완전 모형 일치 기법을 이용한 분산 준최적 H_2 필터링

論 文
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Decentralized Suboptimal H_2 Filtering : An Exact Model Matching Approach

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Abstract - In this paper, the decentralized suboptimal H_2 filtering problem is considered. An additional term is added to the centralized optimal H_2 filter so that the whole filter is decentralized. We derive a necessary and sufficient condition for existence of proposed decentralized filters. By employing the solution procedure for the exact model matching problem, we obtain a set of decentralized H_2 filters, and choose a suboptimal filter from this set of decentralized H_2 filters.

Key Words : H_2 problem, Decentralized Filtering, Linear Fractional Transformation, Exact model matching problem

1. INTRODUCTION

The problem of designing filters for large-scale systems has been of great interest and many interesting result have been reported in the literature. When designing a filter for large-scale systems, the centralized scheme often fails to hold due to either lack of the overall information or lack of the centralized computing capability. An approach in [1,2] is to first construct a set of local filters for the independent subsystems and then to incorporate appropriate compensatory signals in order to account for the presence of interconnections among the subsystems. In [3], the unknown input observer theory was used to deal with the interconnection effect in designing local estimators. These schemes require the exchange of state estimates among the subsystem observers, or impose severe restriction on the decentralized system structure, specially on the interconnection pattern. To avoid these difficulties, a knowledge of the interconnection pattern is exploited in [4].

In this paper we present a new approach for the design of decentralized filter using the whole knowledge about the dynamics of overall systems. Our approach is motivated by the suboptimal H_2 controller parameterization result[5] and is based on the exact model matching problem[6]. We insert an auxiliary term into the centralized H_2 optimal filter so that the decentralized filtering is possible. Then, we will show that this decentralization problem is reduced to the exact model matching problem, and derive a existent condition of this filter. Using the result in [6] we obtain the set of proposed decentralized filters and reduce the H_2 norm of the filter by reducing that of design parameter only.

The plan of this paper is as follows: In Section 2, we propose a suboptimal H_2 filter which is stable and contains some design parameter. In Section 3, it is shown that the proposed suboptimal H_2 filter reduced to decentralized suboptimal H_2 filter when the design parameter satisfies some conditions. In Section 4, we present a design procedure for this decentralized suboptimal H_2 filter using the solution procedure for the exact model matching problem. In Section 5, an example is given, which illustrates the proposed decentralized H_2 filtering. Finally, we present our conclusions in Section 6.

Notations

(1) A transfer matrix in terms of state-space data is denoted

$$\left[\begin{array}{c:c} A & B \\ \hdashline C & D \end{array}\right] = C(sI-A)^{-1}B + D.$$

(2) Let $P(s)$ be a partitioned matrix with a state-space realization given by

$$P(s) = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}.$$

Then a linear fractional transformation of the partitioned matrix P and a matrix K is defined as

$$F(P,K) = P_{11} + P_{12}K(I-P_{22}K)^{-1}P_{21}.$$

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2. Suboptimal H2 filtering

In this section, we consider suboptimal H_2 filtering of LTI system driven by noise process w with unit variance:

$$\dot{x} = Ax + Bw \tag{2.1}$$

$$y = Cx + Dw, \tag{2.2}$$

where A, B, C and D are, respectively, $n\times n$, $n\times m$, $p\times n$ and $p\times m$ matrices, and assume

$$\begin{bmatrix} B \\ D \end{bmatrix} D^T = \begin{bmatrix} 0 \\ I \end{bmatrix}. \tag{2.3}$$

We also assume that (A, B) is stabilizable and (C, A) is detectable.

In state estimation problem, we seek to estimate a linear combination of the state vector defined by

$$z = Lx \tag{2.4}$$

where L is a $q\times n$ matrix.

Let $\hat{z}$ be an estimate of z generated from the observation y by a filter $K(s)$, that is,

$$\hat{z} = K(s)y. \tag{2.5}$$

The estimation error is

$$e = z - \hat{z}. \tag{2.6}$$

Then, we can write

$$\begin{bmatrix} e \\ y \end{bmatrix} = \left[\begin{array}{c:cc} A & B & 0 \\ \hdashline L & 0 & -I \\ C & D & 0 \end{array}\right] \begin{bmatrix} w \\ \hat{z} \end{bmatrix}.$$

In H_2 filtering problem, we seek to minimize the H_2-norm of the transfer function $T_{ew}(s)$ from the noise w to the estimation error e. Recall that 2-norm of the system G is the expected root-mean-square (RMS) value of the output when the input is a realization of a unit variance white noise process.

