Robust Control of Positioning Systems with a Bang-Bang Actuator

뱅-뱅 액츄에이터를 가진 위치 제어계의 강인 제어

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요 약 : 본 논문에서는 기계적 위치 제어계에서 다단계 뱅-뱅 액츄에이터의 비선형성에 의한 목표 위치에서의 리미트 사이클을 방지하는 제어 방법을 제안한다. 다단계 뱅-뱅 액츄에이터의 선형화된 모델인 기술함수를 이용하여 비선형성을 보상한다. 강인성을 확보하기 위해 루프 형성 기법에 의한 H_∞ 제어기가 설계된다. 제안된 제어방법은 기존의 선형제어기보다 비선형성이 보상되어 사역대가 작아지므로 최소 제어 가능 구간을 줄일수 있다. 1축 위치 제어계의 실험을하여 제안된 제어 방법이 리미트 사이클을 줄이고 제어정도를 향상 시키는데 유효함을 입증하였다.

Keywords: nonlinear, robust, bang-bang H_∞ control, normalized corprime factorization, deadband, limit cycle

I. Introduction

High-power motors actuate the big reclaimers used to reclaim ore in mines and raw ores yards of the iron-making works. Wall cranes also use these motors to transfer heavy materials in the factories. Most of the motors in these machines are of multi-step bang-bang type. Workers easily operate these motors using several speed control levers. The motors are less expensive than linear motors, and the simple hardware needs less maintenance. One of the main difficulties encountered in automating reclaimers is to overcome the position errors and stable oscillations caused by the bang-bang actuators. The hard nonlinearities of the bang-bang actuator cause a limit cycle in the feedback control system, which tends to cause poor position accuracy. The constant oscillation associated with the limit cycle increases wear, undesirable chattering, and mechanical failures in these systems. In general, a deadband is used in the control system to avoid the problem of the persistent switching due to limit cycle, but it necessarily brings about a steady-state position error. Despite needs for this kind of position control system in the real fields, there have been less researches for synthesizing a reasonable controller. Quasi-linearization methods serve as viable tools in the analysis and synthesis of nonlinear systems with hard nonlinearities. In a previous research, Taylor and Strobel[1] designed a fully nonlinear PID compensator for hard nonlinear systems via a sinusoidal input describing function. Beaman[2] proposed the quasi-linear quadratic Guassian(QLQG) control, and Kim[3] proposed the quasi- linear quadratic Gaussian control with loop transfer recovery (QLQG/LTR). The QLQG method includes the optimal estimation and control for statistically linearized systems, and the QLQG/LTR method is the integration of nonlinear systems, target filter loop design, loop transfer recovery, and the inverse random input describing function method. The QLQG method can not address the performance and stability robustness problem, but

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In this paper, a nonlinear robust control scheme is proposed. The design method is not complicated and addresses stability robustness issues. The method is used to prevent limit cycle due to the nonlinearity of the multi-step bang-bang actuator in a mechanical position control system. Using a linearized model, sinusoidal input describing function, for the multi-step bang-bang actuator, an H_∞ robust controller is designed to control position. The proposed scheme in this study needs a smaller deadband as a result of compensating the nonlinearity of the bang-bang actuator. The controller attenuates the nonlinear effects of the multi-step bang-bang actuator by introducing the describing function inverse, and simultaneously suppresses the persistent contact switching of the actuator. The position controller is synthesized by loop shaping techniques with normalized coprime factorization(NCF) stabilization[4] to address the robustness. The control system has been implemented in a single axis servo system with a 2-step bang-bang amplifier. The phase portraits for step references verify that the designed control system is effective to suppress limit cycle caused by nonlinear effects of multi-step bang-bang actuator in the vicinity of the reference position. Experimental results show that the controller achieves the desired performance and is robust to variations in the loop gain.

