Bilinear Model Predictive Control Methods for Chemical Processes

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1. Introduction

Design of a control system implies selection of suitable measurements, identification of the process models and choice of the appropriate control law. Much research in the field of process control has been primarily focused on the design of a control system capable of maintaining the process at its optimal steady state despite changing various operating conditions. But there has been a great need of a more effective control technique due to the increasing energy and raw materials costs and the increasing complexity of processes. Moreover, many processes are intrinsically nonlinear, and the use of linear control methods to handle the nonlinear processes has been restricted within a certain region. Thus it is essential to develop a new control technique applicable to nonlinear systems.

Recently, rapid development of digital computer technology has made it possible to implement more sophisticated control methods. The main advantages of the computer control over classical analog control lie in the operating cost and the flexibility. A model predictive control method(MPC) has recently received much attention as one of the computer control techniques which meet today's need for more effective control strategy. The MPC method is characterized by two features ; the use of a proper plant model in the control system structure and the predictive nature of a control algorithm. It employs the model to predict the process conditions which are used to compute proper control actions. The MPC method was subsequently developed [1-6] and applied successfully to several industrial processes involving multivariable process dynamics[7].

In practice, operating conditions change with time, and physical parameter and dynamic characteristics of processes are poorly known. As a promising strategy which is potentially applicable in these situations, adaptive control techniques has been developed. In the adaptive control system the model parameters are adjusted at each sampling time to compensate for the significant changes in process characteristics. So far, a great many adaptive control methods have been proposed, but only a few of them use the great advantage of MPC scheme. Moreover, since most of the adaptive model predictive control(AMPC) methods developed so far are based on the linear system models, they cannot handle nonlinear situations which arise especially in the control of chemical engineering processes.

Recently, the class of bilinear model has been introduced as a useful tool for examining many nonlinear phenomena. The bilinear models are nonlinear jointly with respect to the state and the input but linear separately, and their structural properties are similar to those of linear models. Many successful application results summarized by Mohler and Kalodzies[8] illustrate the effectiveness of the use of bilinear models as approximations of nonlinear systems. This paper endeavours to present various bilinear models and predictive control schemes developed so far. Couple of bilinear controller design methods and examples of chemical process controls are shown to help understanding.

2. Bilinear Systems

Linear systems are described by linear differential or algebraic equations and the principle of superposition applies. Nonlinear systems are described by complex nonlinear differential equations and linear approximation methods have been used in the control of the nonlinear systems. However, the intrinsic limits of the use of linear models appear more and more evident. Usually linear approximation of a nonlinear system is possible only when the behavior of the system is confined in the region around some normal operating levels. As an approximation to general nonlinear plant, the bilinear model can provide a more accurate representation than linear one[6]. It has been reported[9] that for a general plant in which the control appears linearly, dynamically equivalent bilinear model can be found. The recent applications of bilinear models studied so far were summarized in Table 1.

Table 1. Recent applications of bilinear models.

Process	Control algorithm	Reference
Waste water treatment	Self-Tuning minimum variance	Goodwin et al., 1982 [10] Goodwin and Sin, 1984 [11]
Blood pressure	Self-Tuning pole assignment	McInnis et al., 1985 [12]
pH process	Self-Tuning pole assignment	Gilles and Laggoune, 1985 [13,14]
Fermentation	Self-Tuning minimum variance	Dochain and Bastin, 1984 [15]
Cancer drugs	Optimal control	Biran and McInnis, 1979 [16]
Solar system	Optimal control	Wang and Dorato, 1983 [17]
CSTR	Optimal control	Cebuhar and Costanza, 1984[15]

2.1 Review on Bilinear Models

Recently extensive studies have been done on the structure of bilinear systems. One of many appealing features of a bilinear structure is that its equations(e.g., state-space or autoregressive moving average(ARMA) equations) describe approximately those systems whose dynamic behavior shows linear dependence on the states or inputs and on the system parameters. Thus certain attractive characteristics of linear optimal controllers can be applied to the bilinear control systems. A typical bilinear model has the form

$$\dot{X}(t) = AX(t) + \sum_{i=1}^{m} B_i U_i(t) X(t) + CU(t) \tag{1}$$

where $X(t) \in R^{n \times l}$ and $U(t) \in R^{m \times l}$ are state and input vectors respectively. The block diagram of (1) is shown in Figure 1. The model consists of multiplicative and additive terms.

Much effort has been given to the bilinearization of nonlinear systems with input appearing linearly. Ruberti et al.[19] summarized the theoretical develop-

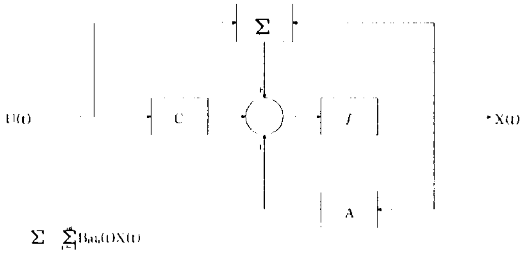

Figure 1. Bilinear System Structure.

