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전력 계통 정적 전압 안정도 해석법의 동일 근거에 관한 연구

A Study on the Identical Basis of Static Voltage Stability Analysis Methods in Power Systems

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Abstract – The voltage stability problem has recently been dealt with in the literature from various points of view. The diverse theories have been established in voltage stability analysis because of the complicates of power systems and diverse phenomena of voltage collapse. Through rigorous mathematical operations, this paper shows that all the major methods used in static voltage stability analysis, i.e. – Jacobian method, voltage sensitivity method, real and reactive power loss sensitivity method and energy function method – have an identical background in theory. The results from the test in sample systems have shown the validity of this verification.

Key Words : Voltage stability, Voltage sensitivity, Loss sensitivity, Energy function, Stable Equilibrium Point, Unstable Equilibrium Point.

1. Introduction

The voltage stability problem is a relatively new research subject in power system analysis. The incidents of large scale blackouts caused by voltage instability in 1970's in Europe and America gave an impetus for the theoretical development of voltage stability analysis. Moreover, the blackout in Tokyo in 1987 attracted increasing attentions throughout the world. However, diverse theories of voltage stability analysis have been resulted from diverse phenomena of voltage collapse. Various approaches have been presented by analyzing only local phenomenon of voltage collapse. The main difference between conventional system analysis and voltage stability analysis is due to the fact that voltage instability is a structural instability of system caused by parameter variation. Consequently, it is very difficult to formalize and solve the problem. Jacobian method[1, 2], voltage sensitivity method[3, 4], real and reactive power loss sensitivity method[5, 6] and energy function method[7, 8] are the major methods used in static voltage stability analysis up to now. In this paper, it is proven that all the above-mentioned major methods have an identical background in theory. Finally, two illustrative examples are given for several sample systems, which will show the validity of this verification.

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2. Classification of Static Voltage Stability Analysis Methods

In this section, various methods of voltage stability analysis are examined to show that all of the methods have an identical background in theory.

2.1 Jacobian Method

Voltage instability is due to the structural instability of the system resulted from the change of system parameters. Some special case of parameter changes the system state transfers to the saddle node. If this case happens, small variations of the parameters can make the system state branch off into two kinds of nodes, i.e. – stable node and unstable node(Bifurcation Phenomenon). The Jacobian method exploits the condition of saddle node bifurcation by examining power flow solutions with the changes in bus injection powers. Let the given system be represented as below.

$$\dot{x} = f(x, u) \tag{1}$$

It is well known that the saddle node bifurcation condition is given as follows[9].

$$\left| \frac{\partial f}{\partial x} \right|_{x = x^u} = 0 \tag{2}$$

For a power system, the bifurcation condition is given by

$$|J| = \begin{vmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial V} \end{vmatrix} = 0 \text{ with } \left|\frac{\partial P}{\partial \theta}\right| \neq 0 \qquad (3)$$

On the basis of the Jacobian method, the determinant or the least absolute value of the eigenvalues is sometimes utilized as a performance index of voltage collapse.

2.2 Voltage Sensitivity Method

The voltage sensitivity method is the most direct approach using the voltage sensitivity to system parameters. The voltage sensitivity goes to infinity at the collapse point. This collapse condition should be identical with that of the Jacobian method, which will be proven here. From the power flow equations, we can get the following differential relationship.

$$\begin{bmatrix} dP \\ dQ \end{bmatrix} = \begin{bmatrix} H & N \\ M & L \end{bmatrix} \begin{bmatrix} d\theta \\ dV \end{bmatrix} \qquad (4)$$

where $H=\frac{\partial P}{\partial \theta}$, $N=\frac{\partial P}{\partial V}$, $M=\frac{\partial Q}{\partial \theta}$, $L=\frac{\partial Q}{\partial V}$, which are the sub matrices of the Jacobian of the power flow equations. Here, it should be noted that all the sub matrices H, N, M and L are nonsingular in the normal operation state of the power system since they have dominant diagonal elements. From Eq.(4), $d\theta$ and dV are calculated as follows.

$$\begin{bmatrix} d\theta \\ dV \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} dP \\ dQ \end{bmatrix} \qquad (5)$$

where

$$A = (H-NL^{-1}M)^{-1}$$

$$B = -H^{-1}N(L-MH^{-1}N)^{-1}$$

$$C = -L^{-1}M(H-NL^{-1}M)^{-1}$$

$$D = (L-MH^{-1}N)^{-1}$$

Eq.(5) gives

$$dV = C\cdot dP + D\cdot dQ \qquad (6)$$

In the above equations, $\|C\|$ or $\|D\|$ should be infinite in order to have the infinite voltage sensitivity at a certain bus. That is, matrices $C^{-1} = -(H-NL^{-1}M)M^{-1}L$ or $D^{-1} = L-MH^{-1}N$ should be singular at a voltage collapse point. Regarding the singularity of matrices C^{-1} and D^{-1}, we have developed the following theorems.

<u>Theorem 2.1</u>

Let matrices H, N, M and L be nonsingular. If any of the matrices A^{-1}, B^{-1}, C^{-1} and D^{-1} is singular, then the others are also singular.

<u>Proof</u>

Matrix A^{-1} can be rewritten as follows.

$$\begin{aligned} A^{-1} = H-NL^{-1}M &= (I-NL^{-1}MH^{-1})H \\ &= N(N^{-1}-L^{-1}MH^{-1})H \\ &= NL^{-1}(LN^{-1}-MH^{-1})H \\ &= NL^{-1}(L-MH^{-1}N)N^{-1}H \end{aligned}$$

Hence,

$$det(A^{-1}) = det(D^{-1})\cdot det(L^{-1})\cdot det(H) \qquad (7)$$

Similarly, we can obtain the following equations.

$$\begin{aligned} det(B^{-1}) &= -det(L-MH^{-1}N)\cdot det(N^{-1}H) \\ &= -det(D^{-1})\cdot det(N^{-1})\cdot det(H) \end{aligned} \qquad (8)$$

$$\begin{aligned} det(C^{-1}) &= -det(H-NL^{-1}M)\cdot det(M^{-1}L) \\ &= -det(A^{-1})\cdot det(M^{-1}L) \\ &= -det(D^{-1})\cdot det(M^{-1})\cdot det(H) \end{aligned} \qquad (9)$$

Hence, when matrix D^{-1} is singular, matrices A^{-1}, B^{-1} and C^{-1} are also singular. The other cases can be proved in a similar way.