II. Describing Function for the Bang-Bang Actuator

In the describing function analysis, it is assumed that only the fundamental harmonic component of the output is significant. Such an assumption is often valid since the higher harmonics in the output of a nonlinear element are often of smaller amplitude than the amplitude of the fundamental harmonic component. In addition, most control systems have the characteristics of low-pass filters, with the result that the higher harmonics are very much attenuated compared with the fundamental harmonic component. The sinusoidal input describing function of a nonlinear element is defined to be the complex ratio of the fundamental harmonic component of the output to the input[5]. If no energy-storage element is included in the nonlinear elements, then the describing function N is a function of only the amplitude of the input to the element. The input-output relationship for a multi-step bang-bang actuator is plotted in Fig. 1 with x_i and g_i denoting the input and output step. Since this nonlinearity is single-valued, the describing function is a real function of the input amplitude M when the input signal *x(t)* is *Msin(wt)*. The describing function for the multi-step bang-bang actuator is given by[6]

$$N(M)=\begin{cases} 0\le M\le x_1 & 0 \\ x_j\langle M\le x_{j+1} & \frac{4}{\pi M}\left[\sum_{j=1}^{j}(g_i-g_{i-1})\sqrt{1-(x_i/M)^2}\right] \\ x_n\langle M & \frac{4}{\pi M}\left[\sum_{j=1}^{n}(g_i-g_{i-1})\sqrt{1-(x_i/M)^2}\right]. \end{cases} \quad (1)$$

A dc motor treated in this paper is of the 2-step bang-bang type with deadband. Its describing function is given as

$$N(M)=\begin{cases} 0\le M\le x_1 & 0 \\ x_j\langle M\le x_2 & \frac{4g_1}{\pi M}\sqrt{1-(x_i/M)^2} \\ x_n\langle M & \frac{4}{\pi M}\left[\sum_{j=1}^{2}(g_i-g_{i-1})\sqrt{1-(x_i/M)^2}\right]. \end{cases} \quad (2)$$

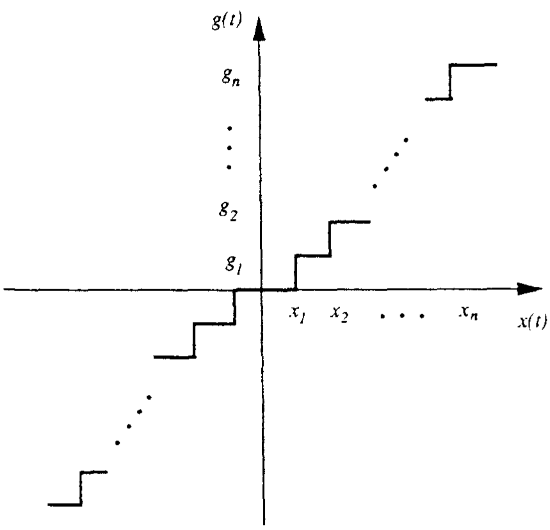


Fig. 1. Input-output relationship of the bang-bang actuator.

III. Design of the Proposed Controller

1. Loop Shaping Techniques

A controller proposed in this paper is designed using a loop shaping scheme by normalized coprime factorization stabilization. This method has remarkable characteristics to synthesize the optimal control directly from the Nehari extension of the normalized right coprime factorization (NRCF) of the nominal system, while other H_∞ syntheses require iterative procedures to obtain optimal or suboptimal solutions to satisfy the H_∞ norm.

The robust stabilization problem is to stabilize the nominal state-space system which is denoted G=[A, B, C, D] with NRCF(P, Q) and the family of systems G_ε defined

$$G_\varepsilon=\left\{(P+\Delta_P)(Q+\Delta_Q)^{-1}: \left\| \begin{matrix}\Delta_P \\ \Delta_Q\end{matrix} \right\| \langle \varepsilon\right\} \quad (3)$$

using a proper feedback control[7].

Definition (NRCF Robust Stabilization): Let (P, Q) be a NRCF of G, then, 1) Find the largest positive number $\varepsilon_{\max}$ called the maximum stability margin, such that (P, Q, $\varepsilon_{\max}$) is robustly stable. 2) For a particular value $\varepsilon \le \varepsilon_{\max}$, synthesize an H_∞controller K_∞ such that (P, Q, K_∞, ε) is robustly stable.

The maximum stability margin is calculated as

$$G_\varepsilon=\left\{(P+\Delta_P)\ (Q+\Delta_Q)^{-1}: \left\| \left\{ \begin{matrix}\Delta_P \\ \Delta_Q\end{matrix} \right\| \langle \varepsilon\right\} \quad (4)$$

where X and Y are the solutions of the control and filter ARE. A state-space realization of the central H_∞ controller satisfying the stability margin is

$$K_\infty=\left[\begin{array}{c|c} A+BB^T+\varepsilon^{-2}W_I^{-1}YC^TC & \varepsilon^{-2}W_I^{-1}YC^T \\ \hline B^TX & 0 \end{array}\right] \quad (5)$$

where $W_I=(1-\varepsilon^{-2})I+YX$.