Since

$$y = C(sI-A)^{-1}Bw + Dw,$$

we have

$$\begin{aligned} e &= L(sI-A)^{-1}Bw - \hat{z} \\ &= L(sI-A)^{-1}Bw - K(s)[C(sI-A)^{-1}Bw + Dw]. \end{aligned}$$

We denote by P the solution of the Riccati equation

$$AP + PA^T - PC^TCP + BB^T = 0,$$

and define

$$F = -PC^T, \quad \hat{A}_0 = A + FC. \tag{2.7}$$

We decompose

$$K(s) = K_0(s) + K_1(s),$$

where K_0 is chosen as

$$K_0(s) = -L(sI-\hat{A}_0)^{-1}F. \tag{2.8}$$

Note that $K(s)$ reduced to the optimal filter, when

$K_1(s) = 0$.

From the fractional representations[7],

$$(sI-A)^{-1}B = [I + (sI-\hat{A}_0)^{-1}FC]^{-1}(sI-\hat{A}_0)^{-1}B.$$

We obtain

$$\begin{aligned} e &= L(sI-A)^{-1}Bw + L(sI-\hat{A}_0)^{-1}FC(sI-A)^{-1}Bw \\ &\quad + L(sI-\hat{A}_0)^{-1}FDw - K_1(s)y \\ &= L\{I + (sI-\hat{A}_0)^{-1}FC\}(sI-A)^{-1}Bw \\ &\quad + L(sI-\hat{A}_0)^{-1}FDw - K_1(s)y \\ &= L(sI-\hat{A}_0)^{-1}Bw + L(sI-\hat{A}_0)^{-1}FDw - K_1(s)y \\ &= L(sI-\hat{A}_0)^{-1}[B + FD]w - K_1(s)y. \end{aligned}$$

From the fractional representations[7],

$$\begin{aligned} &C(sI-A)^{-1}B + D \\ &\quad = [I + C(sI-\hat{A}_0)^{-1}F]^{-1} \\ &\qquad [C(sI-\hat{A}_0)^{-1}(B + FD) + D] \end{aligned}$$

We also have

$$\begin{aligned} K_1(s)y &= K_1(s)[I + C(sI-\hat{A}_0)^{-1}F]^{-1} \\ &\qquad [C(sI-\hat{A}_0)^{-1}(B+FD) + D]w \end{aligned}$$

We let

$$K_1(s) = Q(s)[I + C(sI-\hat{A}_0)^{-1}F],$$

that is,

$$Q(s) = K_1(s)[I + C(sI-\hat{A}_0)^{-1}F]^{-1}. \tag{2.9}$$

In section 3, we will construct a decentralized suboptimal filter $K(s)$ by choosing $Q(s)$ appropriately.

Now, we have

$$K_1(s)y = Q(s)[I + C(sI - \widehat{A}_0)^{-1}F]\, y$$
$$= Q(s)[C(sI - \widehat{A}_0)^{-1}(B + FD) + D]\, w,$$

$$\widehat{z} = K_0(s)\, y + K_1(s)\, y$$
$$= -L(sI - \widehat{A}_0)^{-1}Fy + Q(s)[I + C(sI - \widehat{A}_0)^{-1}F]\, y.$$

Therefore, the filter $K(s)$ can be represented as a linear fractional representation of the partitioned matrix $M(s)$ and $Q(s)$ as in Figure 1,

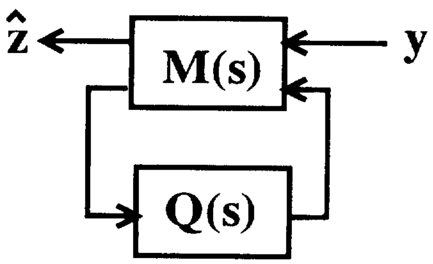

Fig. 1 LFT representation of K(s)

where

$$M(s) = \left[\begin{array}{c:cc} \widehat{A}_0 & F & 0 \\ \hline -L & 0 & I \\ C & I & 0 \end{array}\right] = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix},$$

where

$$M_{11}(s) = -L(sI - \widehat{A}_0)^{-1}F, \quad M_{12}(s) = I,$$
$$M_{21}(s) = I + C(sI - \widehat{A}_0)^{-1}F, \quad M_{22}(s) = 0. \qquad (2.10)$$

With $K(s)$ defined as above, we obtain

$$e = L(sI - \widehat{A}_0)^{-1}[B + FD]w$$
$$- K_1(s)[I + C(sI - \widehat{A}_0)^{-1}F]^{-1}$$
$$[C(sI - \widehat{A}_0)^{-1}(B + FD) + D]w$$
$$= L(sI - \widehat{A}_0)^{-1}[B + FD]w - Q(s)$$
$$[C(sI - \widehat{A}_0)^{-1}(B + FD) + D]w$$
$$= G_f(s)\, w - Q(s)\, U(s)\, w,$$

where

$$G_f(s) = \left[\begin{array}{c:c} \widehat{A}_0 & B + FD \\ \hline L & 0 \end{array}\right], \qquad (2.11)$$

$$U(s) = \left[\begin{array}{c:c} \widehat{A}_0 & B + FD \\ \hline C & D \end{array}\right]. \qquad (2.12)$$

Therefore, it follows that

$$T_{ew}(s) = G_f(s) - Q(s)U(s). \qquad (2.13)$$

Lemma 2.1 $U(s)$ has no invariant zero in CRHP.