Q.E.D.

<u>Theorem 2.2</u>

Let matrices H, N, M and L be nonsingular. If any of the matrices A^{-1}, B^{-1}, C^{-1} and D^{-1} is singular, then the Jacobian is also singular.

<u>Proof</u>

$$J = \begin{bmatrix} H & N \\ M & L \end{bmatrix} = \begin{bmatrix} I & N \\ MH^{-1} & L \end{bmatrix}\begin{bmatrix} H & 0 \\ 0 & I \end{bmatrix}$$

$$= \begin{bmatrix} N & N \\ MH^{-1}N & L \end{bmatrix}\begin{bmatrix} N^{-1} & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} H & 0 \\ 0 & I \end{bmatrix}$$

Hence,

$$\begin{aligned} det(J) &= det\left(\begin{bmatrix} N & N \\ MH^{-1}N & L \end{bmatrix}\right)\cdot det\left(\begin{bmatrix} N^{-1} & 0 \\ 0 & I \end{bmatrix}\right)\cdot det\left(\begin{bmatrix} H & 0 \\ 0 & I \end{bmatrix}\right) \\ &= det\left(\begin{bmatrix} N & 0 \\ MH^{-1}N & L-MH^{-1}N \end{bmatrix}\right)\cdot det(N^{-1})\cdot det(H) \\ &= det(L-MH^{-1}N)\cdot det(N)\cdot det(N^{-1})\cdot det(H) \\ &= det(L-MH^{-1}N)\cdot det(H) \\ &= det(D^{-1})\cdot det(H) \end{aligned} \qquad (10)$$

From Theorem 2.1, when any of the matrices A^{-1}, B^{-1} and C^{-1} is singular, matrix D^{-1} is singular. Consequently, Eq.(10) proves that the Jacobian is singular if any of the matrices A^{-1}, B^{-1}, $C^{-1}$and D^{-1} is singular.

Q.E.D.

By combining Theorem 2.1 and Eq.(6), we can conclude that the voltage collapse condition of the voltage sensitivity method implies that matrices A^{-1}, B^{-1}, $C^{-1}$and D^{-1} are all singular. In reverse, if any of A^{-1}, B^{-1}, $C^{-1}$and D^{-1} is singular, the voltage sensitivities at some buses are infinite. Therefore, we can confirm that both of the Jacobian method and the voltage sensitivity method provide the identical voltage collapse condition by using Theorem 2.1 and 2.2.

2.3 Power Loss Sensitivity Method

The voltage collapse phenomenon is accompanied with the rapid increase in line flows. At the collapse point, the loss increasing ratios $\frac{dP_{loss}}{dP}$, $\frac{dP_{loss}}{dQ}$, $\frac{dQ_{loss}}{dP}$ and $\frac{dQ_{loss}}{dQ}$ go to infinity. This fact provides the theoretical background for the power loss sensitivity method. In this section, it is proven that the voltage collapse condition by the power loss sensitivity method is identical with that by the Jacobian method. Real and reactive power losses of transmission lines can be expressed as a function of system state vectors, V and θ.

$$P_{loss} = f_P(\theta, V)$$

$$Q_{loss} = f_Q(\theta, V)$$

Hence,

$$\begin{bmatrix} dP_{loss} \\ dQ_{loss} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_{loss}}{\partial \theta} & \frac{\partial P_{loss}}{\partial V} \\ \frac{\partial Q_{loss}}{\partial \theta} & \frac{\partial Q_{loss}}{\partial V} \end{bmatrix} \begin{bmatrix} d\theta \\ dV \end{bmatrix}$$

The substitution of Eq.(5) into the above equation gives

$$\begin{bmatrix} dP_{loss} \\ dQ_{loss} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_{loss}}{\partial \theta} & \frac{\partial P_{loss}}{\partial V} \\ \frac{\partial Q_{loss}}{\partial \theta} & \frac{\partial Q_{loss}}{\partial V} \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} dP \\ dQ \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\partial P_{loss}}{\partial \theta} A + \frac{\partial P_{loss}}{\partial V} C & \frac{\partial P_{loss}}{\partial \theta} B + \frac{\partial P_{loss}}{\partial V} D \\ \frac{\partial Q_{loss}}{\partial \theta} A + \frac{\partial Q_{loss}}{\partial V} C & \frac{\partial Q_{loss}}{\partial \theta} B + \frac{\partial Q_{loss}}{\partial V} D \end{bmatrix} \begin{bmatrix} dP \\ dQ \end{bmatrix} \quad (11)$$

In the above equations, it is noted that at least one of the matrices A, B, C and D should have an infinite norm in order to make the differential sensitivity of P_{loss} or Q_{loss} infinity. That is, at least one of the matrices A^{-1}, B^{-1}, $C^{-1}$and D^{-1} should be singular, which makes the Jacobian matrix singular by Theorem 2.2. As a result, it can be concluded that the power loss sensitivity method is identical to the Jacobian method.

2.4 Energy function method

Energy function method is to determine the system stability by comparing the energy difference between unstable equilibrium points(UEPs) and a stable equilibrium point(SEP). If the voltage of SEP and the voltage of UEP coincide, the energy difference between them will be zero and bring about voltage collapse. The saddle node bifurcation phenomenon occurs when UEP coincides with SEP, which also makes the energy difference zero. Consequently, the energy function method provides the same voltage collapse condition as the saddle node bifurcation method. By using the energy function, the saddle node bifurcation conditions can be expressed as follows.

$$\left.\frac{\partial \nu}{\partial x}\right| = 0 \quad (12)$$

$$\left|\frac{\partial^2 \nu}{\partial x^2}\right|_{x = x_{SEP}} = 0 \quad (13)$$

The Eq.(12) describes the equilibrium condition of the system with the use of the energy function. This equation yields a SEP and UEPs as the solutions. Eq.(13) represents the condition that the SEP coincides with an UEP, which is just the voltage collapse condition. In order to keep the consistency in the voltage stability analysis, this condition should agree with the collapse condition of the Jacobian method that the Jacobian matrix is singular at the voltage collapse point.

