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루프쉐이핑과 구조적 특이치를 이용한 견실성능개선

(Analysis of robust performance improvement using loop shaping and structured singular value)

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요 약

본 논문에서는 정규화된 좌소인수분해형태의 불확실성을 가지는 시스템에서 루프쉐이핑과 구조적 특이치를 이용한 견실성능개선 방법을 제안한다. 이를 위해 condition number를 고려한 루프쉐이핑의 하중함수를 선택하고 정규화된 좌소인수분해형태의 구조를 4-블럭구조로 변환한다. 루프쉐이핑함수 선택시 가해진 제약조건으로 인해 좋은 성능을 얻을 수 없으므로 성능개선을 위해 구조적 특이치를 사용하며, 최종적인 성능개선은 D-K 순환의 스켈링요소와 성능개선 요소 W_a를 사용함으로써 가능함을 보인다.

Abstract

In this paper, we present a robust performance improvement method for the NLCF(normalized left coprime factor) uncertain structure using loop shaping and the structured singular value. For this, we select weighting functions for a loop shaping considering condition number, and transform the NLCF uncertain structure into the 4-block structure. However, we can't get a good performance on account of the restriction of weighting functions. To cope with this, we motivate the use of structured singular value in the robust performance improvement procedure. After all, the robust performance improvement can be obtained by a factor W_a and a scaling factor of D-K iteration.

I. Introduction

Robust stability in the presence of uncertainty is an important issue in control system analysis and has attracted considerable interests[1]. And the so-called performance such as tracking of the reference input and the steady state response is another significant problem. Designing a controller satisfying these objectives is very difficult because of the tradeoffs between robustness and performance. Therefore many researchers have been studying the issue of the robust performance. A new analysis framework, based on the structured singular value μ, has been proposed by Doyle[2] to assess the stability and performance robustness of linear time invariant feedback systems in the presence of structured singular value. The design of a feedback system that exhibits closed loop stability and performance in the face of uncertainty is the so-called μ-synthesis problem. The synthesis approach proposed by Doyle[3,4,5] is an iterative scheme, referred to as D-K iteration, that involves a sequence of scaled H^{∞} based feedback design problems.

McFarlane and Glover[6] proposed a loop

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shaping design procedure which combines classical loop shaping ideas with H^{∞} robust stabilization. But a loop shaping design ignores structure of the uncertainty and stabilizes only a shaped plant for which the uncertainty description may not be representative of actual uncertainty [7].

To cope with this, in this paper, we use a structured singular value and the condition number of loop shaping weighting functions. Firstly, we select loop shaping weighting functions considering the condition number and then design a controller. Secondly, we transform the NLCF uncertain structure into the 4-block structure in order to use the structured singular value and then proceed D-K iteration till satisfy robust performance condition. Finally, we select weighting factor W_a to improve robust performance. After all, the robust performance improvement can be obtained by a factor W_a and a scaling factor of D-K iteration.

Ⅱ. Structured singular value and robust performance

The block diagram in Figure 1 is the standard framework for considering the robust feedback design problem. The diagram represents any linear interconnection of inputs, outputs, perturbations, and a compensator. G is the known model that contains the plant to be controlled and K is the compensator to be designed.

The synthesis objective is to find a K to achieve nominal stability and performance of the feedback loop and to provide robustness with respect to the modeling error. The compensator K in Figure 1 is known for the purpose of analysis, and is incorporarated with the plant G via a lower linear fractional transformation to yield the closed loop operator M in Figure 2. And the closed loop transfer function from the inputs d to errors e in Figure 2 is given by

$$e = F_u(M, \Delta)d = [\ M_{22} + M_{21}\Delta(I - M_{11}\Delta)^{-1}M_{12}]\ d. \quad (1)$$

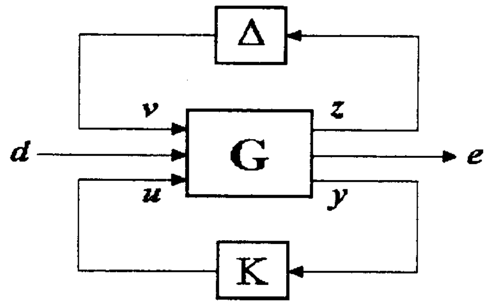

그림 1. 일반적인 견실궤환 설계문제에 대한 구조
Fig. 1. General framework for the robust feedback design problem.