Proof: Suppose that λ is an invariant zero of $U(s)$, then

$$\begin{bmatrix} \lambda I - \widehat{A}_0 & B+FD \\ -C & D \end{bmatrix} \text{ is not full rank.} \qquad (2.14)$$

If this matrix is premultiplied by $\begin{bmatrix} I & -F \\ 0 & I \end{bmatrix}$, then

$$\begin{bmatrix} I & -F \\ 0 & I \end{bmatrix}\begin{bmatrix} \lambda I - \widehat{A}_0 & B+FD \\ -C & D \end{bmatrix} = \begin{bmatrix} \lambda I - A & B \\ C & D \end{bmatrix}.$$

Thus (2.14) implies that there exists nonzero vector $[x_1^* \ x_2^*]$ so that

$$[x_1^* \ x_2^*]\begin{bmatrix} \lambda I - A & B \\ C & D \end{bmatrix} = [0 \ 0]. \qquad (2.15)$$

From $x_1^* B + x_2^* D = 0$, postmultiplying D^T on each side of this equality, we obtain $x_2^* = 0$. Therefore (2.15) is reduced to

$$x_1^*[\lambda I - A \quad B] = [0 \quad 0].$$

Since x_1^* is nonzero, λ is an uncontrollable mode of (A, B). However (A, B) is stabilizable, and hence λ has negative real part.

Q.E.D.

Lemma 2.2 $T_{ew}(s)$ is in RH_2 if and only if $Q(s)$ is in RH_2.

Proof: (Sufficiency) obvious.

(Necessity) Note that $DD^T = I$ requires D is "fat" (i.e. has more columns than rows) and so is $U(s)$.

Since $T_{ew}(s) = G_f(s) - Q(s)U(s)$, we can represent $T_{ew}(s)$ as a LFT of $\widehat{T}(s)$ and $Q(s)$,

$$T_{ew}(s) = F(\widehat{T}, Q),$$

where

$$\widehat{T}(s) = \begin{bmatrix} G_f & -I \\ U & 0 \end{bmatrix} = \left[\begin{array}{c:cc} \widehat{A}_0 & B+FD & 0 \\ \hline L & 0 & -I \\ C & D & 0 \end{array}\right].$$

With a minimal realization of $Q(s)$, Theorem 4.2 in Limebeer[8] implies that $T_{ew}(s)$ has no unstable unobservable mode and every uncontrollable mode in $T_{ew}(s)$ is an invariant zero of $U(s)$. Since $U(s)$ has no invariant zero in CRHP, $T_{ew}(s)$ has no unstable uncontrollable mode. Hence $T_{ew}(s) \in RH_2$ implies that it has no unstable mode and so does $Q(s)$, that is, $Q(s) \in RH_2$.

Q.E.D.

From Lemma 2.2, it can be seen that we only need to consider $Q(s)$ in RH_2.

Theorem 2.3 The family of all filters such that $\| T_{ew} \|_2^2 \le \| G_f \|_2^2 + \varepsilon^2$ is the set of all transfer matrices from y to $\hat{z}$ in Fig 1, where $Q \in RH_2$, $\| Q \|_2^2 \le \varepsilon^2$.

Proof: Note that $\hat{A}_0$ is a stable matrix[5]. From (2.11) and (2.12),

$$
\begin{aligned}
&G_f(s)\, U(s)^{\sim} \\
&= L(sI - \hat{A}_0)^{-1}(B+FD)\, \{C(sI-\hat{A}_0)^{-1}(B+FD) + D\}^{\sim} \\
&= L(sI - \hat{A}_0)^{-1}\{BB^T(sI+\hat{A}_0^T)^{-1}(-C^T) + F \\
&\quad + FF^T(sI+\hat{A}_0^T)^{-1}(-C^T)\} \\
&= L(sI - \hat{A}_0)^{-1}\{BB^T + P(sI+\hat{A}_0^T) \\
&\quad + PC^TCP\}\,(sI+\hat{A}_0^T)^{-1}(-C^T) \\
&= L(sI - \hat{A}_0)^{-1}(sP+PA^T+BB^T)(sI+\hat{A}_0^T)^{-1}(-C^T) \\
&= L(sI - \hat{A}_0)^{-1}(sI-A+PC^TC)P(sI+\hat{A}_0^T)^{-1}(-C^T) \\
&= -LP(sI+\hat{A}_0^T)^{-1}C^T,
\end{aligned}
$$