The loop shaping procedure has the following stages.

(1) Loop Shaping: using a loop compensator W, the magnitude of the nominal system G is shaped to give a desired target loop which determines the open-loop shape of the closed-loop system. The nominal system and the loop compensator W are combined to form the shaped system G_s where $G_s = GW$.

(2) Robust Stabilization: an H_∞ controller K_∞ is synthesized using the NRCF stabilization procedure which robustly stabilizes the NRCF of G_s with stability margin ε.

(3) Synthesis: The feedback controller K is then constructed by combining the H_∞ controller K_∞ with the loop compensator W such that $K=WK_\infty$.

The stability margin ε for the shaped plant is interpreted as a measure of robustness. It provides direct information on the size of the neighborhood of plants close to the shaped plant in the gap metric that are also stabilized by the controller.

2. Design of an H_∞ Controller with Actuator Compensation

Since the electrical time constant is much smaller than the mechanical time constant in the servomotor-driven position control system, the armature inductance effects are neglected. The dynamic model of the system, whether it has rotation joints or translation joints, is, in general, characterized by the 2nd order transfer function, and has no finite zeros. Because the relative degree of the transfer function is 2, the roll-off rate of its magnitude near the crossover frequency is about -40db/dec. Fig.2 shows magnitude plot of the index table. The transfer function for the index table not including the bang-bang actuator is experimentally obtained using frequency response and is approximately given as follows:

$$G=\frac{k}{s(Ts+1)} \quad (6)$$

where k is 1.52 and T is 0.009 sec.

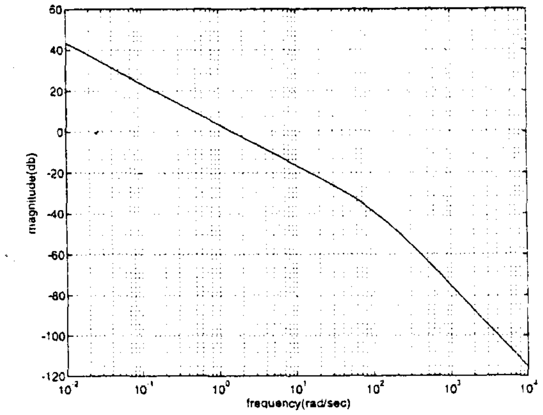


Fig. 2. Magnitude plot of the index table.

The positioning system including the linearized model of the bang-bang actuator is

$$G=\frac{kN(M)}{s(Ts+1)} \tag{7}$$

The loop gain of G_N moves upward or downward according to the magnitude of the control input M. Therefore, the crossover frequency varies, and the bandwidth of the system is dependent on the control input. The varying bandwidth means that the position control system may have bad servo performance when the M is large, because the nonlinearity dominates the response dynamics. The plant dynamics is shaped to look like $k/s(Ts+1)$ by dividing $N(M)$. This simplifies the solution of an H_∞ controller. Then, the $k/s(Ts+1)$ is shaped to the desired form with a loop compensator to meet control specifications.

To guarantee a good phase margin, it is necessary for the target loop to have a -20db/dec slope near the crossover frequency. To accomplish this, a finite zero with an appropriate proportional gain is used. The roll-off rate near the crossover frequency is -20 db/dec. If a large proportional gain is used, the loop gain increases and the roll-off rate near the crossover frequency will be -40 db/dec. The insertion of this zero near the crossover frequency guarantees target loop to have -20db/dec slope. The shaped plant G_s is obtained by inserting a loop compensator $k_p(s/\omega_z+1)$ where ω_z =105 rad/sec. The k_p of the loop compensator is chosen as 10. If an ω_z is larger than 1/0.009, the desired roll-off can not be obtained. If an ω_z smaller than 1/0.009 is used, the bandwidth of the closed-loop system is reduced.