Table 2. Single-Input single-output(SISO) bilinear models.

Type		Model	Reference
deterministc	continuous, discrete, state-space,	* continuous $x^{(n)} + \sum_{i=1}^{m} (a_i + vl_{n-i}) x^{(i-1)} = \sum_{i=0}^{m} b_i u^{(i+m)} + c\, f(x, f)$ * discretd $x(k+n) + \sum_{i=1}^{m} (a_i + v\, l_{n-1}) x(k+n-i) = \sum_{i=1}^{m} b_i u(k+i) + c\, f(x, k)$	Ionesuc, 1977, [21]
	discrete, parametric	$y(k+1) = \sum_{j=1}^{n} [\, a_j y(k+1_j) + b_j y(k+1-j) u(k+1-j) + c_j u(k+1-j) \,]$	Ohkawa and Yonezawa, 1983 [22]
	discrete, parametric,	$y(t) = \sum_{i=1}^{n} a_j y(t-i) - \sum_{j=0}^{P} c_j y(t-d-j) u(t-d-j) + \sum_{k=0}^{m} b_k u(t-d-k) + \eta(t)$ where, a : time delay $(d \geq 1)$ $\eta(t) = w(t) - \sum_{i=1}^{n} a_i w(t-i) - \sum_{j=0}^{P} c_j w(t-d-j) u(t-d-j)$	Tao and Ioannou, 1989 [23]
stochastic	discrete, polynomial,	$A(z) y(t) = \wedge(z) e(t)$ $[\, B(z) + \sum_{i=0}^{n} [\, C^i(z) y(t-k) \,] z^i \,]\; u(t-k)$ where, k-1 : time delay $(k \geq 1)$ $zy(t) = y(t-1)$ $A(z) = 1 + a_1 z + \cdot \cdot \cdot + a_{k+n} z^{k+n}$ $B(z) = b_0 + b_1 z + \cdot \cdot \cdot + b_n z^n$ $C^i(z) = c_0^i + c_1^i z + \cdot \cdot \cdot + C_n^i z^n$ $\wedge(z) = 1 + \lambda_1 z + \cdot \cdot \cdot + \lambda_{k+n} z^{k+n}$	Svoronos, Stephanopoulos and Aris, 1981 [24]
	discrete, state-space,	$x(t+1) = Px(t) + Qx(t) u(t) + Ru(t) + \leq(t)$ $w(t) = Sx(t)$ $y(t) = w(t) + e(t)$	Dai, Sinha and Puthenpura, 1989 [25]
	discrete, parameteric, polynomial,	* parametric $y_i = \sum_{j=1}^{dA} a_j y_{i-j} + \sum_{j=0}^{dC} c_j e_{i-1} + \sum_{k=0}^{P} \sum_{l=1}^{Q} \beta_{kl} e_{i-k} y_{i-1}$ * polynomial $A(B) y_i = C(B) e_i + \sum_{k=0}^{P} \sum_{l=1}^{Q} \beta_{kl} e_{i-k} y_{i-1}$	Bielinska, 1990 [26]

Type		Model	Reference
stochastic	discrete parameteric polynomial	where, B : bilinear delay operator $By_i = y_{i-1}$ $B^n y_i = y_{i-n}$ $(B^n y_i)y_i = y_{i-n}y_i = (y_i)B^n y_i$ $A(B) = 1 + a_1 B + a_2 B^2 + \cdot\cdot\cdot + a_{dA}B^{dA}$ $C(B) = 1 + c_1 B + c_2 B^2 + \cdot\cdot\cdot c_{dC}B^{dC}$ If dA=0, the model is homogeneous only in the output If dC=0, the model is homogeneous only in the input If dA=0 and dC=0, the model is homogeneous both in the output and in the input Superdiagonal : $\beta_{kl} = 0$ for all $i<k$ Subdiagonal : $\beta_{kl} = 0$ for all $i<k$ Diagonal : $\beta_{kl} = 0$ for all $i<k$	Bielinska, 1990 [26]
	discrete, polynomial, state-space,	* polynomial $A(q^{-1})x(t) = q^{-k}B(q^{-1})u(t)$ $+ [C(q^{-1}) - A(q^{-1})]e(t)$ $+ q^{-k}\sum_{i=0}^{n}\sum_{j=1}^{m} x(t-i)u(t-j-i+1)d_{ij}$ $y(t) = x(t) + e(t)$ where, k-1 : time delay $(k \geq 1)$ $q^{-1}y(t) = y(t-i)$ $A(q^{-1}) = 1 + a_1 q^{-1} + \cdot\cdot\cdot a_{na}q^{-na}$ $B(q^{-1}) = b_0 + b_1 q^{-1} + \cdot\cdot\cdot b_{nb}q^{-nb},\ b_0 \neq 0$ $C(q^{-1}) = 1 + c_1 q^{-1} + \cdot\cdot\cdot c_{nc}q^{-nc}$ * state-space $x(t+1) = Px(t) + Qu(t) + Re(t)$ $+ \sum_{i=1}^{m} u(t-i+1)D_i x(t)$ $y(t) = Hx(t) + e(t)$	King, Burnham and James, 1990 [27]

ments during the sixties, and numerous theoretical results on the bilinear systems have been reported. Much of the development up to the late seventies can be found in the survey paper by Mohler and Kolodziej [8]. Inagaki and Funahashi[20] studied the bilinear realization problems for inhomogeneous systems. The various types of bilinear model used in the studies of nonlinear systems are summarized in Table 2 and Table 3.