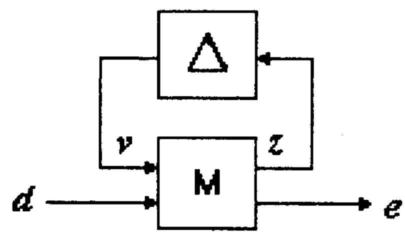

그림 2. 분석된 블럭 선도
Fig. 2. Analyzed block diagram.

The following notations will be used throughout:

$\bar{\sigma}$: largest singular value

$\underline{\sigma}$: smallest singular value

ρ: spectral radius

Δ_ν: set of all complex perturbations with a specific block diagonal structure and spectral norm less than ν

$$\Delta_\nu = \{\Delta = \text{diag}[\ \Delta_1, \Delta_2, \cdots, \Delta_m] \ \mid \bar{\sigma}(\Delta_i) \le \nu\}$$

U: set of block diagonal unitary transformation matrix

$$\text{U} = \{\text{diag}[\ U_1, U_2, \cdots, U_m] \ \text{ such that } U_i \in C^{k_i \times k_i},\ U_i^* U_i = I_{k_i}\}$$

D: set of diagonal scaling matrix

$$\text{D} = \{\text{diag}[\ d_1 I_{k_1}, d_2 I_{k_2}, \cdots, d_n I_{k_n}] \ \text{ where } d_i \in R,\ d_i > 0\}$$

Definition 1 [5].

The structured singular value of M, $\mu(M)$, is defined such that $\mu^{-1}(M)$ is equal to the smallest $\bar{\sigma}(\Delta)$ needed to make $(I+M\Delta)$ singular, i.e.,

$$\mu^{-1}(M) = \min_{\Delta \in \Delta_\nu} \{\bar{\sigma}(\Delta) \mid \det(I+M\Delta)=0\}. \quad (2)$$

If no $\Delta \in \Delta_\nu$ exists such that $\det(I+M\Delta)=0$, then $\mu(M)=0$.

Definition 1 is not typically useful in computing μ since the implied optimization problem is cumbersome. Therefore we use equation (3)

$$\max_{U \in \mathrm{U}} \rho(UM) \le \mu(M) \le \inf_{D \in \mathrm{D}} \bar{\sigma}(DMD^{-1}) \quad (3)$$

in the optimization procedure.

Theorem 1 [5]. (Robust performance condition)

The robust performance is said to be achieved if and only if $\mu(M)$ with respect to the structured uncertainty $\Delta \in \Delta_{\nu=1}$, satisfies

$$\mu(M) < 1. \quad (4)$$

Theorem 1 may be interpreted as a 'generalized small gain theorem' which also takes into account the structure of Δ.

Ⅲ. Loop shaping in NLCF

A normalized left coprime factorization of nominal plant G is a left coprime factorization $(\tilde{N}, \tilde{M})$ of G which satisfies

$$\tilde{N}\tilde{N}^* + \tilde{M}\tilde{M}^* = I, \text{ for all } s \in jR \quad (5)$$

or equivalently, $[\tilde{N}, \tilde{M}]$ is co-inner, where R is the field of real numbers.