Robust stability is, in general, interpreted in terms of a combination of the robustness for the multiplicative and additive uncertainties. By small gain theorem, the allowable multiplicative and additive uncertainties is as follows:

$$\| \Delta_m \|_\infty \le \| T_c \|_\infty^{-1} \tag{8}$$

$$\| \Delta_a \|_\infty \le \left\| \frac{k}{1+GK} \right\|_\infty^{-1} \simeq \| K \|_\infty^{-1} \text{ in the high frequency region} \tag{9}$$

where T_c, Δ_m, and Δ_a are the complementary sensitivity function, the multiplicative uncertainties, and the additive uncertainties, respectively. The controller can tolerate the multiplicative uncertainties of 100 % below the frequency of the bandwidth, as known from the magnitude of the T_c. If the gain of the controller K becomes larger, the allowable additive uncertainties will be decreased. Therefore, the magnitude of the controller gain has to be selected by considering robust stability as well as design specifications. The shaped transfer function with the loop compensator is

$$G_s=\frac{kk_p(s/\omega_z+1)}{s(Ts+1)} \tag{10}$$

Sufficiently large gain and phase margins are necessary for the system to have good relative stability which can be, to some degree, robustness measure in SISO LTI systems. The gain and phase margins of the shaped system are db and about 91 degrees as shown in Fig. 3, which are greatly large margins.

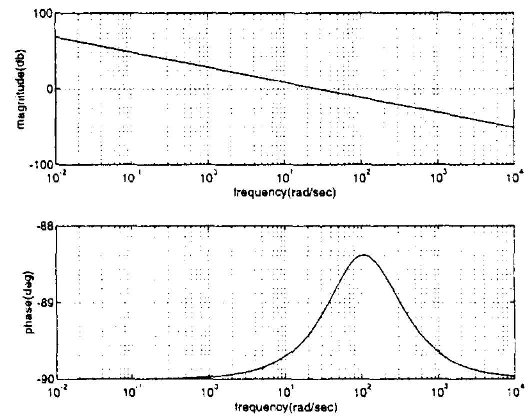


Fig. 3. Bode plot of the shaped system.

In the next design step, an H_∞ controller K_∞ is synthesized using the NRCF stabilization procedure which robustly stabilizes the NRCF of G_s, with stability margin ε. Since the describing function is an approximation of the bang-bang actuator neglecting the higher harmonic components, the real system have modeling errors which exist in the high-frequencies regions. Therefore, the H_∞ controller is designed to address robustness for the errors. The stability margin ε is also a factor to determine the magnitude of the H_∞ controller K_∞. ε near to ε_{max} gives the good recovery, but has a disadvantage to introduce fast poles. As ε becomes smaller, the deterioration in the target shape is enlarged. The maximum stability margin calculated in this design is 0.72, but ε =0.5 is adopted to prevent fast poles from being introduced into the controller.

The H_∞ controller K_∞ by the NRCF stabilization procedure is obtained.

$$K_\infty=\frac{93.77(s+111.02)}{(s+122.47)(s+98.09)} \tag{11}$$

The feedback controller K is given as follows by combining the loop compensator and the K_∞ in the last loop shaping stage:

$$K_\infty = \frac{93.77k_p(s/\omega_z+1)(s+111.02)}{(s+122.47)(s+98.09)} \quad (12)$$

Fig. 4 shows the magnitude Bode plot for the target loop, the designed loop shape and the controller K_∞.

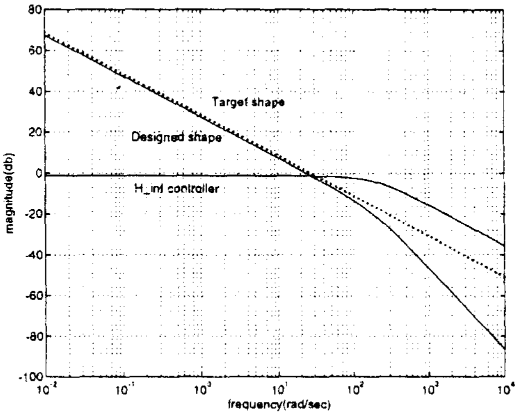


Fig. 4. Magnitude plot of the designed system.