2.2 Development of Bilinear Models for Chemical Processes

Many processes, particularly chemical engineering processes, are by nature bilinear systems. For example, in a heat transfer problem a bilinear system arises if the heat transfer coefficient becomes a control variable. Energy balances commonly contain products of flows and temperature. As an illustration, bilinear approximation of continuous stirred tank reactor(CSTR) system is introduced in this manuscript. Comparison between linear and bilinear approximation with and without parameter estimation is also presented. The simulated CSTR process consists of an irreversible, exothermic reaction A → B, in a constant volume reactor cooled by a single coolant stream which can be modeled by the following equations[31].

$$\frac{dC_A(t)}{dt} = a_1[C_{A0} - C_A(t)] - k_0 C_A(t)\exp\left[-\frac{a_3}{T(t)}\right] \qquad (2)$$

Table 3. Multi-Input multi-output(MIMO) bilinear models.

Type		Model	Reference
deterministic	continuous, state-space	$\dot{y}(t) = (A + \sum_{i=1}^{m} B_i u_i(t))y(t)$ $z(t) = (C + \sum_{i=1}^{a} D_i v_i(t))y(t)$	Lo, 1975 [9]
	discrete, state-space	$x(t+1) = Ax(t) + Bu(t) + u(t)y^T(t)\rho$ $x(0) = x_0$ $y(t) = Cx(t)$	De La Sen, 1986 [28]
	continuous, state-sapce	$\dot{x} = Ax + \sum_{i=1}^{m} u_i A_i x + A_0 u$	Benallou and Mellichamp, 1988 [29]
stochastic	discrete, parametric, state-space	* polynomial $X_t = \sum_{i=1}^{na} A_i X_{t-i} + U_t + \sum_{j=1}^{nb} B_j U_{t-j} + \sum_{i=1}^{na} \sum_{j=1}^{nb} \sum_{k=1}^{nc} C_{ijk} X_{t-i} U^k_{t-j}$ Superdiagonal : $k=1$ and $C_{ij} = 0$ for each i>j Diagonal : $k=1$ and $C_{ij} = 0$ for each i>j Subdiagonal : $k=1$ and $C_{ij} = 0$ for each i>j * state-space $X_{t+1} = DX_t + EU_t + U_t FX_t$ $Z_t = GX_t + Hv_t$	Lessi, 1990 [30]

$$\frac{dT(t)}{dt} = a_1[T_0 - T(t)] + a_2 C_A(t)\exp[\frac{a_3}{T(t)}] + a_4 C_A(t) q_c [T_{c0} - T(t)] - a_4 q_c \exp[\frac{a_5}{q_c}][T_{c0} - T(t)] \quad (3)$$

where

$a_1 = (q/V)$

$a_2 = [(-\triangle H k_0)/(\rho C_p V)]$

$a_3 = -E/R$

$a_4 = [(\rho_c C_{pc})/(\rho C_p V)]$

$a_5 = [(-hA)/(\rho_c C_{pc})]$

Linearization around the steady state values of reactor temperature (T_s) and coolant flow rate (q_{cs}) is carried out for the exponential terms, $\exp[\frac{a_3}{T}]$ and $q_c \exp[\frac{a_5}{q_c}]$.

Using Taylor's series expansion yields

$$\exp[\frac{a_3}{T(t)}] = \exp[\frac{a_3}{T_s}] - [T(t) - T_s][\frac{a_3}{T_s^2}]\exp[\frac{a_3}{T_s}] \quad (4)$$

$$q_c \exp[\frac{a_5}{q_c}] = q_{cs}\exp[\frac{a_5}{q_{cs}}] + a_6[q_c - q_{cs}] \quad (5)$$

where

$$a_6 = [1 - (\frac{a_5}{q_{cs}})]\exp[\frac{a_5}{q_{cs}}]$$

Substitution of equation (4) and (5) into equation (2) and (3) and rearranging will yield the following simple bilinear model for CSTR.