The perturbed plant G_Δ given by

$$G_\Delta = (\tilde{M}+\Delta_M)^{-1}(\tilde{N}+\Delta_N), \quad (6)$$

can be stabilized by a single controller K given by

$$\inf_{\text{stabilizing } K} \left\| \begin{bmatrix} S \\ KS \end{bmatrix} \tilde{M}^{-1} \right\|_\infty = \gamma_{\min} = \varepsilon_{\max}^{-1}, \quad (7)$$

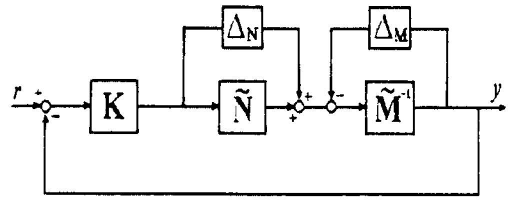

그림 3. 정규화된 LCF 구조
Fig. 3. The structure of NLCF.

where $S=(I+GK)^{-1}$ and K is chosen over all stabilizing controllers. The maximum stability margin $\varepsilon_{\max}$ is given by

$$\varepsilon_{\max} = (1 - \|\tilde{N}, \tilde{M}\|_H^2)^{1/2}, \quad (8)$$

where $\|\cdot\|_H$ means hankel norm and defined as follows.

Assume that (A, B, C, D) be the minimal realization of $G \in H^\infty$, and let $P=P^T$, and $Q=Q^T$ be the solutions of the Lyapunov equations

$$AP + PA^T + BB^T = 0 \quad (9)$$

and

$$A^TQ + QA + C^TC = 0 \quad (10)$$

where P and Q are the controllability and observability gramians, respectively. Then the hankel norm of G is defined as

$$\|G\|_H = \bar{\sigma}[(PQ)^{1/2}]. \quad (11)$$

The robust stabilization problem of (7) does not directly address performance. To overcome this, Mcfarlane and Glover proposed that performance and robustness tradeoffs are addressed by implementing a loop shaping solution and a brief outline is as follows.

(i) The nominal plant, G, and 'shaping functions' W_1, W_2 are combined to form the 'shaped plant', G_s where $G_s = W_2GW_1$. We assume that W_1 and W_2 are such that G_s contains no hidden unstable modes.

(ii) Calculate $\varepsilon_{\max}$. If $\varepsilon_{\max} \ll 1$ return to (i) and adjust W_1 and W_2.

(iii) Select $\epsilon \le \epsilon_{max}$, then synthesize a feedback controller K_∞ which robustly stabilizes the normalized left coprime factorization of G_s.

(iv) The final feedback controller K is then constructed by combining the H^∞ controller K_∞, with the shaping functions W_1 and W_2 such that $K = W_1 K_\infty W_2$.

Ⅳ. Robust performance improvement

In this section, we describe the key features of the controller design technique for robust performance improvement in NLCF. In order to do this, we consider the 4-block structure (12)

$$\inf_{\text{stabilizing } K} \left\| \begin{matrix} S & SG \\ KS & KSG \end{matrix} \right\|_\infty = \inf_{\text{stabilizing } K} \| M \|_\infty = \gamma_{min} = \epsilon_{max}^{-1} \quad (12)$$

which is equivalent to equation (7). The equivalence of equation (7) and (12) can be seen by noting the fact that infinity norm is invariant under right multiplication by a co-inner matrix and then multiplying $\begin{bmatrix} S \\ KS \end{bmatrix} \tilde{M}^{-1}$ by $[\ \tilde{M}\ \ \tilde{N}\ ]$ which is co-inner[6]. Also, equation (12) can be derived by considering robust stabilization in the face of simultaneous input and output uncertainty shown in Figure 4. The class of perturbed plants which is stabilized by a single controller obtained from equation (12) can be represented by

$$G_\Delta = (I + \Delta_1)^{-1} G (I + \Delta_2);\ \|\Delta\|_\infty = \left\| \begin{matrix} \Delta_1 & 0 \\ 0 & \Delta_2 \end{matrix} \right\|_\infty < \epsilon. \quad (13)$$