which implies that $G_f U^{\sim} \in RH_2^{\perp}$. Hence we have,

$$
\begin{aligned}
&\langle Q(s)\, U(s)\, ,\, G_f(s) \rangle^2 \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty} trace[\, U^{\sim}(jw)Q^{\sim}(jw)\; G_f(jw)]dw \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty} trace[\, Q^{\sim}(jw)\; G_f(jw)U^{\sim}(jw)]dw \\
&= \langle Q(s)\, ,\, G_f(s)\, U^{\sim}(s) \rangle^2 \\
&= 0.
\end{aligned}
\tag{2.16}
$$

Similarly, it can be shown that

$$\langle G_f(s)\, ,\, Q(s)U(s) \rangle = 0. \tag{2.17}$$

From (2.12),

$$
\begin{aligned}
UU^{\sim} &= \{\, C(sI-\hat{A}_0)^{-1}(B+FD)+D\} \\
&\quad (B^T+D^TF^T)(-sI-\hat{A}_0^T)^{-1}C^T + D^T \\
&= C(sI-\hat{A}_0)^{-1}(B+FD) \\
&\quad (B^T+D^TF^T)(-sI-\hat{A}_0^T)^{-1}C^T \\
&\quad + C(sI-\hat{A}_0)^{-1}(B+FD)D^T \\
&\quad + D(B^T+D^TF^T)(-sI-\hat{A}_0^T)^{-1}C^T + I \\
&= C(sI-\hat{A}_0)^{-1}\{BB^T+FF^T+P(sI+\hat{A}_0^T) \\
&\quad -(sI-\hat{A}_0)P\}(-sI-\hat{A}_0^T)^{-1}C^T + I \\
&= C(sI-\hat{A}_0)^{-1}\{\, AP+PA^T-PC^TCP+BB^T\} \\
&\quad (-sI-\hat{A}_0^T)^{-1}C^T + I \\
&= I,
\end{aligned}
\tag{2.18}
$$

which implies that U is coinner.

Thus,

$$
\begin{aligned}
\| T_{ew} \|_2^2 &= \langle T_{ew}, T_{ew} \rangle \\
&= \langle G_f(s) - Q(s)U(s)\, ,\, G_f(s) - Q(s)U(s) \rangle \\
&= \langle G_f(s)\, , G_f(s) \rangle - \langle G_f(s)\, , \\
&\quad Q(s)U(s) \rangle - \langle Q(s)U(s)\, , G_f(s) \rangle \\
&\quad + \langle Q(s)U(s)\, , Q(s)U(s) \rangle \\
&= \| G_f \|_2^2 + \| Q \|_2^2.
\end{aligned}
\tag{2.19}
$$

Q.E.D.

3. Decentralized H2 filtering.

In this section, we consider a decentralized filtering problem. We partition

$$\hat{z} = \begin{bmatrix} \hat{z}_1 \\ \hat{z}_2 \end{bmatrix}, \quad y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}. \tag{3.1}$$

We seek to find a decentralized filter $K(s) = block-diag\,\{K_{11}(s),\ K_{22}(s)\}$ so that

$$\hat{z}_1 = K_{11}(s)y_1, \quad \hat{z}_2 = K_{22}(s)y_2. \tag{3.2}$$

while satisfying $\| T_{ew} \|_2^2 \le \| G_f \|_2^2 + \varepsilon^2$.

From Fig 1, it follows that

$$
\begin{aligned}
K(s) &= F(M, Q) \\
&= M_{11} + QM_{21},
\end{aligned}
\tag{3.3}
$$

where

$$M_{11} = -L(sI-\hat{A}_0)^{-1}F,$$

$$M_{21} = C(sI-\hat{A}_0)^{-1}F + I.$$

We first determine whether there exists a decentralized filter $K(s)$ so that $T_{ew} \in RH_2$.

We partition

$$L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}, \quad F = [F_1 F_2],$$

$$C=\begin{bmatrix} C_1 \\ C_2 \end{bmatrix}, \quad Q(s) = \begin{bmatrix} Q_1(s) \\ Q_2(s) \end{bmatrix} \tag{3.4}$$

where L_1, L_2, C_1 and C_2 are, respectively, $q_1 \times n$, $q_2 \times n$, $p_1 \times n$ and $p_2 \times n$ matrices.