$$\frac{dC_A(t)}{dt} = b_{c1} C_A(t) + b_{c2} C_A(t) + b_{vs} \quad (6)$$

$$\frac{dT(t)}{dt} = b_1 T(t) + b_2 C_A(t) + b_3 C_A(t) T(t) + b_5 q_c(t) + b_6 \qquad (7)$$

where

$$b_1 = -a_1 + a_4 q_{cs} \exp(a_5/q_{cs}) - a_5 a_6 q_{cs}$$

$$b_2 = a_2 \exp(a_3/T_s) + (a_2 a_3/T_s) \exp(a_3/T_s)$$

$$b_3 = -(a_2 a_3/T_s^2) \exp(a_3/T_s)$$

$$b_4 = -a_4 + a_5 a_6$$

$$b_5 = a_4 T_{c0} - a_5 a_6 T_{c0}$$

$$b_6 = a_1 T_0 - a_4 q_{cs} \exp(a_5/q_{cs}) + a_5 a_6 q_{cs} T_{c0}$$

$$b_{c1} = -a_1 - k_0 \exp(a_3/T_s) - (k_0 a_3/T_s) \exp(a_3/T_s)$$

$$b_{c2} = (k_0 a_3/T_s^2) \exp(a_3/T_s)$$

$$b_{c3} = a_1 C_{A0}$$

The parameters of equation (6) and (7) are estimated at each sampling period using a recursive least square method. The coolant flow rate was changed from an initial values of 100 l/min., to 110, to 100, to 90 and back to 100, at 7 min. interval.

Figures 2 to 5 show the concentration and temperature response to step changes in the coolant flow rate (q_c) with and without recursive parameters estimation. The results for the simulated CSTR, the bilinear models and the linear models are explicitly shown in these figures.. It can be seen from the figures that bilinear models describe the dynamics of the CSTR more accurately than linear models. Considerable improvement in the predictions of bilinear models is observed when parameter estimation algorithm is implemented.

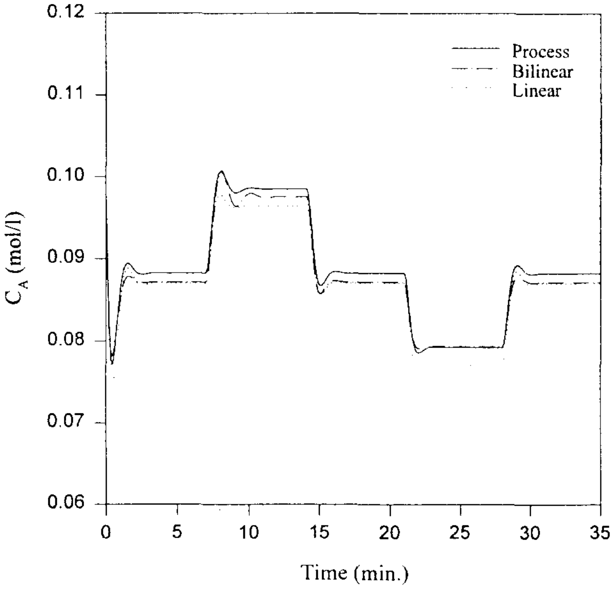

Figure 2. Concentration response of CSTR without parameter estimation.

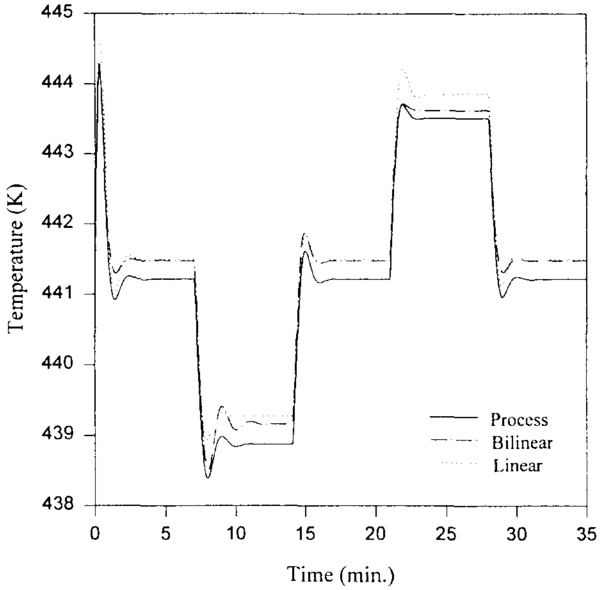

Figure 3. Temperature response without parameter estimation.

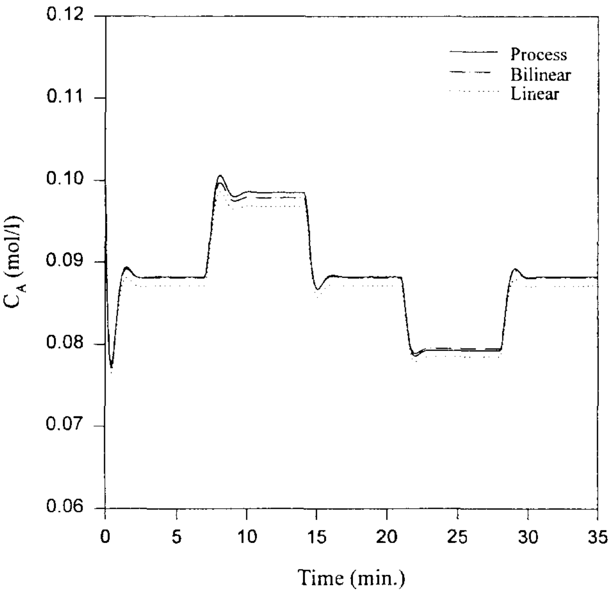

Figure 4. Concentration reponse of CSTR with parameter estimation.