Although the uncertainty of Figure 4 represents simultaneous input multiplicative and inverse output multiplicative uncertainty, it can still be considered to be a left coprime factor uncertainty:

$$G_\Delta = \tilde{M}_\Delta^{-1}\ \tilde{N}_\Delta, \quad (14)$$

$$\tilde{M}_\Delta = \tilde{M}(I + \Delta_M),\ \tilde{N}_\Delta = \tilde{N}(I + \Delta_N), \quad (15)$$

i.e., input multiplicative uncertainties on each of the left coprime factors of G, so that any perturbed plant can be expressed in terms of stable uncertainties.

It is apparent that H^∞ robust stabilization may not be appropriate for the procedure of loop shaping. First, it ignores the obvious structure of the uncertainty. Second, the loop shaping design procedure robustly stabilizes only a shaped plant for which the uncertainty description may not be representative of actual uncertainty. In this case we design against uncertainty in the loop shaping weights.

Let $\Delta_s = \begin{bmatrix} \Delta_{s_1} & 0 \\ 0 & \Delta_{s_2} \end{bmatrix}$ be the shaped uncertainty, and $\Delta_a = \begin{bmatrix} \Delta_{a1} & 0 \\ 0 & \Delta_{a2} \end{bmatrix}$ be the actual uncertainty against which we would like the resulting design to be robust stable. These uncertainties are shown in Figure 4, 5, respectively.

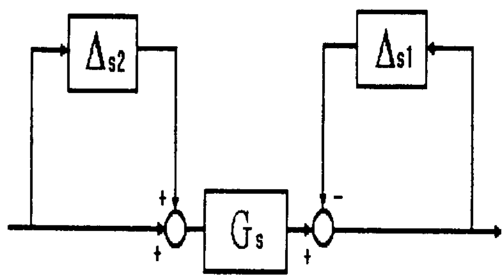

그림 4. 4-블럭문제를 가지는 섭동구조
Fig. 4. Uncertainty structure for the 4-block problem.

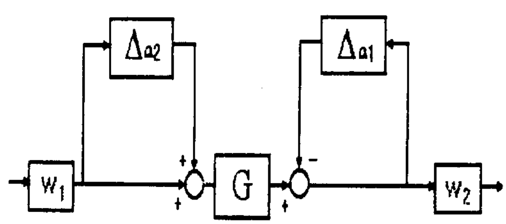

그림 5. 루프쉐이핑에 대한 실질적인 섭동구조
Fig. 5. Actual perturbation for loop shaping design procedure.

By equating uncertainty at plant input and output, we have

$$\begin{bmatrix} \Delta_{a1} & 0 \\ 0 & \Delta_{a2} \end{bmatrix} = \begin{bmatrix} W_2^{-1}\Delta_{s_1}W_2 & 0 \\ 0 & W_1\Delta_{s_2}W_1^{-1} \end{bmatrix}. \quad (16)$$

And we can deduce that the closed loop system will be robustly stable to any actual perturbations such that

$$\|\Delta_a\|_\infty < \frac{\varepsilon}{\max\{\sup_\omega c(W_1), \sup_\omega c(W_2)\}}, \quad (17)$$

where c denotes the condition number, i.e., $\bar{\sigma}(\cdot)/\underline{\sigma}(\cdot)$. If the loop shaping weights are not well conditioned, then the actual uncertainty bound will be less than that indicated by loop shaping design procedure. Therefore we select weighting functions considering condition number, and then design a controller satisfying equation (7). This controller does not give rise to a good performance due to the restriction of loop shaping weighting functions. To get a good performance, we motivate the use of structured singular value.