The H_∞ controller has a higher roll-off rate in the high frequency region. The designed loop shape also shows higher roll-off rate in the high frequency region and therefore robustness to the neglected higher harmonics of the bang-bang actuator and measurement noise. The magnitude of the H_∞ controller is slightly below 0 db in the low frequency region because of using a smaller ϵ. This magnitude will gradually approach to 0 db, if we adopt a larger e. The stability margin adopted in the design appears to give the good loop recovery, because the designed loop has slightly lower magnitude than the target loop below the crossover frequency. The designed loop shows that it gently crosses the 0 db with -20db/dec, and it is thought that it will have a nice transient response.

The final controller Kf is obtained by combining the feedback controller K and the describing function inverse $N(M)^{-1}$.

$$K_\infty = \frac{93.77k_p(s/\omega_z+1)(s+111.02)}{(s+122.47)(s+98.09)} N(M)^{-1} \quad (13)$$

The final controller Kf can be considered an adaptive controller with varying gain which is dependent on the magnitude of the control input. The position control system with the final controller is shown in Fig.5.

The final controller takes a positional error e and an amplitude M of the input to the actuator as input signals. Therefore, the M should be generated to implement $N(M)^{-1}$ into the controller. Colgren[8] took the absolute value of the control input to the actuator m and filtered it through a low pass filter. Powers[9] used the similar method in a pitch stick shaping circuit on the space shuttle. An amplitude estimator in the design is also accomplished by taking the absolute value of the actuator input signal and filtering it with a first-order low pass filter, $1/Ts+1$. The time constant T of the filter is chosen as 0.008 sec, considering the index table. If $M \le x_1$, $N(M)^{-1}$ is not calculated to prevent it from going to ∞.

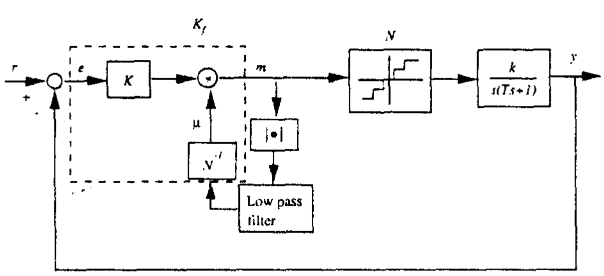


Fig. 5. The position control system with the actuator compensation.

Ⅳ. Experimental Results

Fig. 6 shows a testbed for the position control system with a 2-step bang-bang actuator. The system consists of two main elements: the servomotor-driven index table and VME computer with interface. The position sensor used is an incremental encoder. The motor used is DC servo motor, and the controller is implemented using MVME143 computer that has VME bus and 32 bit microprocessor, MC68030, running at 25MHz. The computer is interfaced to a power amp via a DAC board and a encoder through a counter board. The H_∞ controller designed in the continuous time domain is transformed to the discrete time domain controller using a bilinear transformation. The digital codes written in C are generated for actual control, and executed every 2 msec under assistance of real time operating system, VMEexec, that is important requirement for a microprocessor based control system, and has real time kernel and other tools for application software development.

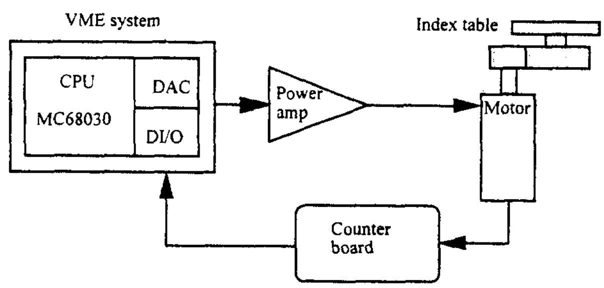
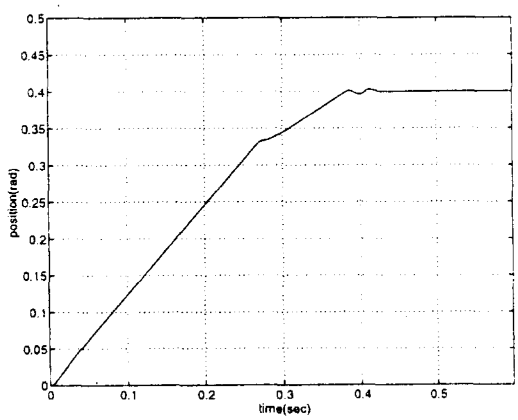
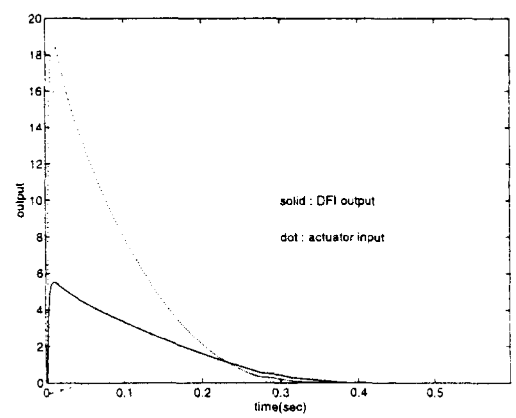