Define

$$X_1 = -L_1(sI-\hat{A}_0)^{-1}F_2 \tag{3.5}$$

$$Y_1 = C(sI-\hat{A}_0)^{-1}F_2 + \begin{bmatrix} 0 \\ I_{p_2} \end{bmatrix} \tag{3.6}$$

$$X_2 = -L_2(sI-\hat{A}_0)^{-1}F_1 \tag{3.7}$$

$$Y_2 = C(sI-\hat{A}_0)^{-1}F_1 + \begin{bmatrix} I_{p_1} \\ 0 \end{bmatrix}. \tag{3.8}$$

Then, we have

Lemma 3.1 There exists a decentralized filter $K(s)$ such that $T_{ew} \in RH_2$ and $\| T_{ew} \|_2^2 \le \| G_f \|_2^2 + \varepsilon^2$ if the exact model matching problems (3.9) and (3.10) are solvable under the constraint that $Q(s) \in RH_2$, $\| Q(s) \|_2^2 \le \varepsilon^2$.

$$X_1 + Q_1(s) Y_1 = 0 \tag{3.9}$$

$$X_2 + Q_2(s) Y_2 = 0 \tag{3.10}$$

Proof: Since

$$\hat{z}_1 = K_{11}y_1 + K_{12}y_2,$$

$$\hat{z}_2 = K_{21}y_1 + K_{22}y_2,$$

there exists a decentralized filter if and only if $K_{12} = 0$ and $K_{21} = 0$. From (3.3) and (3.4),

$$\begin{aligned} K(s) &= M_{11} + QM_{21} \\ &= -\begin{bmatrix} L_1 \\ L_2 \end{bmatrix}(sI-\hat{A}_0)^{-1}[F_1 F_2] \\ &\quad + \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}\left[C(sI-\hat{A}_0)^{-1}[F_1 F_2] + \begin{bmatrix} I_{p_1} & 0 \\ 0 & I_{p_2} \end{bmatrix}\right] \\ &= \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}. \end{aligned}$$

It follows that

$$\begin{aligned} K_{12}(s) &= -L_1(sI-\hat{A}_0)^{-1}F_2 + Q_1 \\ &\quad \left\{ C(sI-\hat{A}_0)^{-1}F_2 + \begin{bmatrix} 0 \\ I_{p_2} \end{bmatrix}\right\} \\ &= X_1 + Q_1(s) Y_1 \end{aligned}$$

$$\begin{aligned} K_{21}(s) &= -L_2(sI-\hat{A}_0)^{-1}F_1 + Q_2 \\ &\quad \left\{ C(sI-\hat{A}_0)^{-1}F_1 + \begin{bmatrix} I_{p_1} \\ 0 \end{bmatrix}\right\} \\ &= X_2 + Q_2(s) Y_2. \end{aligned}$$

Hence, $K_{12} = 0$ and $K_{21} = 0$ if and only if

$$X_1 + Q_1(s) Y_1 = 0,$$

$$X_2 + Q_2(s) Y_2 = 0.$$

Q.E.D.

4. Decentralized Filter Design

Now we consider a practical design method for the suggested decentralized filter. The algorithm introduced below to design such filter utilizes results from the literature on exact model matching problem. So we introduce some results of exact model matching problem.

4.1 Model matching problem

The exact model matching problem (EMMP) is defined as follows[6]: Given proper rational $p \times m$ matrix P with $rank[P] = p \le m$ and proper stable $p \times q$ matrix T, find a proper stable rational matrix M such that the equation

$$PM = T \tag{4.1}$$

holds.

For a proper and stable solution M to exist, P and T have to satisfy certain conditions.

Theorem 4.1 Given proper P and proper stable T with $rank[P] = rank[T] = p$, there exist a proper stable solution M if and only if T has as its zeros all the RHP finite zeros and all the zeros at infinity of P together with their associated structure.

Proof: See [6].

Remark : If P is minimum phase and has no zeros at infinity, then there always exist proper stable solutions.

Since the Theorem 4.1 does not provide a convenient and direct way to determine whether T is appropriate, there is a need for simple and direct conditions which will help the designer to determine whether T contains the unavoidable unstable zeros together with the appropriate structure.

Theorem 4.2 Let $P=ND^{-1}$ be a right coprime polynomial factorization of P, and $T=N_T D_T^{-1}$ be a right coprime polynomial factorization of T. Then, the unstable zeros z_i, $i=1,\ldots,l$ of P together with their structure will appear in T if and only if

$$a_i N_T(z_i) = 0, \; i=1,\ldots,l \tag{4.2}$$

where a_i are determined from

$$a_i N(z_i) = 0. \; i=1,\ldots,l \tag{4.3}$$

Proof : See [9].