In here, $\varepsilon_{\max}$ always satisfies $\varepsilon_{\max} < 1$ and $\mu(M) = \varepsilon_{\max}^{-1} = \gamma_{\min}$ for some shaped plant, equation (12) never satisfies $\mu(M) < 1$. To overcome this, we motivate the use of structured singular value, and then use μ-synthesis technique. The μ-synthesis problem does not yet as complete a solution as does the H^∞ synthesis problem. A reasonable approach would be to try to find a stabilizing controller K and scaling D so that

$$\|DMD^{-1}\|_\infty < 1. \quad (18)$$

One method to do this is to alternately minimize the above expression for either K and D while holding the other constant. For fixed D, the left-hand side of equation (18) is just an H^∞ control problem and can be solved using H^∞-optimization controller design algorithms. For fixed K, the left-hand side of equation (18) can be minimized at each frequency as a convex optimization problem in D. This is often referred to as the D-K iteration and then, repeat the process till satisfy equation (18). If designed controller satisfies robust performance condition, then we use the weighting factor W_a in order to improve robust performance.

Theorem 2.

Let $M = \begin{bmatrix} S & SG \\ KS & KSG \end{bmatrix}$, $M^a = \begin{bmatrix} W_aS & SG \\ W_aKS & KSG \end{bmatrix}$ where W_a is real rational weighting functions in H^∞, and satisfies $\|W_a\|_\infty \le 1$. If the upper bound of the structured singular value $\mu(M)$ satisfies $\inf_{D\in \mathbf{D}} \bar{\sigma}(DMD^{-1}) < 1$, then $\mu(M^a)$ satisfies robust performance condition.

(Proof)

$$\begin{bmatrix} D_1 & 0 \\ 0 & D_2 \end{bmatrix}\begin{bmatrix} W_aS & SG \\ W_aKS & KSG \end{bmatrix}\begin{bmatrix} D_1 & 0 \\ 0 & D_2 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} D_1 & 0 \\ 0 & D_2 \end{bmatrix}\begin{bmatrix} S & SG \\ KS & KSG \end{bmatrix}\begin{bmatrix} W_aI & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} D_1 & 0 \\ 0 & D_2 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} D_1 & 0 \\ 0 & D_2 \end{bmatrix}\begin{bmatrix} S & SG \\ KS & KSG \end{bmatrix}\begin{bmatrix} D_1 & 0 \\ 0 & D_2 \end{bmatrix}^{-1}\begin{bmatrix} W_aI & 0 \\ 0 & I \end{bmatrix}$$

$$\Rightarrow \bar{\sigma}(DM^aD^{-1}) \le \bar{\sigma}(DMD^{-1})\bar{\sigma}\begin{bmatrix} W_aI & 0 \\ 0 & I \end{bmatrix}$$

$$\Leftrightarrow \bar{\sigma}(DM^aD^{-1}) \le \bar{\sigma}(DMD^{-1})$$

$$\Rightarrow \inf_{D\in \mathbf{D}} \bar{\sigma}(DM^aD^{-1}) \le \inf_{D\in \mathbf{D}} \bar{\sigma}(DMD^{-1})$$

If $\inf_{D\in \mathbf{D}} \bar{\sigma}(DMD^{-1}) < 1$,

then $\inf_{D\in \mathbf{D}} \bar{\sigma}(DM^aD^{-1}) \le \inf_{D\in \mathbf{D}} \bar{\sigma}(DMD^{-1}) < 1$

$$\Rightarrow \mu(M^a) \le \inf_{D\in \mathbf{D}} \bar{\sigma}(DM^aD^{-1}) \le \inf_{D\in \mathbf{D}} \bar{\sigma}(DMD^{-1}) < 1$$

$$\Rightarrow \mu(M^a) < 1.$$ ■

In theorem 2, weighting factor W_a can be looked upon as a weighting function of inverse output multiplicative uncertainty(or equivalently output disturbance). In other words, we can select weighting factor W_a as weighting function of general H^∞-optimization problem. For example, we use W_a to improve performance such that steady state error, overshoot, rising time, and tracking of the reference input. After all, the designed final controller satisfies

$$\mu(M^a) \le \inf_{D\in \mathrm{D}} \bar{\sigma}(DM^a D^{-1}) \le \inf_{D\in \mathrm{D}} \bar{\sigma}(DMD^{-1}) < 1, \qquad (19)$$

and the robust performance improvement can be obtained by a scaling factor of D-K iteration and weighting factor W_a.