3. Non-Adaptive Bilinear Model Predictive Control

A need for development of a more effective control method to control more complex chemical processes often arises where the traditional techniques are not well adapted. Recent development of digital computer

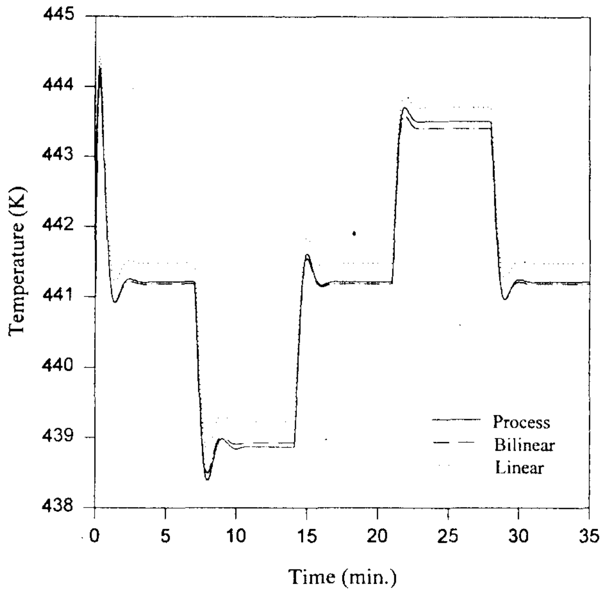

Figure 5. Temperature response with parameter estimation.

technology has made it possible to implement more sophisticated control method.

Predictive control techniques have lately drawn much attention as more effective and powerful control schemes. A general predictive control technique is characterized by

1. an objective function to be optimized
2. a desired output trajectory
3. a model structure for predictions

Usually, an objective function consists of the deviation of the predicted outputs from the desired output trajectory and a penalty on the control input. The purpose is then to find the control law which minimizes the objective function over a certain time horizon in the future. The inclusion of control actions in the objective function is necessary to prevent excessive control action or to maintain stability.

The minimization problem is formulated to calculate a sequence of future control inputs. In practice, at each sampling instant, only the present input is calculated and implemented. The inclusion of future inputs in the formulation improves controller performance, although these inputs are neither calculated nor implemented. By increasing the weighting factors for the error terms in the objective function, we can include implicit constraints on the outputs. Explicit constraints on the inputs can also be included, and the resulting optimization problem is a quadratic programming problem.

In the last decade, model predictive control(MPC) method has been employed as an advanced process control technique at many factories and has been applied to solve several industrial process control problem. Many reports have been addressing the industrial successes on MPC in many chemical process control applications. For further development of MPC, it is a matter of interest that one should understand a state of the arts in the industrial applications of MPC as well as the evaluation of MPC by the industrial practitioners. However, in spite of the fact that real-world processes have nonlinearities, almost all the MPC method so far applied to them use linear models of the processes. The nonlinear MPC is theoretically one of the interesting research area, but many difficulties still exsist.

Table 4. Applications of MPC algorithm(linear and bilinear).

Process	Type	Reference
Heat exchanger	SISO	Montague et al., 1985 [35]
Engine test beds	SISO	Tacey, 1987 [36]
Heating systems	SISO/MIMO	Jota, 1987 [37]
Robot manipulators	SISO	Lambert, E., 1987 [38]
Extruder	SISO	M'Saad at al., 1987 [39]
Tracking system	MIMO	Favier, 1987 [40]
Dryers	SISO/MIMO	Lambert, E., [38]
Cement mill	SISO	Al-Assaf, 1987 [41]
Semi-batch reactor	SISO	Sanchez Del Rio et al., 1990 [42]
Stirred tank heater	SISO	McIntosh et al., 1990 [43]
Distillation column	MIMO	Coelho et al., 1990 [44]

Several MPC's based on the parametric input-output models have been developed from the concept of adaptive controllers[32-34]. Clarke *et al.*[1-2] developed the generalized predictive controller(GPC) based on the controlled auto-regressive integrated moving-average (CARIMA) model which are known to useful to eliminate the offset. The applications of MPC algorithm were summarized in Table 4.

3.1 Review on Bilinear Control Method

Most of the bilinear controller designed so far are derived from or rely on stabilization theory or optimization theory. In general an optimal control law results from the minimization of some specific cost function. Wei and Person[45] considered the optimal control problem for commutative bilinear systems and obtained constant optimal input vectors for a time invariant system by using a quadratic cost function. Grasselli et al.[46] described the output regulation problem of bilinear systems suject to constant disturbances under the assumption that the system is reachable and observable. Their controller consists of additive and multiplicative control terms. The additive term can be regarded as a compensator for disturbances. Derse and Noldus[47] developed an optimal controller for bilinear systems where the inputs are either purely additive or multiplicative. By solving the optimization problem of a modified quadratic cost function they could obtain a quadratic controller. The stability of the control system was verified using a positive quadratic Lyapunov function. Longchamp[48] developed a feedback controller for bilinear systems and used the Lyapunov stability criterion to prove the stability of the control system. His design is based on the switching hyperplane which is dependent on the behavior of the state vector, and thus the feedback control law is discontinuous. Derese and Noldus[49] also designed a feedback controller for bilinear systems. By solving an algebraic Riccati equation, they could obtain a stabilizing constant matrix, called the feedback amplifier, which depends on a few parameters to be specified. In the design of a nonlinear controller for bilinear systems, Derese and Noldus[47] used full order observers and chose a quadratic Lyapunov function consisting of the observation error. Many of the bilinear controllers proposed so far are based on Lyapunov stability theory and a state-space representation.