Remark : (4.2) is always equivalent to $a_i T(z_i) = 0$ since er T is stable, and (4.3) can be written as $a_i P(z_i) = 0$ when P does not have any poles at z_i.

4.2 Decentralized Filter Design

To find when there exists a decentralized filter, we rewrite (3.9) and (3.10) as follows:

$$Y_1^T Q_1^T = -X_1^T \tag{4.4}$$

$$Y_2^T Q_2^T = -X_2^T \tag{4.5}$$

From Theorem 4.1, it can be seen that the transmission zeroes of Y_1^T, Y_2^T play an important role in determining the existence of stable decentralized filters. Hence we first determine where the zeroes of Y_1^T and Y_2^T come from.

Lemma 4.3 The invariant zeroes of Y_1^T (respectively, Y_2^T) are the unobservable modes of $\{A, C_1\}$ (respectively, $\{A, C_2\}$).

Proof: Note that Y_1 and Y_1^T have same invariant zeros, since Y_1 and Y_1^T have same Smith-Mcmillan form.

The invariant zeros[10] of Y_1 are those for which

$$rank \begin{bmatrix} \lambda I_n - \widehat{A}_0 & F_2 \\ -C & \begin{bmatrix} 0 \\ I_{p_2} \end{bmatrix} \end{bmatrix} < n+\min(p, p_2) = n+p_2.$$

If this matrix is premultiplied by $\begin{bmatrix} I_n & -F \\ 0 & -I_p \end{bmatrix}$, then we have,

$$\begin{bmatrix} I_n & -F \\ 0 & -I_p \end{bmatrix} \begin{bmatrix} \lambda I_n - \widehat{A}_0 & F_2 \\ -C & \begin{bmatrix} 0 \\ I_{p_2} \end{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} \lambda I_n - \widehat{A}_0 + FC & 0 \\ C & -\begin{bmatrix} 0 \\ I_{p_2} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} \lambda I_n - A & 0 \\ \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} & \begin{bmatrix} 0 \\ -I_{p_2} \end{bmatrix} \end{bmatrix}.$$

Thus, the last matrix has reduced rank (less than $n+p_2$) only if λ is a zero of Y_1. Note, however, that the λ which reduces the rank of this matrix are exactly those which reduce the rank of $\begin{bmatrix} \lambda I_n - A \\ C_1 \end{bmatrix}$, which are exactly the unobservable poles of $\{A, C_1\}$.

Q.E.D

Since the "A" matrix of Y_1 is stable, the decoupling zeros of Y_1 are stable [10]. Moreover, since the set of invariant zeros is composed by the set of transmission zeros plus some decoupling zeros[10], if some invariant zeroes of Y_1 are in RHP, then this invariant zeroes must be transmission zeros. Hence we have

Lemma 4.4 For $k=1,2$, let z_i^k $(i=1,\ldots,l_k)$ be RHP unobservable modes of $\{A, C_k\}$. Then there exists a decentralized filter $K(s)$ such that $T_{ew} \in RH_2$ if and only if

$$a_i^k X_k^T(z_i^k) = 0, \quad \forall\, i, k \tag{4.6}$$

where a_i^k are determined from

$$a_i^k Y_k^T(z_i^k) = 0, \tag{4.7}$$

and X_k, Y_k are given by (3.5), (3.6), (3.7), (3.8).

Proof : Note that $Y_1^T(\infty) = [\,0 \;\; I\,]$ has full row rank, which implies that Y_1^T has right inverse. Since the "D" matrix of Y_1 is $\begin{bmatrix} 0 \\ I \end{bmatrix}$, Y_1 is biproper, and hence Y_1 has no transmission zero at infinity. Similarly, Y_2^T has full row rank, and has no transmission zero at infinity.

Since the "A" matrix of Y_k is $\widehat{A}_0$, Y_k is stable and Y_k has no poles at z_i^k, $\forall\, i, k$. It follows from Theorem 4.2 and Lemma 4.3 that for $k=1,2$, X_k^T has as its zeros all the RHP finite zeros of Y_k^T, together with their associated structure. Hence the Theorem 4.1 guarantees the existence of $Q(s)$ which is stable and satisfies (3.9) and (3.10).

