. \quad (2)$$

In the equation the subscript i stands for incident beam and the argument x is to represent the electric field distribution at the interface. The electric field can also be given by (x, z) coordinates after the coordinate transformation; $(\xi-\xi_1)=-x\sin\theta$ and $\eta=x\cos\theta$ (see Fig. 1). Since, however, the transformation can be done at any later time of our convenience, it will be ignored for the moment.

When the reflection coefficient varies as a function of position at the dielectric interface, we can assume that the electric field distribution of the reflected beam at the interface is given by that of the incident beam multiplied by the reflection coefficient $\rho(x)$;

$$\varepsilon_{nsr}(x)=\rho(x)\ \varepsilon_i(x). \quad (3)$$

Here the subscript nsr represents nonspecular reflection and the argument x represents that the corresponding functional values are taken at the interface. For simplicity in calculation we ignore the variations of both the electric field and the reflection coefficient in y direction.

III. THE NONSPECULAR REFLECTION AT THE INTERFACE

It can be seen from Fig. 1 that the coordinate system (ξ', η') can be obtained from (ξ, η) coordinate system by rotating it about a point (ξ_1, O) by an angle of $\Delta\theta$ followed by translations of ΔL and ΔF in the transverse and longitudinal directions, respectively. Therefore (ξ', η') coordinates are given in terms of (ξ, η) coordinates as

$$\xi'=\xi_1+\cos(\Delta\theta)(\xi-\xi_1)-\sin(\Delta\theta)\ \eta-\Delta F, \quad (4)$$

$$\eta'=\sin(\Delta\theta)(\xi-\xi_1)+\cos(\Delta\theta)\ \eta-\Delta L \quad (5)$$

Here we assume ξ' axis to represent the propagation direction of the beam reflected from the dielectric interface where the reflection coefficient varies as a complex function of position. In other words, when the electric field distribution at the interface is given as in (3), we assume, emerging from the interface is a Gaussian beam of fundamental mode that is rotated by $\Delta\theta$ relative to the direction $\hat{\xi}$ predicted by geometrical optics, displaced by ΔL and ΔF in the transverse and longitudinal directions, changed in the Rayleigh length by $\Delta\xi_o$, and changed in the magnitude and phase of the complex amplitude. By so doing we can avoid solving the wave equation with complicated boundary conditions. It can be shown that the six operations on a Gaussian beam still make the beam a solution of the wave equation. Therefore the beam after the nonspecular reflection can be assumed to be of the form of

$$\varepsilon_{nsr}(\xi', \eta')=\frac{1}{2}E_o'\exp\left[i\frac{k\eta'^2}{2q'}-\ln\left(1+i\frac{\xi'}{\xi_o'}\right)+ik\xi'\right]\exp(-i\omega t)+c.c. \quad (6)$$

In the equation E_o' represents the changed magnitude and phase in the complex amplitude, ξ_o' the changed Rayleigh length, ξ' the changed beam direction and position as can be seen from (4) and (5), and q' the changed Gaussian beam parameter given by $\xi'-i\xi_o'$.

Since the electric field (6) is assumed to be a solution of the wave equation after the nonspecular reflection, what we need next is to consider boundary conditions. When $\Delta\theta$ is much smaller than unity, $\sin(\Delta\theta)$ and $\cos(\Delta\theta)$ in (4) and (5) can be approximated by $\Delta\theta$ and unity, respectively. After inserting (4) and (5) into (6) we expand all the primed quantities of (6) in Taylor series and collect the zero and the first order terms. It is noted in this case that the zero order terms are given by (ξ, η) coordinates and the first order terms are given by the six quantities representing the six nonspecular effects: rotation $\Delta\theta$, lateral shift ΔL, focal shift ΔF change in Rayleigh length $\Delta\xi_o$, magnitude and phase changes ΔE_o in the complex amplitude. For simplicity in calculation ΔE_o is rewritten as $\exp(A_r+iA_i)$. Then the electric field distribution after the nonspecular reflection is obtained at the interface as

$$\begin{aligned}\varepsilon_{nsr}(x)=&\frac{1}{2}E_o exp\left\{i\frac{k}{2q}(\xi-\xi_1)^2\cot^2\theta\right.\\&\left.-\ln\left(1+i\frac{\xi}{\xi_o}\right)+ik\,\xi\right\}\\&\cdot\exp\left\{-i\frac{k}{q}\Big[(\xi-\xi_1)(\Delta\theta)-\Delta L\Big](\xi-\xi_1)\cot\theta\right\}\\&\cdot\exp\left\{i\frac{k}{2q^2}\Big[-(\Delta\theta)(\xi-\xi_1)\cot\theta+\Delta F+i(\Delta\xi_o)\Big]\right.\\&\left.(\xi-\xi_1)^2\cot^2\theta\right\}\\&\cdot\exp\left\{\left(\frac{1}{q}-ik\right)\Big[-(\Delta\theta)(\xi-\xi_1)\cot\theta+\Delta F\Big]\right.\\&\left.+\left[i\frac{1}{q}+\frac{1}{\xi_o}\right](\Delta\xi_o)\right\}\\&\cdot\exp(A-i\omega t)+c.c.\end{aligned}\tag{7}$$