Yeo and Williams[50] developed a bilinear model predictive controller based on ARMA model. By predicting the future outputs from bilinear model under the assumption that time delay and disturbance are known, they could obtain a stable predictive controller for bilinear systems.

3.2 Design of Control Law

In the GPC algorithm, the future output prediction and the predictive control law are developed by solving the Diophantine equation of the CARIMA model, which increases the computational burden, especially, in the case of MIMO processes. Moreover, if the processes are time-varing or show severe nonlinear behavior, the model parameter adaptation is required to trace the process dynamic behavior and the numerous Diophantine equations should be solved whenever the process model is updated. For the above reason, the application of GPC to the practical chemical processes are few. In this manuscript, we use the ARMA model. The predictive control algorithm based on ARMA model is practically useful and easy to be implemented to real processes.

The multivariable system to be controlled is assumed to be described by a discrete, bilinear ARMA model of the form

$$\begin{aligned} \boldsymbol{Y}^*(\boldsymbol{k}) = \sum_{i=1}^{N}[\ & \boldsymbol{A}_i^* \boldsymbol{Y}(\boldsymbol{k}-\boldsymbol{i}) + \sum_{j=1}^{m} \boldsymbol{B}_{ij}^* \boldsymbol{Y}(\boldsymbol{k}-\boldsymbol{i}) u_j(k-i-T) \\ & + \boldsymbol{C}_i^* \boldsymbol{U}(\boldsymbol{k}-\boldsymbol{i}-\boldsymbol{T})] \end{aligned} \tag{8}$$

T is the known time delay, but we do not need the exact knowledge of the plant structure. We will simplify the problem by considering one-step ahead prediction.

The computation involving the iterations of large dimension matrices causes numerical difficulties. The prediction of the future outputs $Y^*(k+1),\ldots, Y^*(k+T)$ does not require future inputs. Since the present output error vector $E(k)$ given by (9) is known, these predicted future values can be obtained by successive substitutions.

$$\boldsymbol{E}(\boldsymbol{k}) = \boldsymbol{Y}(\boldsymbol{k}) - \boldsymbol{Y}^*(\boldsymbol{k}) \tag{9}$$

The computations involving the iterations of large dimension matrices cause numerical difficulties. The objective function is given by

$$J = [Y_d(k+T+1) - Y^*(k+T+1)]^T \Gamma [Y_d(k+T+1) - Y^*(k+T+1)] + U^T(k)BU(k) \quad (10)$$

where

$$\Gamma = diag\{\gamma_1^2, \ldots, \gamma_n^2\}$$

$$B = diag\{\beta_1^2, \ldots, \beta_n^2\}$$

Minimization of (10) yields[51]

$$U(k) = \underline{W}[Y_d(k+T+1) - A_1^* Y^*(k+T) - \{Y(k) - Y^*(k)\} - \sum_{i=2}^{N}\{A_i^* Y^*(k+T+1-i) + \sum_{j=1}^{m} B_{ij}^* Y^*(k+T+1-i)u_j(k+1-i) + C_i^* U(k+1-i)\}] \quad (11)$$

where

$$\underline{W} = [(\underline{R} + C_1^*)^T(\underline{R} + C_1^*) + B]^{-1}(\underline{R} + C_1^*)^T \Gamma \quad (12)$$

$$\underline{R} = [B_{11}^* Y^*(k+T) \ldots B_{1m}^* Y^*(k+T)] \quad (13)$$

3.3 Design of an Offset Compensator

At steady-state, eqn.(8) gives

$$Y_s^* = \lim_{k\to\infty} Y^*(k) = A_s^* Y_s + (B_s^* + C_s^*) U_s \quad (14)$$

where

$$A_s^* = \sum_{i=1}^{N} A_i^*, \quad B_s^* = \sum_{i=1}^{N} \overline{B_i}, \quad C_s^* = \sum_{i=1}^{N} C_i^*$$

$$\overline{B_i} = [B_{i1}^* Y_s \ldots B_{im}^* Y_s] \quad ; \ 1 \le i \le N$$

and Y_s and U_s are steady-state values of output and input variables respectively. Sustitution of (14) into (11) yields upon rearrangement

$$\{(\overline{B_1} + C_1^*)^T \Gamma (\overline{B_1} + C_1^*) + B\} U_s = (\overline{B_1} + C_1^*)^T \Gamma (Y_{ds} - Y_s) + (\overline{B_1} + C_1^*)^T \Gamma (\overline{B_1} + C_1^*) U_s \quad (15)$$

where $\overline{B_1} = [B_{11}^* Y_s .. B_{1m}^* Y_s]$. From (15), we can see that there is no offset if $B=0$, and that nonzero β_i always gives offset. We now introduce a constant offset compensation matrix $K \in R^{n \times m}$ such that (11) becomes

$$U(k) = \underline{W}[KY_d(k+T+1) - (A_1^* - A_s^* + KA_s^*) Y^*(k+T) - K\{Y(k) - Y^*(k)\} - \sum_{i=2}^{N}\{A_i^* Y^*(k+T+1-i) + \sum_{j=1}^{m} B_{ij}^* Y^*(k+T+1-i)u_j(K+1-i) + C_i^* U(k+1-i)\}] \quad (16)$$