The energy function includes several path-dependent integral terms. To solve the path-dependency problem, we assume that P_{mi}, P_{Li}, Q_{Ci} and Q_{Li} are all constants and

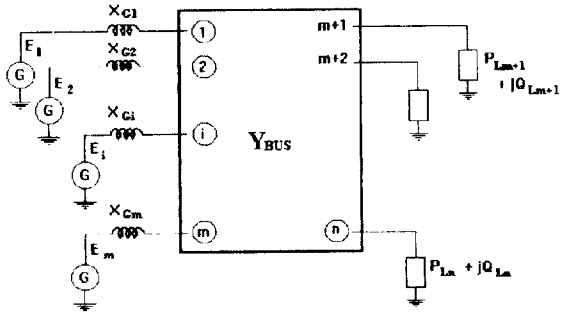


Fig. 1 Multiport Representation of Power Systems

that line resistances are small enough to be ignored. The power systems can be represented by the multiport network (Fig. 1).

The energy function of the power system is given by[10]:

$$\nu = \frac{1}{2}\sum_{j=1}^{n}\Big\{\sum_{i\neq j}^{n} b_{ij}(V_i^2 - V_{i0}^2) + \sum_{i\neq j}^{n} b_{ij}(V_{i0}V_{j0}\cos\theta_{ij0} - V_iV_j\cos\theta_{ij})\Big\} + \sum_{i=1}^{n} P_{Li}(\theta_i - \theta_{i0}) - \sum_{i=1}^{n}(Q_{Ci} - Q_{Li})\log\frac{V_i}{V_{i0}} + \frac{1}{2}\sum_{i=1}^{m} b_{Gi}(E_i^2 - E_{i0}^2) + \sum_{i=1}^{m}\{b_{Gi}(E_{i0}V_{i0}\cos(\delta_{io} - \theta_{i0}) - E_iV_i\cos(\delta_i - \theta_i))\} + \frac{1}{2}\sum_{i=1}^{m} M_i\,\omega_i^2 - \sum_{i=1}^{m} P_{mi}(\delta_i - \delta_{i0}) - \sum_{i=1}^{m}\int_{E_{i0}}^{E_i}\frac{Q_{Gi}}{E_i}dE_i \quad (14)$$

where

b_{ij} : Line susceptance between bus i and bus j

b_{Gi} : Generator internal susceptance

M_i : Generator inertia , ω_i : Angle velocity

E_i : Generator internal voltage, V_i : Bus Voltage

P_{Mi} : Generator mechanical output, P_{Li}: Real power load

Q_{Gi} : Generator reactive power output

Q_{Ci} : Reactive power compensation, Q_{Li}: Reactive power load

m : Number of generators, n : Number of buses

With the use of energy function Eq.(14), the partial differentials with respect to V_i and E_i are given by :

$$\frac{\partial\nu}{\partial V_i} = \sum_{j\neq i} b_{ij}V_i - \sum_{j\neq i} b_{ij}V_j\cos\theta_{ij} - \frac{Q_{Ci} - Q_{Li}}{V_i} - b_{Gi}E_i\cos(\delta_i - \theta_i) = 0 \quad (i = 1, 2, \ldots, n) \quad (15.a)$$

$$\frac{\partial\nu}{\partial E_i} = b_{Gi}E_i - b_{Gi}V_i\cos(\delta_i - \theta_i) - \frac{Q_{Gi}}{E_i} = 0 \quad (i = 1, 2, \ldots, m) \quad (15.b)$$

Multiplying Eq. (15.a) and Eq. (15.b) by V_i and E_i respectively gives :

$$Q_{Gi} - Q_{Li} = \sum_{j\neq i} b_{ij}V_i^2 - \sum_{j\neq i} b_{ij}V_iV_j\cos\theta_{ij} - b_{Gi}E_iV_i\cos(\delta_i - \theta_i) \quad (i = 1, 2, \ldots, n) \quad (16.a)$$

$$Q_{Gi} = b_{Gi}E_i^2 - b_{Gi}E_iV_i\cos(\delta_i - \theta_i) \quad (i = 1, 2, \ldots, m) \quad (16.b)$$

If the above Q_{Gi} is viewed from the system bus i, this will be converted as follows.

$$Q_{Gi}' = -b_{Gi}V_i^2 + b_{Gi}E_iV_i\cos(\delta_i - \theta_i) \quad (i = 1, 2, \ldots, m) \quad (16.c)$$

By substituting Eq.(16.c) into Eq.(16.a) for all generator buses, we can obtain the following reactive power equation at system bus i.

$$Q_{Gi}' + Q_{Ci} - Q_{Li} = \sum_{j\neq i} b_{ij}V_i^2 - \sum_{j\neq i} b_{ij}V_iV_j\cos\theta_{ij} - b_{Gi}V_i^2 \quad (17)$$

Here we will assume that E_i is constant for the consistency with the other voltage stability methods, which are derived in the basis of bus voltage and angle. In a similar way, differentiating Eq.(14) with respect to $\delta_i - \theta_i$ yields the following equations.

$$\frac{\partial\nu}{\partial\delta_i} = b_{Gi}E_iV_i\sin(\delta_i - \theta_i) - P_{mi} = 0 \quad i\leq m \quad (18.a)$$

$$\frac{\partial\nu}{\partial\theta_i} = P_{Li} + \sum_{j\neq i} b_{ij}V_iV_j\sin\theta_{ij} - b_{Gi}E_iV_i\sin(\delta_i - \theta_i) = 0 \quad i\leq m \quad (18.b)$$

$$\frac{\partial\nu}{\partial\theta_i} = P_{Li} + \sum_{j\neq i} b_{ij}V_iV_j\sin\theta_{ij} = 0 \quad i > m \quad (18.c)$$