In the equation replacements of ξ with ($\xi_1-x\sin\theta$) has been postponed for some later time. The first exponential term results from the zero order terms in Taylor series and describes the amplitude and phase distributions at the interface in the absence of any nonspecular effect. The second exponential term results from the perturbation in η in (6). The third exponential term results from the perturbation in q and the fourth term from the perturbations in ξ and ξ_o in (6).

The boundary condition requires that the electric field of the reflected beam at the interface (7) should be the same with that of the incident beam multiplied by the reflection coefficient as in (3). Here we assume that the complex reflection coefficient $\rho(x)$ can be rewritten as $\exp[\ln\rho(x)]$ and that $\ln\rho(x)$ can be expanded in Taylor series. We note that this condition can be satisfied at all the incident angle except for Brewster angle and the critical angle of total reflection.

$$\ln\rho(x)=\ln\rho(0)+\frac{1}{\rho}\frac{d\rho}{dx}\bigg|_{at\,x=0}\cdot x\equiv R+Lx \tag{8}$$

To match the boundary condition (2), (7) and (8) are inserted into (3) and then ($\xi-\xi_1$) is replaced by $-x\sin\theta$. In this case the Gaussian beam parameter q is rewritten as $(\xi-\xi_1)+(\xi_1-i\xi_o)$.

Next we collect terms with the same power of x, which are terms of constant, x and x^2. Since we are dealing with complex numbers, the real and imaginary parts of the terms with the same power of x should satisfy the equality of (3) independently. Therefore we obtain six simultaneous equations with six unknowns. After a lengthy algebra we obtain the exact solutions as

$$\Delta\theta=-\frac{1}{(k\xi_o-1)\cos\theta}\left[L_r\xi_1+L_i\left(\xi_o+\frac{2\tan^2\theta}{k}\right)\right], \tag{9}$$

$$\Delta\xi_o=\frac{2\sin\theta}{k\cos^2\theta}[L_r\xi_1+L_i\,\xi_o], \tag{10}$$

$$\Delta F=\frac{2\sin\theta}{k\cos^2\theta}[L_r\,\xi_0-L_i\,\xi_1], \tag{11}$$

$$\Delta L\frac{1}{(k\,\xi_o-1)\cos\theta}\left[L_r\left(\xi_1^2+\xi_0^2+\xi_0\frac{2\tan^2\theta-1}{k}-\frac{2\tan^2\theta}{k^2}\right)+L_i\xi_1\frac{2\tan^2\theta+1}{k}\right] \tag{12}$$

$$A_r=R_r-\frac{2\sin\theta}{k\cos^2\theta}L_r\frac{\xi_1}{\xi_o}, \tag{13}$$

$$A_i=R_i+\frac{2\sin\theta}{k\cos^2\theta}[L_r(k\,\xi_o-1)-L_ik\,\xi_1], \tag{14}$$

Here the subscripts r and i represent real and imaginary parts, respectively. It should be noted here that no term is assumed to be small and neglected in solving the six simultaneous equations.

IV. THE REFLECTION COEFFICIENT AT THE DIELECTRIC INTERFACE

Since the Gaussian beam incident to the dielectric interface has a curved phase front, the incident angle of the phase front varies at the interface. When the reflection coefficient is given as a function of incident angle, it can be converted into a function of position as follows. First, we obtain propagation directions of different points in the phase front. These points will then correspond to different points at the interface, which will have different values of reflection coefficient due to the different incident angle of the phase front. Since the electric field (1) is obtained by assuming the reflection coefficient of unity, the reflection angle of the phase front at a given point of the interface should be the same with the corresponding incident angle of the phase front. Therefore, at the interface, the variation of the incident angle of the phase front can be obtained from (1) by taking gradient of its phase term. When the condition $\pi^2 W_o^2 \gg \lambda^2$ is satisfied, where W^2 is the beam waist, the incident (or reflection) angle ϕ of the phase front (see Fig. 2) can be obtained at a point $(x, 0)$ of the interface as

$$\phi \approx \theta - \frac{(\xi_1 - x\sin\theta)x\cos\theta}{[(\xi_1 - x\sin\theta)^2 + \xi_o^2]} \tag{15}$$

Here θ is the incident angle of the Gaussian beam measured between the beam center and the surface normal, or z-axis. The angle ϕ is also measured with respect to the surface normal.