Fig. 6. Experimental setup of a servomechanism.

Input steps of the 2-step bang-bang actuator, x_1 and x_2, were set to 0.0003 and 0.4 by adjusting the input terminal of the motor amp. Output steps, g_1 and g_2, were set to 0.4 and 0.7. The magnitude of the deadband which is 0.0003 was appropriately determined to be the minimum that does not bring about persistent switching. Step response experiments were carried out to demonstrate the effectiveness of the proposed controller. Step responses provide useful information to examine the existence of the limit cycle in the vicinity of the target position. The index table was given 0.4 radian step command. The position response shows good performance despite the hard nonlinearity of the actuator as shown in Fig. 7(a). The output of the describing function inverse m , and the final controller output m are plotted in the Fig. 7(b). The actuator switches only once near the reference position, which is encouraging for the real mechanical systems as illustrated in Fig. 8(a). The phase portrait in Fig. 8(b) shows the nature of the system response and corresponding actuator switching behavior for the position control system. The phase portrait in the vicinity of the reference position represent the stable convergence and gentle switching. The index table seems to take the negative force after the actuator stoped near the reference position. That phenomenon is thought to occur due to some compliances in the joint



(a) position response.

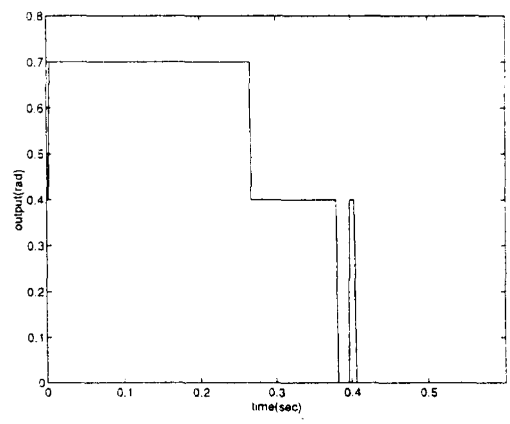


(b) control signals.

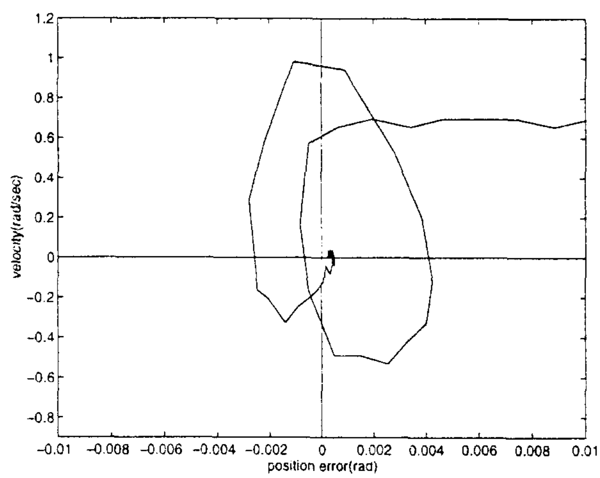
Fig. 7. Time responses for the step reference.

The proposed nonlinear controller is compared with the linear H_∞ controller not including the nonlinear compensator for the bang-bang actuator. A wider deadband whose value is 0.015 is used in this linear zcontroller. The phase portraits of the step reference for the two controllers are plotted in Fig. 9. The linear controller yields larger steady-state error than the nonlinear controller as a result of the wider deadband. It also produces more switchings though it has a wider deadband. The linear controller shows worse switching behaviors as seen in the phase portrait. The deadband of the nonlinear controller is much smaller than that of the linear controller.