Q.E.D

Remark : If $\{A, C_1\}$ and $\{A, C_2\}$ are detectable, then there exists no unstable unobservable mode, hence it follows from Lemma 4.4 that there exists a decentralized filter $K(s)$ such that $T_{ew} \in RH_2$

Now the natural question arises : If the conditions of Lemma 4.4 are satisfied, how to find $Q_1(s), Q_2(s)$ satisfying (4.4) and (4.5) ? When these $Q_1(s), Q_2(s)$ have been found, we can construct decentralized filter as in the proof of Lemma 3.1, i.e.

$$K_{11}(s) = -L_1(sI-\widehat{A}_0)^{-1}F_1 + Q_1(s)\left\{C(sI-\widehat{A}_0)^{-1}F_1+\begin{bmatrix}I\\0\end{bmatrix}\right\},$$

$$K_{22}(s) = -L_2(sI-\widehat{A}_0)^{-1}F_2 + Q_2(s)\left\{C(sI-\widehat{A}_0)^{-1}F_2+\begin{bmatrix}0\\I\end{bmatrix}\right\}.$$

To answer this question, we rely on the EMMP. The basic idea of EMMP is that $Q_1^T = -(Y_1^T)_n X_1^T$ clearly satisfies $Y_1^T Q_1^T = -X_1^T$. However it is not simple to construct this $Q_1(s)$, so we present a algorithm to construct $Q_1(s)$ and $Q_2(s)$ by using the results from the literature of EMMP[6]. In this algorithm, right inverses are first determined, and then $Q_1(s)$ and $Q_2(s)$ are determined, and finally $K_{11}(s)$ and $K_{22}(s)$ are determined. Note that, in this algorithm, the poles of the proper right inverse $(Y_k^T)_{ri}$ will dconsist of *1) a set of* $\widehat{n}_k$ *poles equal to the zeros of* Y_k *and 2) a set of* $(n-\widehat{n}_k)$ *poles arbitrarily assignable via linear state feedback* H[11].

Decentralized Filter Design Algorithm (DFDA):

Step 1 : Find an irreducible state-space realization of Y_1^T as $\{A, B, C, E\}$, where A, B, C, and E are $n\times n$, $n\times m$, $p\times n$ and $p\times m$ real matrices, respectively.

Step 2 : Find an $m \times m$ nonsingular matrix M such that $EM = [I \quad 0]$.

Step 3 : Set $[\underline{B}_1, \underline{B}_2] = BM$ and calculate $A - \underline{B}_1 C$.

Step 4 : Find a matrix H such that $(n-\widehat{n}_1)$ eigenvalues of $A-\underline{B}_1C+\underline{B}_2H$ are in the LHP, where $\widehat{n}_1$ is the number of zeros of Y_1. The existence of such H is always guaranteed[11].

Step 5 : The desired proper right inverse is

$$(A_{ri}, B_{ri}, C_{ri}, E_{ri}) = \left(A+BM\begin{bmatrix}-C\\H\end{bmatrix},\ BM\begin{bmatrix}I\\0\end{bmatrix},\ M\begin{bmatrix}-C\\H\end{bmatrix},\ M\begin{bmatrix}I\\0\end{bmatrix}\right),$$

where H was determined in step 4.

Step 6:Calculate $(Y_1^T)_{ri} = C_{ri}(sI-A_{ri})^{-1}B_{ri} + E_{ri}$.

Step 7:Calculate $Q_1^T(s) = -(Y_1^T)_{ri}X_1^T$.

Step 8:Repeat step 1 - step 7 with Y_2^T instead of Y_1^T to obtain $Q_2^T(s)$.

Step 9:Construct a decentralized filter :

$$K_{11}(s) = -L_1(sI-\widehat{A}_0)^{-1}F_1 + Q_1(s)\left\{C(sI-\widehat{A}_0)^{-1}F_1+\begin{bmatrix}I\\0\end{bmatrix}\right\},$$

$$K_{22}(s) = -L_2(sI-\widehat{A}_0)^{-1}F_2 + Q_2(s)\left\{C(sI-\widehat{A}_0)^{-1}F_2+\begin{bmatrix}0\\I\end{bmatrix}\right\}.$$

With Lemma 4.4 and DFDA, we are ready to present our main result in the following theorem.

Theorem 4.5 For given ε, if one can find $Q_1(s)$ and $Q_2(s)$ satisfying (4.4), (4.5) and $\left\|\begin{matrix}Q_1\\Q_2\end{matrix}\right\|_2 < \varepsilon$ by DFDA, then there exists a decentralized filter such that $\|T_{ew}\|_2^2 < \|G_f\|_2^2 + \varepsilon^2$

Remark : Since we can assign arbitrarily all poles of Y_1 and Y_2 except for unobservable modes of $\{A, C_1\}$ and $\{A, C_2\}$ in step 4 of DFDA, we can reduce the H_2 norm of $Q(s)$ by assigning remaining $(n-\widehat{n}_k)$ poles at appropriate places in LHP. If one cannot find Q_1 and Q_2 satisfying (4.4), (4.5) and $\left\|\begin{matrix}Q_1\\Q_2\end{matrix}\right\|_2 < \varepsilon$ by DFDA, then a larger upper bound on the norm is required.