Using Fresnel formula of $\rho(\phi)$ and the chain rule $d\rho(\phi)/dx = (d\rho(\phi)/d\phi)(d\phi/dx)$ we obtain the real and imaginary parts of L for σ polarization, where the electric field is perpendicular to the plane of incidence, as

$$L_r = -\frac{2n_1\cos\theta\sin\theta}{\sqrt{n_2^2 - n_1^2\sin^2\theta}}\frac{\xi_1}{(\xi_1^2 + \xi_o^2)}, \tag{16}$$

$$L_i = \frac{2n_1\cos\theta\sin\theta}{\sqrt{n_1^2\sin^2\theta - n_2^2}}\frac{\xi_1}{(\xi_1^2 + \xi_o^2)}, \tag{17}$$

Similarly for π polarization, where the electric field is in the plane of incidence, we obtain

$$L_r = -\frac{2n_1 n_2^2\cos\theta\sin\theta}{\sqrt{n_2^2 - n_1^2\sin^2\theta}}\frac{1}{[(n_1^2 + n_2^2)\sin^2\theta - n_2^2]}\cdot\frac{\xi_1}{(\xi_1^2 + \xi_o^2)}, \tag{18}$$

$$L_i = \frac{2n_1 n_2^2\cos\theta\sin\theta}{\sqrt{n_1^2\sin^2\theta - n_2^2}}\frac{1}{[(n_1^2 + n_2^2)\sin^2\theta - n_2^2]}\frac{\xi_1}{(\xi_1^2 + \xi_o^2)}, \tag{19}$$

In the equations the refractive index n_1 is assumed to be larger than n_2. The square root in the denominator of (16)-(19) becomes zero at the critical angle of total reflection and the bracket in the denominator of (18)-(19) becomes zero at Brewster angle. This means blowing up of L_r and L_i at these points. It should be noted that, at an incident angle less than the critical angle of total reflection, $L_i = 0$ and L_r is given by (16)

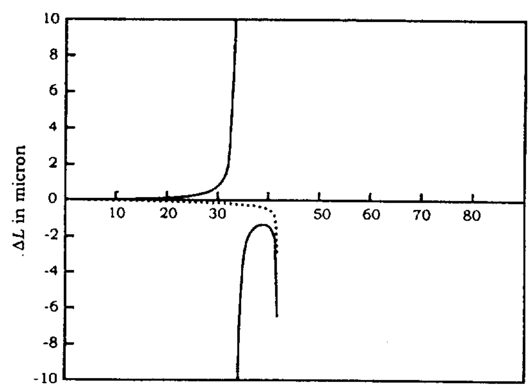

Fig. 3. The lateral shift ΔL is plotted versus the incident angle θ in degree for refractive indices $n_1 = 1.5$, $n_2 = 1.0$, beam waist $w_o = 1$ mm, and wavelength $\lambda = 632.8$ nm. In this case Brewster angle $\theta_B = 33.6899°$ (0.58800 radian) and the critical angle of total reflection $\theta_c = 41.8104°$ (0.72973 radian). Solid line represents π-polarized beam while dotted line σ-polarized beam. The plot stops when the incident angle becomes closer to Brewster angle or the critical angle by less than 0.0573° (1 mrad).

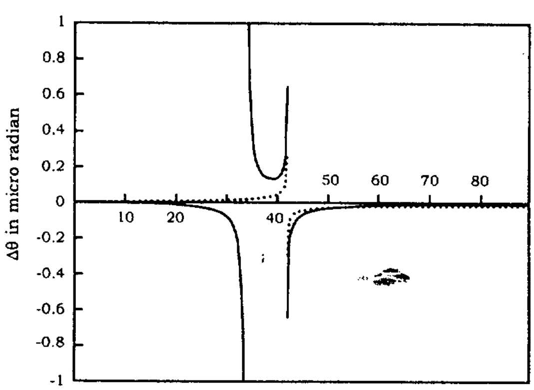

Fig. 4. The rotation $\Delta\theta$ versus the incident angle θ.

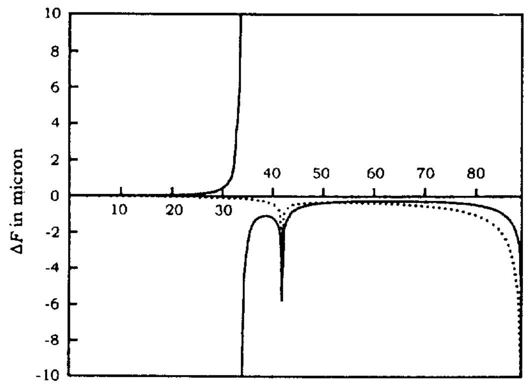

Fig. 5. The focal shift ΔF versus the incident angle θ.