By substituting Eq.(18.a) into Eq.(18.b) and combining Eq.(18.c), we can obtain following real power equation.

$$\frac{\partial\nu}{\partial\theta_i} = -P_{mi} + P_{Li} + \sum_{j\neq i} b_{ij}V_iV_j\sin\theta_{ij} = 0 \quad (i = 1, 2, \ldots, n) \quad (19.a)$$

where $P_{mi} = 0$ if $i > m$

By using Eq.(16.c), we can eliminate $\cos(\delta_i - \theta_i)$ in Eq.(15.a) and then Eq.(15.a) can be rewritten as :

$$\frac{\partial\nu}{\partial V_i} = \sum_{j\neq i} b_{ij}V_i - \sum_{j\neq i} b_{ij}V_j\cos\theta_{ij} - \frac{Q_{Gi}' + Q_{Ci} - Q_{Li}}{V_i} - b_{Gi}V_i \quad (i = 1, 2, \ldots, n) \quad (19.b)$$

Obviously the above equation is the reactive power equation in the power flow problem. The second order partial differential of the energy function can be obtained by differentiating Eqs.(15.b), (18.a), (19.a) and (19.b), which yields the following voltage collapse condition.

$$\left|\frac{\partial^2 \nu}{\partial x^2}\right|_{x=x^0} = \begin{vmatrix} \frac{\partial^2 \nu}{\partial \omega^2} & \frac{\partial^2 \nu}{\partial \omega \partial \delta} & \frac{\partial^2 \nu}{\partial \omega \partial \theta} & \frac{\partial^2 \nu}{\partial \omega \partial V} \\ \frac{\partial^2 \nu}{\partial \delta \partial \omega} & \frac{\partial^2 \nu}{\partial \delta^2} & \frac{\partial^2 \nu}{\partial \delta \partial \theta} & \frac{\partial^2 \nu}{\partial \delta \partial V} \\ \frac{\partial^2 \nu}{\partial \theta \partial \omega} & \frac{\partial^2 \nu}{\partial \theta \partial \delta} & \frac{\partial^2 \nu}{\partial \theta^2} & \frac{\partial^2 \nu}{\partial \theta \partial V} \\ \frac{\partial^2 \nu}{\partial V \partial \omega} & \frac{\partial^2 \nu}{\partial V \partial \delta} & \frac{\partial^2 \nu}{\partial V \partial \theta} & \frac{\partial^2 \nu}{\partial V^2} \end{vmatrix}_{x=x^0}$$

$$= \begin{vmatrix} K & 0 & 0 & 0 \\ 0 & T & 0 & 0 \\ 0 & 0 & H' & N' \\ 0 & 0 & M' & L' \end{vmatrix}_{x=x^0} = 0 \tag{20}$$

Here, the elements of K, T, H', N', M' and L' are as follows.

$$K_{ii} = M_i, \quad K_{ij} = 0 \tag{21.a}$$

$$T_{ii} = b_{Gi} E_i V_i \cos(\delta_i - \theta_i), \quad T_{ij} = 0 \tag{21.b}$$

$$H_{ij}' = H_{ij}, \quad N_{ij}' = N_{ij}, \quad M_{ij}' = M_{ij} / V_i, \quad L_{ij}' = L_{ij} / V_i \tag{21.c}$$

Since $|K| \neq 0$ and $|T| \neq 0$, the Eq.(20) can be rewritten as follows.

$$\frac{V_{slack}}{(V_1 V_2 \cdots V_n)} \begin{vmatrix} H & N \\ M & L \end{vmatrix} = 0 \tag{22}$$

This shows that the voltage collapse condition derived from the energy function is exactly the same as that of the Jacobian method.

3. Illustrative Examples

To verify the results of this study, we choose the 2-bus and 5-bus sample systems. The voltage collapse point of 2-bus system is analytically derived and the voltage collapse condition of 5-bus system is numerically examined.

3.1 2-bus System

Consider the 2-bus sample system with an infinite bus shown in Fig. 2.

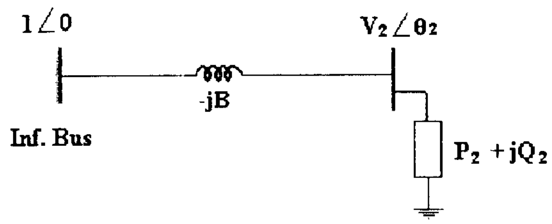


Fig. 2 One-line Diagram of 2-bus System

The power flow equation in the load bus is given as follows.

$$P_2 = V_2 B \sin\theta_2 \tag{23}$$

$$Q_2 = -BV_2\cos\theta_2 + V_2^2 B \tag{24}$$

These equations are very basic equations in voltage stability analysis for the 2-bus system, which will be invoked in all kinds of the voltage stability analysis methods.

3.1.1 Jacobian Method

By differentiating the above equations, we have the following equations.

$$\begin{bmatrix} dP_2 \\ dQ_2 \end{bmatrix} = \begin{bmatrix} V_2 B\cos\theta_2 & B\sin\theta_2 \\ V_2 B\sin\theta_2 & -B\cos\theta_2 + 2V_2 B \end{bmatrix} \begin{bmatrix} d\theta_2 \\ dV_2 \end{bmatrix} = J \begin{bmatrix} d\theta_2 \\ dV_2 \end{bmatrix} \tag{25}$$

Voltage collapse condition is $|J| = 0$, so

$$2V_2\cos\theta_2 = 1 \tag{26}$$

From Eqs.(23) and (26) eliminating θ_2 yields the next collapse voltage.