The procedure of robust performance improvement in NLCF are as follows:

step 1. Select loop shaping weighting functions so that $c(W_1)\approx 1$ and $c(W_2)\approx 1$.

step 2. Design a controller using a procedure of loop shaping.

step 3. Transform the NLCF uncertain structure into the 4-block structure.

step 4. Design the controller using D-K iteration and determine scaling factor.

step 5. Choose the weighting factor W_a and design the final controller.

The upper bound of $\mu(M^a)$ relating the singular value of element M and the condition number of loop shaping weighting functions are very important. Therefore we investigate some facts from the following theorem and lemma.

Theorem 3 [8].

Consider the $M=\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22}\end{bmatrix}$ and the structured singular value defined by definition 1. At each frequency, $\mu(M)$ satisfies the following bounds:

$$\mu(M) \le \{\bar{\sigma}(M_{12})\bar{\sigma}(M_{21})\}^{1/2} + \max\{\bar{\sigma}(M_{11}), \bar{\sigma}(M_{22})\}$$

(Proof) See reference [8].

Lemma 1.

The upper bound of $\mu(M^a)$ is

$$\mu(M^a) \le \varepsilon^{-1}\{(\frac{1}{1+\underline{\sigma}^2(G_s)})^{1/2} c(W_2) + (\frac{\bar{\sigma}^2(G_s)}{1+\bar{\sigma}^2(G_s)})^{1/2} c(W_1)\}$$

$$+ \{\bar{\sigma}(M_{12})\bar{\sigma}(M_{21})\}^{1/2}, \text{ if}$$

$\mu(M) \le \max\{\bar{\sigma}(M_{11}), \bar{\sigma}(M_{22})\} + \{\bar{\sigma}(M_{12})\bar{\sigma}(M_{21})\}^{1/2}$, where

$G_s = W_2 G W_1,\ \varepsilon = \gamma^{-1}$.

(Proof)

$$\mu(M) \le \max\{\bar{\sigma}(M_{11}), \bar{\sigma}(M_{22})\} + \{\bar{\sigma}(M_{12})\bar{\sigma}(M_{21})\}^{1/2}.$$

$$\Rightarrow \mu(M) \le \bar{\sigma}(M_{11}) + \bar{\sigma}(M_{22}) + \{\bar{\sigma}(M_{12})\bar{\sigma}(M_{21})\}^{1/2}$$

where $\bar{\sigma}(M_{11}) = \bar{\sigma}(S) \le \gamma\{\bar{\sigma}(\tilde{M}_s)c(W_2)\}$, and $\bar{\sigma}(M_{22}) = \bar{\sigma}(KSG) \le \gamma\{\bar{\sigma}(\tilde{N}_s)c(W_1)\}$ (by theorem [6])

$$\Rightarrow \mu(M) \le \gamma[\bar{\sigma}(\tilde{M}_s)c(W_2) + \bar{\sigma}(\tilde{N}_s)c(W_1)] + \{\bar{\sigma}(M_{12})\bar{\sigma}(M_{21})\}^{1/2}$$