Robust characteristics of the controller have to be maintained, though faced to modeling perturbation and variations of the controller gain. Robustness for the load uncertainty which causes a parametric modeling error was investigated. An additional 6 kg mass, which corresponds to 4/5 of the allowable maximum load attachable to the index table, was analyzed. Fig. 10(a) shows the error response with the added load. The error responses for an angle command of 0.4 radian after 0.35 seconds illustrate robust performance to the load perturbation. The steady-state errors approach nearly the same value despite the additional load. The actuator outputs also demonstrates a reduction in switching as shown in Fig. 10(b).



(a) control inputs.



(b) phase portrait.

Fig. 8. Actuator and states responses for the step reference.

The width of the deadband which does not give rise to persistent switchings in the linear controller without the nonlinearity compensation is highly dependent on the controller gain. The deadband in the linear controller without the nonlinearity compensation, in general, has approximately linear proportional relationship to the controller gain. Since the proposed nonlinear compensator is dependent on the magnitude of the input signal, any controller gain variations will change the bandwidth of the control loop, and therefore impact tracking ability. Therefore, it is necessary to examine the effects of gain changes on the performance.

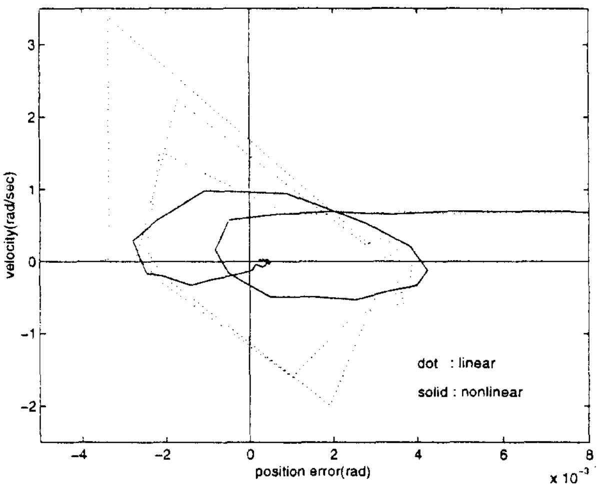
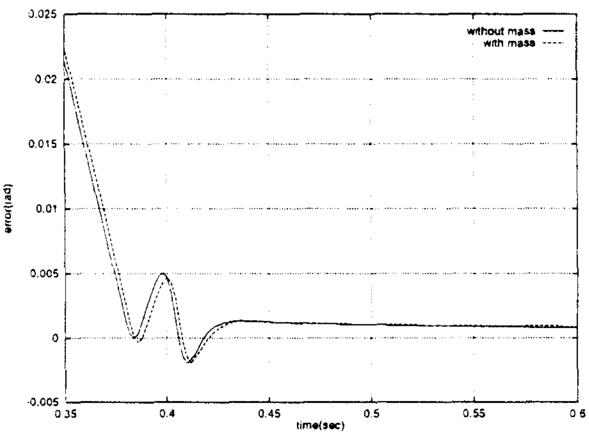
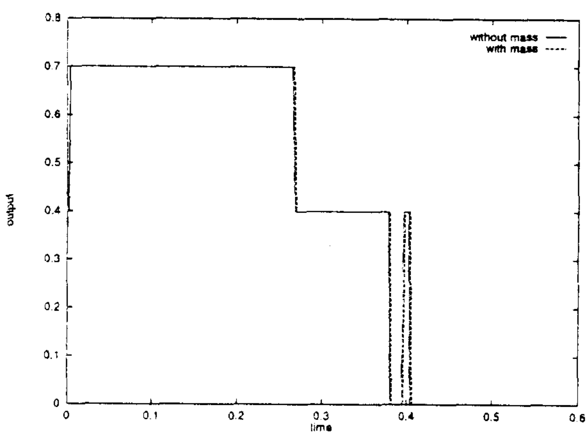


Fig. 9. Phase portraits by the linear and nonlinear controller.



(a) errors.



(b) control inputs.

Fig. 10. Robustness for the load variations.