$$V_2 = \sqrt{\frac{1}{4} + \frac{P_2^2}{B^2}} \tag{27}$$

3.1.2 Voltage Sensitivity Method

The power flow equations for the system in Fig.1 are given by Eqs.(23) and (24). From these two equations eliminating θ_2 yields

$$P_2^2 + (Q_2 - V_2^2 B)^2 = V_2^2 B^2 \tag{28}$$

Differentiating and arranging the above equation yields the voltage sensitivities as follows.

$$\frac{dV_2}{dP_2} = \frac{P_2 + (Q_2 - V_2^2 B)\frac{dQ_2}{dP_2}}{V_2 B^2 - 2V_2^3 B^2 + 2V_2 BQ_2} \tag{29}$$

$$\frac{dV_2}{dQ_2} = \frac{P_2\frac{dP_2}{dQ_2} + Q_2 - V_2^2 B}{V_2 B^2 - 2V_2^3 B^2 + 2V_2 BQ_2} \tag{30}$$

In the observation of the above two equations, it is obvious that the voltage sensitivities become infinite when common denominator is zero, that is,

$$Q_2 = V_2^2 B - \frac{1}{2}B \tag{31}$$

Substituting Eq.(31) into Eq.(28), we can obtain the collapse voltage same as given Eq.(27).

3.1.3 Power Loss Sensitivity Method

Since there is no active power loss in the given system, we will examine only the reactive power loss. The reactive power loss of the system is given as follows.

$$Q_{loss} = B - 2V_2 B\cos\theta_2 + V_2^2 B \tag{32}$$

From Eq.(24) and Eq.(32) eliminating θ_2 yields

$$Q_{loss} = B + 2Q_2 - V_2^2 B \tag{33}$$

Differentiating the above equation yields the reactive power loss sensitivity as follows.

$$\frac{dQ_{loss}}{dQ_2} = 2 - 2V_2 B\frac{dV_2}{dQ_2} \tag{34}$$

Here, $\frac{dV_2}{dQ_2}$ should be infinite to make the above sensitivity infinity, which is the same condition as given in Eq.(30). Consequently, the collapse voltage is given same as in Eq.(27). On the other hand, the reactive power loss sensitivity to real power increase can be calculated as follows. From Eq.(23) and Eq.(32) eliminating θ_2 yields

$$\left[\frac{P_2}{V_2}B\right]^2 + \left[\frac{\frac{1}{2}(B + V_2^2 B - Q_{loss})}{V_2 B}\right]^2 = 1 \tag{35}$$

Differentiating the above equation with respect to P_2 yields the following loss sensitivity.

$$\frac{dQ_{loss}}{dP_2} = -\frac{(V_2^3 B^2 + \frac{1}{2}V_2 B^2 - V_2 B Q_{loss})\frac{dV}{dP_2} + 2P_2}{\frac{1}{2}(Q_{loss} - B - V_2^2 B)} \tag{36}$$

Here, it is necessary that either the denominator should be zero or $\frac{dV_2}{dP_2}$ should be infinite to make the above sensitivity infinite. First, when the denominator is zero, the next equation is derived by using Eq.(32).

$$V_2\cos\theta_2 = 0 \tag{37}$$

But the above equation corresponds to the condition that $|H| = 0$, which is the condition of angle instability. Hence, this should be excluded. When $\frac{dV_2}{dP_2}$ is infinite, this condition is the same as in Eq.(31). Consequently, the collapse voltage is given same as in Eq.(27).

3.1.4 Energy Function Method

The static energy function for the 2-bus system can be derived by using Eq.(14).

$$\nu(\theta_2, V_2) = \frac{1}{2}B(V_2^2 - V_{20}^2) - B(V_2\cos\theta_2 - V_{20}\cos\theta_{20}) - P_2(\theta_2 - \theta_{20}) - Q_2\log\frac{V_2}{V_{20}} \tag{38}$$

First, applying Eq.(12) to Eq.(38) yields the next power flow equations as mentioned earlier.

$$P_2 = V_2 B\sin\theta_2 \tag{23}$$

$$Q_2 = -BV_2\cos\theta_2 + V_2^2 B \tag{24}$$

Next, applying Eq.(13) to Eq.(38) yields

$$\begin{vmatrix} \frac{\partial^2\nu}{\partial\theta_2^2} & \frac{\partial^2\nu}{\partial\theta_2\partial V_2} \\ \frac{\partial^2\nu}{\partial V_2\partial\theta_2} & \frac{\partial^2\nu}{\partial V_2^2} \end{vmatrix} = \frac{1}{V_2}\begin{vmatrix} V_2 B\cos\theta_2 & B\sin\theta_2 \\ V_2 B\sin\theta_2 & -B\cos\theta_2 + 2V_2 B \end{vmatrix}$$

$$= \frac{1}{V_2}|J| = 0 \tag{39}$$

The above condition is the same as in Eq.(26). Consequently, the collapse voltage is given same as in Eq.(27).

As mentioned earlier, both UEP and SEP of energy function become the solutions of power flow equations. The relationship between both solutions is shown in Fig.3 and Fig.4. Here, the following assumptions were adopted. 1) Load has lagging power factor. 2) Reactive load is 2 p.u. 3) Reactance of transmission line is 0.04 p.u. As the real load P_2 increases, two solutions of power flow equations approach each other and finally coalesce at the load P_2 = 10.35 in Fig.3. At this moment, we can observe that both

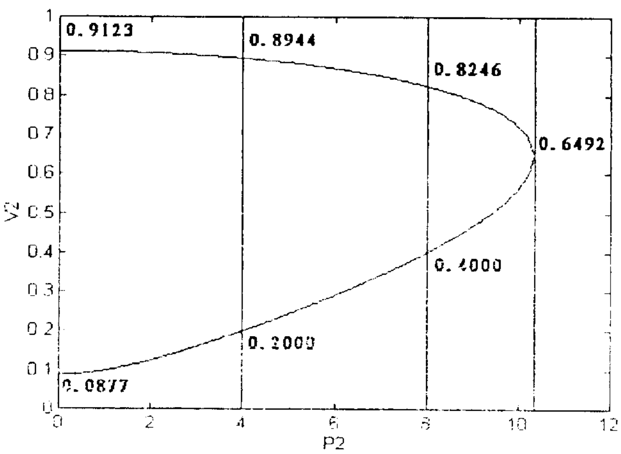


Fig. 3 Transition of Multiple Power Flow Solutions with Real Power Increase

UEP and SEP of energy function coalesce and energy difference between them becomes zero in Fig.4. Here, it is noted that the voltage collapse occurs when the system loses the stable equilibrium point by the change of system parameters. Fig.4 illustrates this fact in manifest. Hence, the collapse voltage is $V_2 = 0.6492$ and this value is exactly the same that is calculated by using Eq.(27).