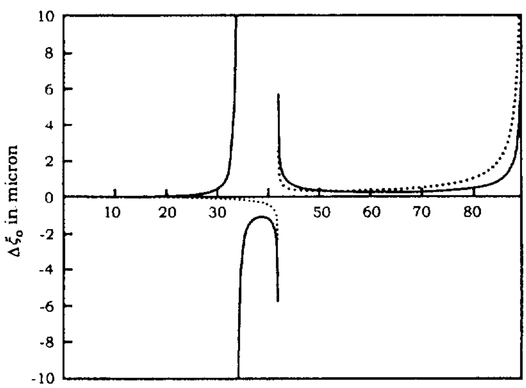

Fig. 6. The change in Rayleigh length $\Delta\xi_o$ versus the incident angle θ.

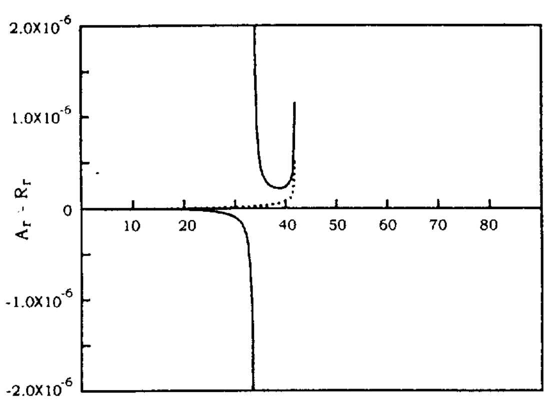

Fig. 7. The exponential of (A_r-R_r) represents a change in the beam amplitude from that predicted in geometrical reflection.

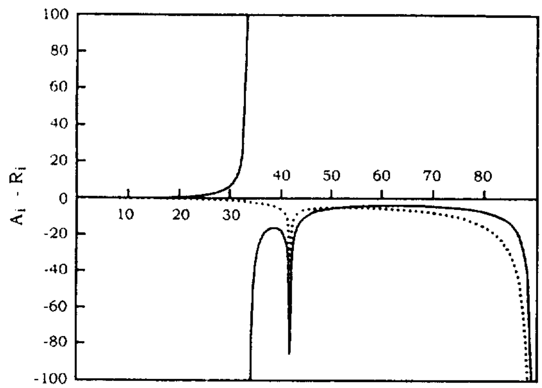

Fig. 8. The phase change of the complex amplitude versus the incident angle θ.

or (18) depending on the beam polarization. Similarly, at an incident angle larger than the critical angle, $L_r=0$ and L_i is given by (17) or (19) depending on the beam polarization.

The six nonspecular effects are plotted in Fig. 3-Fig. 8 as functions of the incident angle θ in degree. In the plots a Gaussian beam of 632.8 nm is assumed to have propagated a distance of ξ_o from its beam waist before it reaches the dielectric interface, which means $\xi_1=\xi_o$. Also assumed in the plots are the beam waist $w_o=1$ mm, refractive indices $n_1=1.5$ and $n_2=1.0$.

V. CONCLUSION

When a Gaussian beam of fundamental mode is incident at a dielectric interface, the different propagation direction of the phase front at the interface is converted into different reflection coefficient by Fresnel formula. Then the electric field distribution of the reflected beam at the interface is assumed to be given by that of the incident beam multiplied by the complex reflection coefficient varying as a function of position. For this boundary value problem we assume a Gaussian beam of fundamental mode to emerge from the interface. In this case the emerging beam is assumed to contain the six nonspecular effects. By matching the electric field distribution of the emerging beam with the boundary value we obtain the exact values of the six nonspecular effects simultaneously. Our calculation shows that a linear variation of ln $\rho(x)$ at the interface produces all the six nonspecular effects at the same time.

It appears that our results (9)-(14) are different from those of Falco and Tamir[9] in two respects. First, in our derivations, there is no ignorance of the longitudinal amplitude and phase changes in the expression of the incident and reflected Gaussian beams. Therefore, our results are given as functions of ξ_1 which represents the distance between the incident beam waist and the dielectric interface. Second, our results are given as functions of the incident angle θ in addition to the angular dependence involved in the reflection coefficient, L_r and L_n.

Since, in our calculation, the complex reflection coefficient $\rho(x)$ is rewritten as $\exp[\ln \rho(x)]$ as in (8), the calculation result is good for all values of the incident angle except for the Brewster angle and the critical angle of total reflection, where $\exp[\ln \rho(x)]$ diverges. In the figures drawing has stopped when the incident angle is closer to the Brewster or critical angle by less than 0.001 radian (or 0.0573 degree). Therefore the figures show that the nonspecular effects diverge faster near the Brewster angle than near the critical angle.

We believe that the divergence of our result at the Brewster angle results from the fact that a Gaussian beam of fundamental order is assumed to emerge from the interface in our calculation even if higher order Gaussian beams may be excited in this case. More study is being done and its result is planned to be published elsewhere.

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