In equation (5), we have

$$\tilde{M}_s\tilde{M}_s^* = I - \tilde{N}_s\tilde{N}_s^* = I - \tilde{M}_s W_2 G W_1 W_1^* G^* W_2^* \tilde{M}_s^*$$

$$\Rightarrow \tilde{M}_s^*\tilde{M}_s = (I + W_2 G W_1 W_1^* G^* W_2^*)^{-1}$$

$$\Rightarrow \bar{\sigma}(\tilde{M}_s) = (\frac{1}{1+\underline{\sigma}^2(W_2 G W_1)})^{1/2}$$

$$\tilde{N}_s\tilde{N}_s^* = G_s^*\tilde{M}_s^*\tilde{M}_s G_s$$

$$= (W_1^* G^* W_2^*)(W_2 G W_1)(I + W_2^* G^* W_1^* W_1 G W_2)^{-1}$$

$$\Rightarrow \bar{\sigma}(\tilde{N}_s) = (\frac{\bar{\sigma}^2(W_2 G W_1)}{1+\bar{\sigma}^2(W_2 G W_1)})^{1/2}.$$

$$\therefore\ \mu(M) \le \gamma[(\frac{1}{1+\underline{\sigma}^2(W_2 G W_1)})^{1/2} c(W_2)$$

$$+ (\frac{\bar{\sigma}^2(W_2 G W_1)}{1+\bar{\sigma}^2(W_2 G W_1)})^{1/2} c(W_1)] + \{\bar{\sigma}(M_{12})\bar{\sigma}(M_{21})\}^{1/2}$$

$$\Rightarrow \mu(M) \le \gamma[(\frac{1}{1+\underline{\sigma}^2(G_s)})^{1/2} c(W_2) + (\frac{\bar{\sigma}^2(G_s)}{1+\bar{\sigma}^2(G_s)})^{1/2}$$

$$c(W_1)] + \{\bar{\sigma}(M_{12})\bar{\sigma}(M_{21})\}^{1/2}$$

$$\Rightarrow \mu(M^a) \le \mu(M) \le \gamma[(\frac{1}{1+\underline{\sigma}^2(G_s)})^{1/2} c(W_2) + (\frac{\bar{\sigma}^2(G_s)}{1+\bar{\sigma}^2(G_s)})^{1/2}$$

$$c(W_1)] + \{\bar{\sigma}(M_{12})\bar{\sigma}(M_{21})\}^{1/2}$$

$$\Rightarrow \mu(M^a) \le \gamma[(\frac{1}{1+\underline{\sigma}^2(G_s)})^{1/2} c(W_2) + (\frac{\bar{\sigma}^2(G_s)}{1+\bar{\sigma}^2(G_s)})^{1/2}$$

$$c(W_1)] + \{\bar{\sigma}(M_{12})\bar{\sigma}(M_{21})\}^{1/2}$$

$$\Rightarrow \mu(M^a) \le \frac{1}{\varepsilon}[(\frac{1}{1+\underline{\sigma}^2(G_s)})^{1/2} c(W_2) + (\frac{\bar{\sigma}^2(G_s)}{1+\bar{\sigma}^2(G_s)})^{1/2}$$

$$c(W_1)] + \{\bar{\sigma}(M_{12})\bar{\sigma}(M_{21})\}^{1/2} \qquad \blacksquare$$

Lemma 1 shows that the upper bound of $\mu(M^a)$ concerns the condition number of loop shaping weighting functions. If we choose weighting functions with large condition number, then the upper bound of $\mu(M^a)$ becomes larger. Hence constrained loop shaping weighting functions, such that $c(W_1)\approx 1$ and $c(W_2)\approx 1$, give a tight upper bound of $\mu(M^a)$. Conclusively, since the loop shaping weighting functions affect robust performance in NLCF, robust performance can be improved by proposed method.