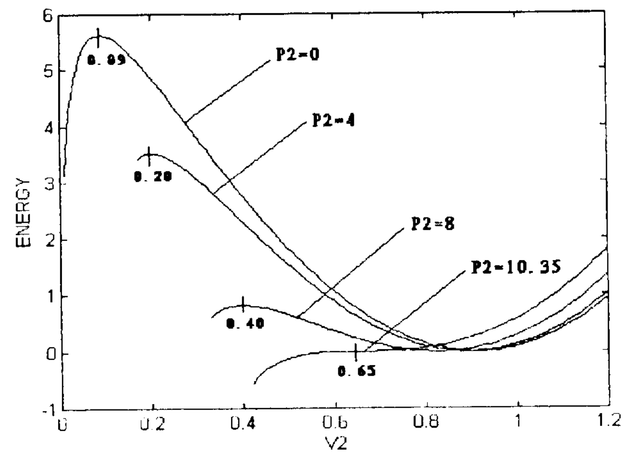


Fig. 4 Transition of UEP & SEP with Real Power Increase

3.2 5-bus System

The Stagg & El-Abiad's 5-bus system[11] is taken as a sample system for an illustrative example. We considered two scenarios of load increase for the 5-bus system. The first scenario(Fig.6) is scheduled so as to increase all of the loads with identical rates. The second scenario(Fig.7) is scheduled so as to increase only the load at No.5 bus while the loads at the other buses remain constant. In both scenarios, the power factor of each load is assumed to be constant, and the load increasing rate K is defined as the ratio of the increased load per the normal load.

In Fig.6 and Fig.7, (a) represents the Jacobian's singularity by using the minimum singular value of J. (b),(c),(d) and (e) represent the bus voltages, the voltage sensitivities and the power loss sensitivities respectively.

Fig.6 shows that the voltage collapse occurs at the load increasing rate K=3.035. At this point the Jacobian matrix

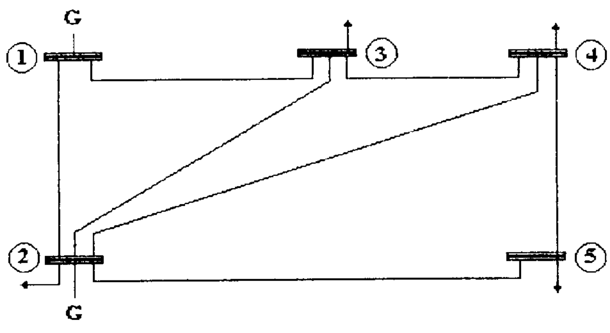
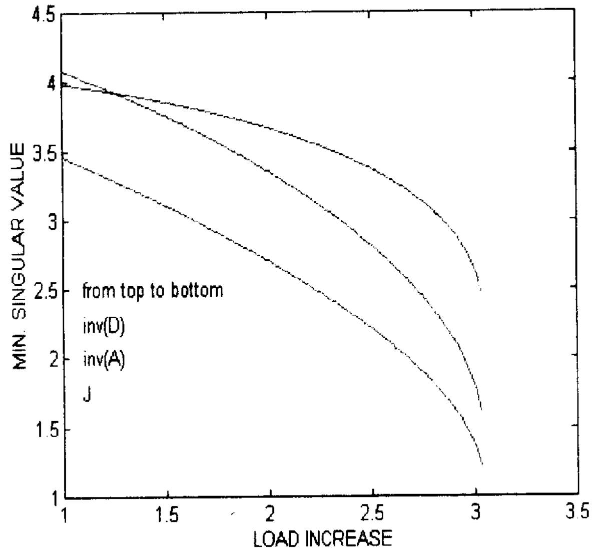
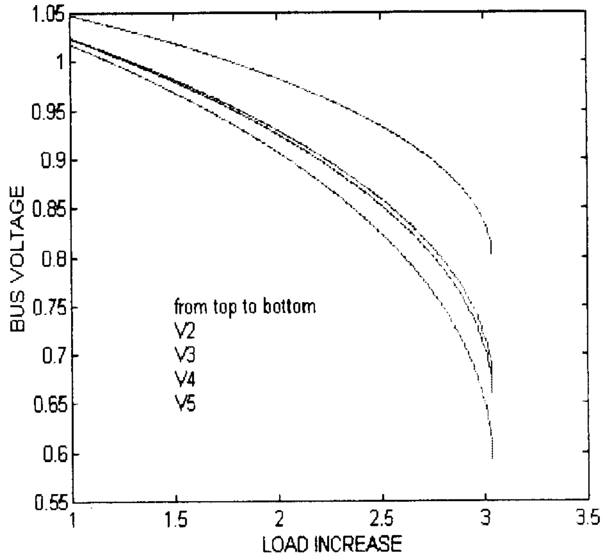