At steady-state, (16) becomes

$$\{(\overline{B_1} + C_1^*)^T \Gamma (\overline{B_1} + C_1^*) + B\} U_s = (\overline{B_1} + C_1)^T \Gamma [K(Y_{ds} - Y_s) + \{(K - I)(B_s^* + C_s^*) + (\overline{B_1} + C_1^*)\} U_s] \quad (17)$$

Rearrangement of (17) gives

$$(\overline{B_1} + C_1^*)^T \Gamma (Y_{ds} - Y_s) = \{B - (\overline{B_1} + C_1^*)^T \Gamma (K - I)(B_s^* + C_s^*)\} U_s \quad (18)$$

It is clear from (18) that zero offset is achieved if

$$B - (\overline{B_1} + C_1^*)^T \Gamma (K - I)(B_s^* + C_s^*) = O$$

or

$$B + (\overline{B_1} + C_1^*)^T \Gamma (B_s^* + C_s^*) = ((B_1 + C_1^*)^T \Gamma K (B_s^* + C_s^*) \quad (19)$$

If $n=m$ (i.e., input and output vectors have the same dimensions), K has the explict form given by

$$K = I + \{(\overline{B_1} + C_1^*)^T \Gamma\}^{-1} B (B_s^* + C_s^*)^{-1} \quad (20)$$

4. Adaptive Bilinear Model Predictive Control

In many practical situations, the operating conditions vary with time, and it is very difficult to obtain any information about the parameters of the plant to be controlled. The adaptive model predictive control (AMPC) methods are believed to be the most promising strategy allpicable in these situations. The AMPC system is a combination of both feedback control and identification.

4.1 Review on Adaptive Control(linear and bilinear)

Many adaptive controllers have been proposed in the literature, but insufficient design methods and the complexity of the adaptive controllers have restricted the practical implementation of the adaptive controllers. Recently, the rapid development of digital computer has made the adaptive controllers technically possible.

Many effort have been devoted to the extension of existing adaptive control systems to predictive control system. Lee and Lee[32] described the adaptive control scheme for disturbance-free systems using a long-term predictor. Their predictive controller allows direct transmission case, which is unusual, but requires the perfect knowledge of the plant structure such as time delay and system order. Martin-Sanchez et al.[52] proposed a stable AMPC system. They used an equation error identification method and proved several stability properties. The applications of AMPC algorithm were summarized in Table 5.

Table 5. Applications of AMPC algorithm.

Process	Type	References
Environmental Test Chamber	MIMO	Dion et al., 1991 [53]
Semi-batch reactor	SISO	Defaye et al., 1993 [54]
Batch reactor	SISO	Jarupintusophon et al., 1994 [55]
		Cluett et al., 1985 [56]
Industrial bleach plant	SISO	Dumont et al., 1989 [57]
Kamyr digester	SISO	Allison et al., 1990 [58]
pH control	SISO	Zhu et al., 1991, [59]
Distillation column	MIMO	Martin-Sanchez and Shah, 1987 [60]

4.2 Identification Algorithm

Since an identification algorithm is itself an adaptation algorithm in the adaptive control system, the analysis of the identification problem with bounded disturbances has often been coupled with the analysis of adaptive control systems with bounded disturbances. Samson[61] analyzed the identification methods for the discrete-time system subject to bounded disturbances. Identification for bilinear systems has been studied by Frick and Valavi[62], Kubrusly[63], Zhang[64], Wang *et al.*[65]. Yeo and Williams[6] have used ARMA model in the identification of single variable bilinear systems. For illustration, a simple identification scheme is presented here with application to bilinear systems.

A single variable bilinear system can be described by ARMA representation of a form

$$y(k)=p^{T}x(k-1)+d(k) \qquad (21)$$

In order to identify the system parameter vector p, we propose a recursive identification algorithm of the form

$$p^{*}(k)=p^{*}(k-1)+\xi(k-1)x(k-1)e^{*}(k) \qquad (22)$$

where

$$e(k)=y(k)-y^{*}(k|k)$$
$$e^{*}(k)=y(k)-y^{*}(k|k-1)$$
$$y^{*}(k|k)=p^{*^{T}}(k)x(k-1)$$
$$y^{*}(k|k-1)=p^{*^{T}}(k-1)x(k-1)$$

and the gain $\xi(k\text{-}1)$ is calculated as follows

$$\xi(k-1)=\begin{cases}\dfrac{2\lambda(k)[\ \zeta(k)-1]}{\zeta(k)\|x(k-1)\|^{2}+\theta(k)} & ;\ \zeta(k)>1\\ 0 & ;\ \zeta(k)\le 1\end{cases} \qquad (23)$$

where

$$\zeta(k)=\frac{|e^{*}(k)|}{qD}$$

$$0<\lambda(k)\le 1$$
$$0<\theta(k)<R_{1}<\infty$$
$$1\le q<R_{2}<\infty$$

In this manuscript the above identification algorithm is used and the extension to multivariable bilinear system is relatively straigthforward.