V. Example

This section presents a simple design example to illustrate the proposed method. We take the underwater vehicle model as follows.

$$A=\begin{bmatrix} -0.7203 & 0 & -0.2939 & -2.8025 & 0 \\ -0.001 & -4.5853 & 10.9267 & -0.0142 & 0 \\ 0.0188 & 12.176 & -33.7341 & -0.1663 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -23.148 & 0 \end{bmatrix}$$

$$B=[\ 0\ \ -19.5148\ \ -122.4154\ \ 0\ \ 0]^T \qquad (20)$$

$$C=[\ 0\ \ 0\ \ 0\ \ 0\ \ 1]\quad D=[\ 0]\ .$$

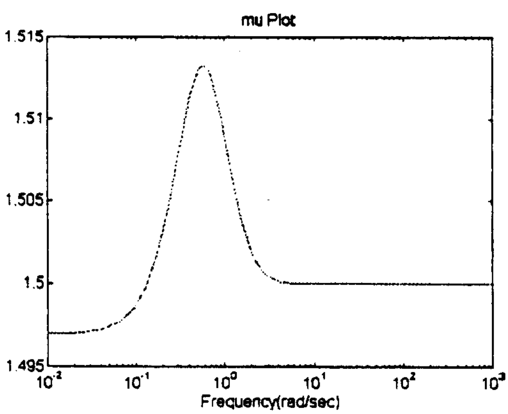

그림 6. 3단계에서의 μ-plot
Fig. 6. μ-plot in step 3.

The simulation of this example is implemented by Matlab and μ-Toolbox[9]. First, we design a controller using step 1, 2, $W_1=(0.9s+2)/(s+1.2)$, $W_2=1$, and then transform to the 4-block structure. However, from figure 6, we know that the designed controller does not satisfy the robust performance condition. To cope with this, we motivate the use of D-K iteration, and the result is shown in figure 7. Finally, to improve robust performance, we select $W_a=(0.8s+1)/(s+5)$, and get a good result as shown in figure 8.

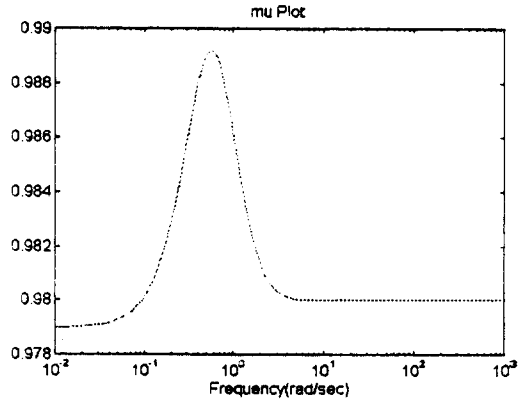

그림 7. 4단계에서의 μ-plot
Fig. 7. μ-plot in step 4.

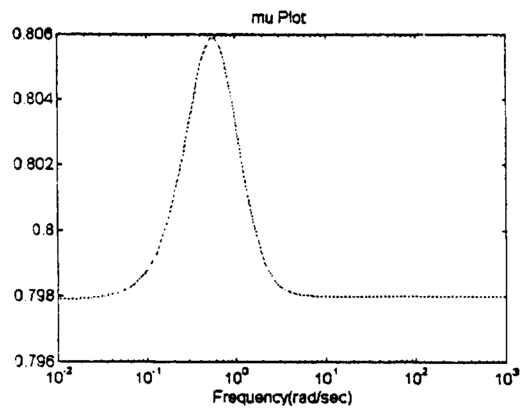

그림 8. 5단계에서의 μ-plot
Fig. 8. μ-plot in step 5.

VI. Conclusion

This paper proposes a method of robust performance improvement in feedback systems which have NLCF uncertainty. For this, we motivate the use of the structured singular value and investigate a condition number of loop shaping weighting functions. In this case we select loop shaping weighting functions of

$c(W_1) \approx 1$ and $c(W_2) \approx 1$, and then design a controller. To improve the robust performance, factor W_a and scaling factor of D-K iteration are used in the final controller design procedure. And, we find the upper bound of $\mu(M^a)$ relating the singular value of element M and the condition number of loop shaping weighting functions.

참 고 문 헌

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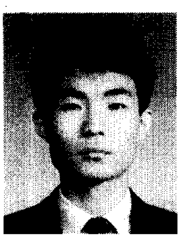

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