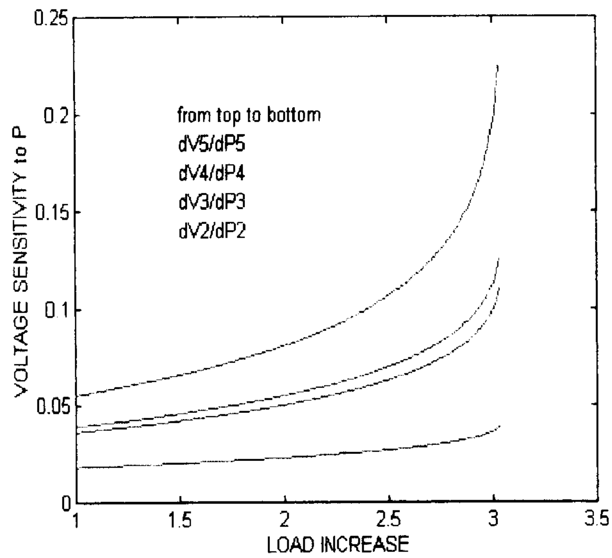
Fig. 5 One Line Diagram of the 5-bus System



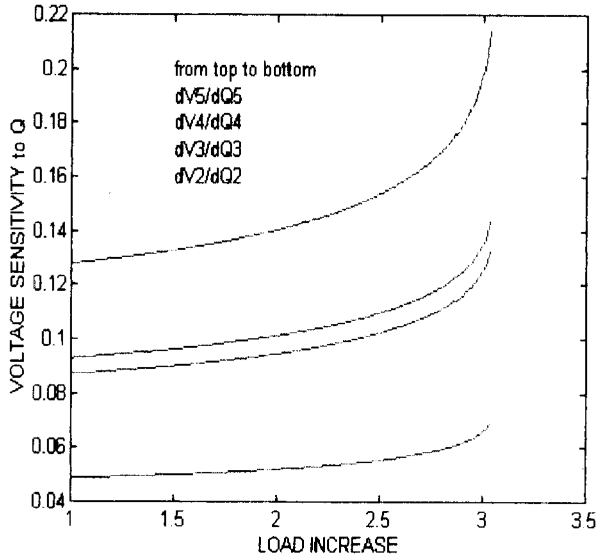
(a) Minimum Singular Values



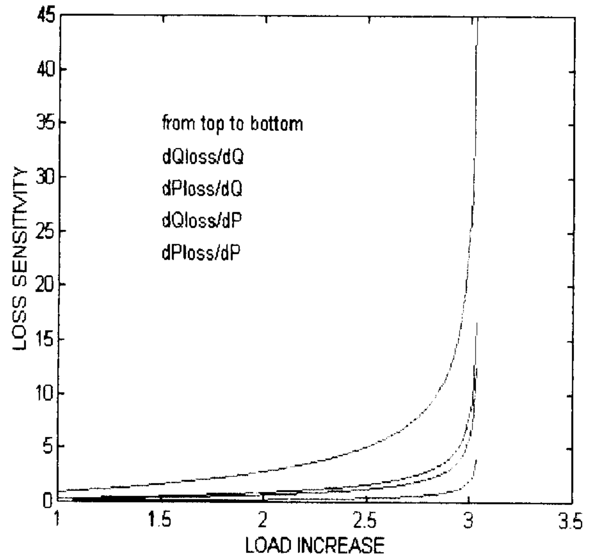
(b) Bus Voltages



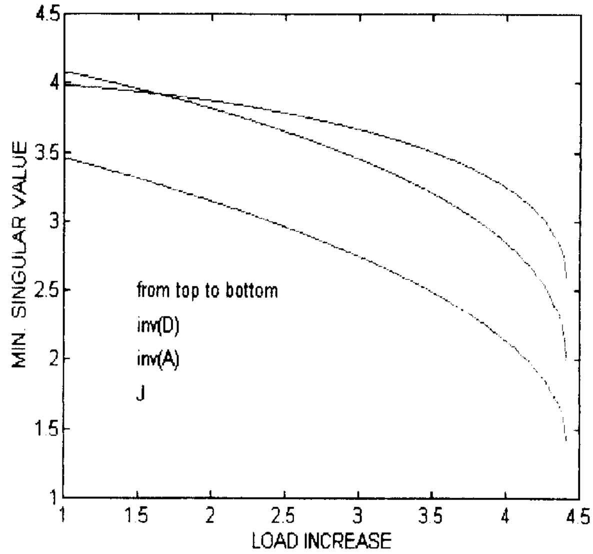
(c)Voltage Sensitivities to P



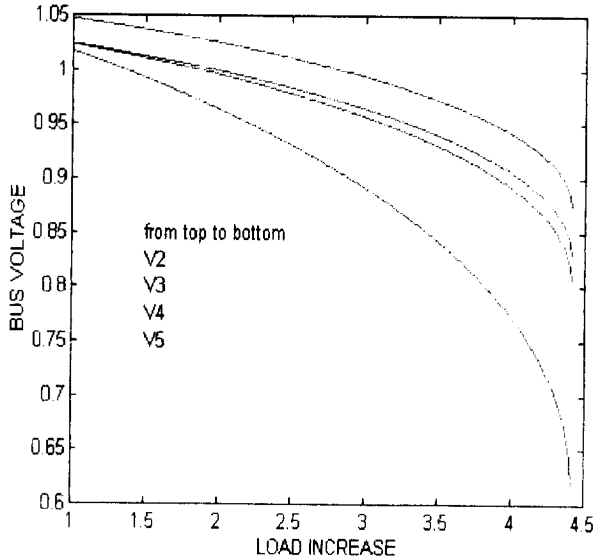
(d)Voltage Sensitivities to Q



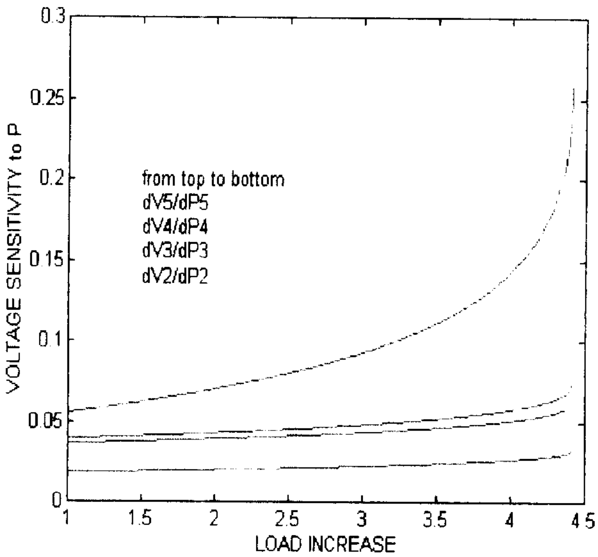
(e)Power Loss Sensitivities



(a) Minimum Singular Values



(b) Bus Voltages

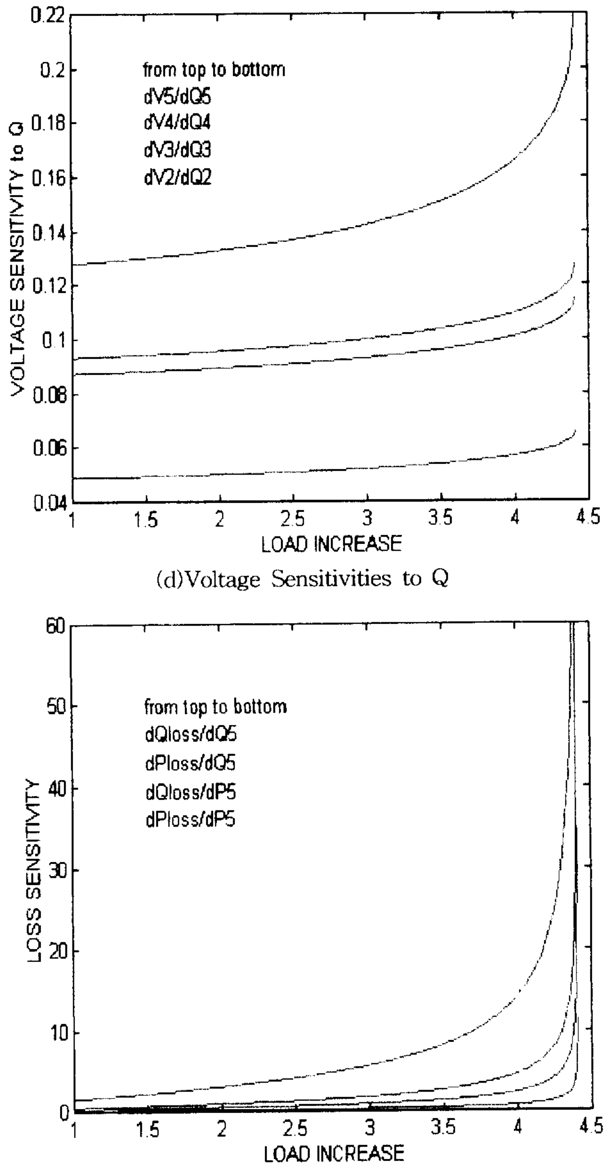


(c)Voltage Sensitivities to P

Fig. 6 In case of All Loads Increasing

becomes singular, and both of the voltage sensitivities and the loss sensitivities go to the infinity. In Fig.7, we can observe that the voltage collapse occurs when the load increasing rate becomes K=4.410 at bus 5.

It is well known that there is a great difficulty in handling path dependent integral terms of the energy function. Here an alternative method is adopted instead of calculating the energy difference directly. As mentioned earlier, both UEPs and a SEP of energy function are the multiple power flow solutions, so if an UEP and a SEP coalesce, the energy difference between them becomes zero obviously. Theoretically, there can be 2^{n-1} power flow solutions for the lightly loaded n-bus system.[12] As the loads increase, the number of solutions decreases and



(d)Voltage Sensitivities to Q

(e)Power Loss Sensitivities

Fig. 7 In case of One Load Increasing

Table 1 Transition of SEP & UEP in Case of Increasing Loads at All Buses

Solutions	Voltage	K=2.50	K=3.00	K=3.03	K=3.035
SEP	V2	0.9337	0.8347	0.8133	0.7999
	V3	0.8591	0.7188	0.6890	0.6704
	V4	0.8525	0.7065	0.6753	0.6556
	V5	0.8239	0.6508	0.6116	0.5863
UEP	V2	0.6891	0.7659	0.7866	0.7999