5. Examples

To illustrate the proposed AMPC method for multivariable bilinear models, we investigated a simple MIMO bilinear system. To correct for the effect of model inaccuracy, we introduce a simple filter given by

$$\boldsymbol{U}(\boldsymbol{k})=(1-\alpha)\,\boldsymbol{U}^{*}(\boldsymbol{k})+\alpha\,\boldsymbol{U}(\boldsymbol{k}-1) \qquad (24)$$

where $U^{*}(k)$ is the unfiltered input vector from the control algorithm (16). The disturbance is assumed to be constant as $D(k)=[0.5\ 0.5]^{T}$. Constant matrices I and B are used in the simulations, i.e., $\gamma_{i}=1(1\le i\le n)$ and $\beta_{i}=\beta(1\le i\le m)$. In the identification, the algorithm given by (22) and (23) with $q=1$, $\theta(k)=1$ and $\lambda(k)=\frac{\zeta(k)}{2[\zeta(k)-1]}$ was used. The process isgiven by (25). The process initially has zero inputs, outputs and disturbances. The results of the AMPC control are shown in Figure 6.

$$\begin{aligned}\boldsymbol{Y}(\boldsymbol{k})=&\begin{bmatrix}-0.2 & 0\\ 0 & 0.2\end{bmatrix}\boldsymbol{Y}(\boldsymbol{k}-1)+\begin{bmatrix}0.3 & 0\\ 0 & 0.4\end{bmatrix}\boldsymbol{Y}(\boldsymbol{k}-2)\\ &+\begin{bmatrix}0.5 & 0\\ 0 & 1.2\end{bmatrix}\boldsymbol{Y}(\boldsymbol{k}-1)u_{1}(k-4)+\begin{bmatrix}-0.12 & 0\\ 0 & 0.3\end{bmatrix}\\ &\boldsymbol{Y}(\boldsymbol{k}-2)u_{1}(k-5)+\begin{bmatrix}4 & 1\\ 2 & 1.5\end{bmatrix}\boldsymbol{U}(\boldsymbol{k}-4)\\ &+\begin{bmatrix}2 & 0.5\\ -1.4 & 0.6\end{bmatrix}\boldsymbol{U}(\boldsymbol{k}-5)+\boldsymbol{D}(\boldsymbol{k})\end{aligned} \qquad (25)$$

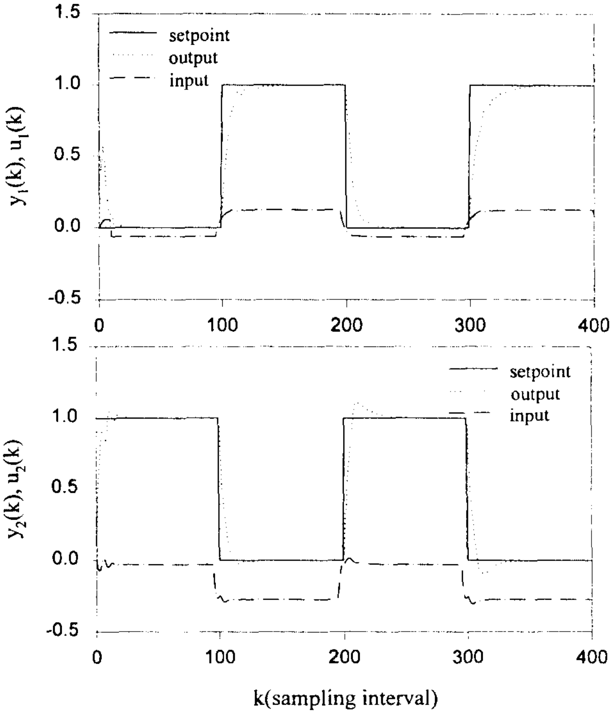

Figure 6. Results of control for example 2(β =3.0, α =0.8)

6. Concluding Remark

In the last decade, the model predictive control methods have enjoyed many industrial applications with successful results. Although the general predictive control methods for nonlinear chemical processes are not yet formulated, the promising features of the model predictive control methods attract attentions of many researchers who are involved with difficult but important nonlinear process control problems.

Recently, the class of bilinear model has been introduced as an useful tool for examining many nonlinear phenomena. Since their structural properties are similar to those of linear models, it is not difficult to develop a robust adaptive model predictive control method based on bilinear model. We expect that the model predictive control method based on bilinear model will expand its region in the world of nonlinear systms.

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