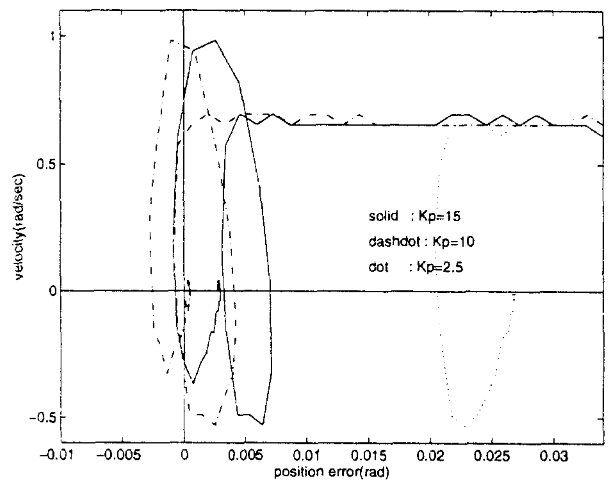
To ascertain the insensitivity to the controller gain variations, three controllers were designed using three loop compensator gains. The loop compensators have the same finite zero ω_z, but the proportional gains k_p of them are chosen as 2.5, 10 and 15. Table 1 represents the designed feedback controllers $K = k_i \prod_i (s - z_i) / \prod_j (s - p_j)$ in zero-pole format for the robustness test. The proportional gains of the feedback controllers somewhat different from those of the loop compensators, because their gains are determined from the gains of the loop compensator W and the H_∞ controller K_∞.

Table 1. Formats of the three final controllers.

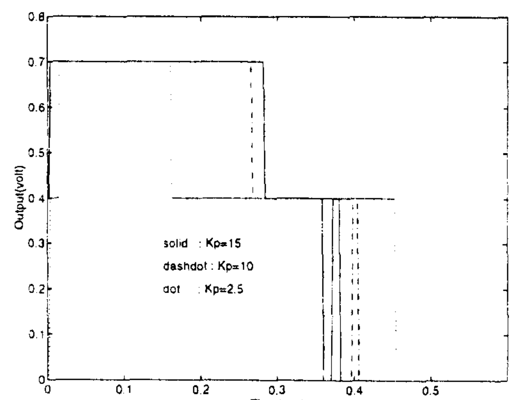
	Gain(k_c)	Poles(p_j)	Zeros(z_i)
Case1	0.37	-111.2, -22.2	-111.1, -105
Case2	8.93	-122.47, -98.09	-111.02, -105
Case3	28.5	-199, -106.4	-110.85, -105

Three designed nonlinear controllers give the similar switching characteristics in spite of the gain changes as shown in Fig. 11. Note that the proposed controller has switching robustness for the wide variation of the controller gain.

It has been shown that the persistent switchings of the actuator are effectively suppressed by the nonlinear compensation. It is believed that the designed controller is also insensitive to perturbations in the load and controller gains.



(a) phase portraits.



(b) control inputs.

Fig. 11. Robustness for the gain variations.

V. Conclusions

A nonlinear control scheme is proposed to prevent persistent actuator switching due to the nonlinearity of the multi-step bang-bang actuator in mechanical position control systems. A robust H_∞ controller for the position control system with the bang-bang actuator is designed by loop shaping techniques with normalized coprime factorization stabilization. The loop shaping method with H_∞ synthesis provides us with the ease and the flexibility in designing the controller to meet specifications.

The nonlinearity of the actuator was compensated by using the describing function inverse. The nonlinear controller was applied to an index table. Experimental results showed that the designed H_∞ controller is effective in suppressing undesirable persistent switching. It was verified that the compensated system with the describing function inverse has gentle actuator switching behaviors in the vicinity of the reference position, and a sufficiently small position error by the narrow deadband. The performance of the controller has been examined through experiments under load uncertainties and various controller gains to address robustness problem. Despite of the dependence of the bang-bang actuator on the input amplitude, the compensated system illustrated the insensitivity to the variations in the controller gain. Moreover, it also showed robustness to model perturbation which real world controller must possess. In the next research, the proposed controller will be applied to 3-phase induction motors of 2-step bang-bang type in reclaimers in the raw ores yards of the iron-making works.

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최 진 태

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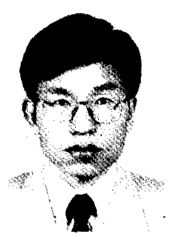

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