	V3	0.5373	0.6237	0.6520	0.6704
	V4	0.5067	0.6062	0.6362	0.6556
	V5	0.3099	0.5190	0.5606	0.5863

Table 2 Transition of SEP & UEP in Case of Increasing Loads at No.5 Bus

Solutions	Voltage	K=3.00	K=4.00	K=4.40	K=4.410
SEP	V2	0.9940	0.9418	0.8852	0.8750
	V3	0.9643	0.9056	0.8414	0.8299
	V4	0.9569	0.8916	0.8200	0.8071
	V5	0.8912	0.7720	0.6420	0.6183
UEP	V2	0.7485	0.8057	0.8648	0.8750
	V3	0.6833	0.7502	0.8182	0.8299
	V4	0.6416	0.7175	0.7940	0.8071
	V5	0.2913	0.4493	0.5943	0.6183

eventually there remain one UEP and one SEP.[8] Table.1 and Table.2 show that heavily loaded system has only a pair of UEP and SEP. As the loads increase, we can see that these two solutions approach each other and coalesce eventually at the same load conditions shown in Fig.6 and Fig.7, i.e., K=3.035 and K=4.410 respectively.

4. Conclusions

Through rigorous mathematical operations and examples, this paper has shown that all of the major methods used in static voltage stability analysis, i.e. - Jacobian method, voltage sensitivity method, real and reactive power loss sensitivity method and energy function method - have an identical background in theory. The results can be summarized as follows.

(1) No matter which method is considered, the voltage collapse condition comes to a conclusion that the Jacobian of the power flow equations is singular.

(2) The identity of the methods of voltage collapse analysis is illustrated with the sample system study.

(3) The results of this study provide some ideas about inter-relationship between various VCPIs(Voltage Collapse Proximity Indices) defined on the basis of various methods.

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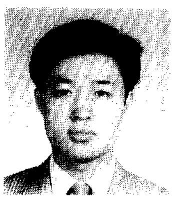

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