5. EXAMPLE

To illustrate the proposed decentralized filter design procedure, we consider a following example. The system matrices A, B, C, D and L are given as

$$A=\begin{bmatrix}-0.6705 & -0.5410 & -0.2115\\ -0.8705 & -0.3410 & -0.2115\\ 0.3295 & -0.5410 & -1.2115\end{bmatrix},$$

$$B=\begin{bmatrix}1.78 & -2.1 & 2.74\\ 1.32 & -1.56 & 2.03\\ 0.43 & -0.51 & 0.66\end{bmatrix},$$

$$C=\begin{bmatrix}-1 & 2 & 5\\ 1 & -2 & -1\end{bmatrix},$$

$$D=\begin{bmatrix}0.408 & -0.577 & -0.707\\ 0.789 & 0.611 & -0.043\end{bmatrix},$$

$$L=\begin{bmatrix}1 & 1 & 1\\ -1.1 & 3.2 & 2.7\end{bmatrix}.$$

We seek to find decentralized filter $K(s) = \begin{bmatrix}K_{11}(s) & 0\\ 0 & K_{22}(s)\end{bmatrix}$

satisfying $\|T_{ew}(s)\|_2 \le 4.5$, whereas the optimal H_2 norm of the closed-loop system is

$$\|G_f\|_2 = 3.7821.$$

Since $\|T_{ew}(s)\|_2^2 = \|G_f(s)\|_2^2 + \|Q\|_2^2$, $Q(s) = \begin{bmatrix} Q_1(s) \\ Q_2(s) \end{bmatrix}$ should be constructed in DPDA such that $\|Q\|_2 \le 2.4383$.

Since the subsystem $\{A, C_1\}$ has unobservable unstable mode at 0.2, it follows from lemma 4.3 that Y_1 has transmission zero at 0.2. Indeed, we obtain from (3.6)

$$A_{y1} = \begin{bmatrix} 3.1999 & -8.2818 & -16.6782 \\ 2.2759 & -6.6338 & -11.5189 \\ 1.1370 & -2.1561 & -5.0839 \end{bmatrix},$$

$$B_{y1} = \begin{bmatrix} 0.7213 \\ 1.1062 \\ 0.0413 \end{bmatrix},$$

$$C_{y1} = \begin{bmatrix} -1 & 2 & 5 \\ 1 & -2 & -1 \end{bmatrix},$$

$$D_{y1} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

so that

$$Y_1^T(0.2) = [\,0 \quad 0\,].$$

Hence we can choose $a_1^1 = 1$ in (4.7)

From (3.5),

$$A_{x1} = \begin{bmatrix} 3.1999 & -8.2818 & -16.6782 \\ 2.2759 & -6.6338 & -11.5189 \\ 1.1370 & -2.1561 & -5.0839 \end{bmatrix},$$

$$B_{y1} = \begin{bmatrix} 0.7213 \\ 1.1062 \\ 0.0413 \end{bmatrix},$$

$$C_{x1} = [-1 \quad -1 \quad -1],$$

$$D_{x1} = 0,$$

we obtain

$$a_1^1 X_1^T(0.2) = 0.$$

Hence the lemma 4.4 guarantees the existence of decentralized filters.

Using the DPDA, we can construct the decentralized filter as

$$K_{11}(s) = \frac{5.96s^4 + 80s^3 + 158.9s^2 + 109.83s + 26.04}{s^5 + 19.12s^4 + 105.5s^3 + 151.57s^2 + 85.65s + 17.21}$$

,

$$K_{22}(s) = \frac{-2.86s^4 - 31.09s^3 - 59.19s^2 - 48.37s - 16.84}{s^5 + 13.32s^4 + 53.89s^3 + 79.43s^2 + 48.76s + 10.9}$$

In step 4 of DPDA, we have placed two nonfixed poles of $(Y_1^T)_{ri}$ at {-0.6, -10} and those of $(Y_2^T)_{ri}$ at {-3.8, -0.9714}, whereas the fixed pole of $(Y_1^T)_{ri}$ is 0.2 and that of $(Y_2^T)_{ri}$ is -1. Since the resulting $\|Q(s)\|_2$ is 2.4104, a prescribed sub-optimal norm bound is satisfied.

Note that the poles of K_{11} are {-0.6, -10, -0.6027± i0.1704, -7.3124} and those of K_{22} are {-1, -3.8, -0.6027± i0.1704, -7.3124}, which implies that the proposed decentralized filter is stable.

The simulation was performed by MATLAB with initial conditions

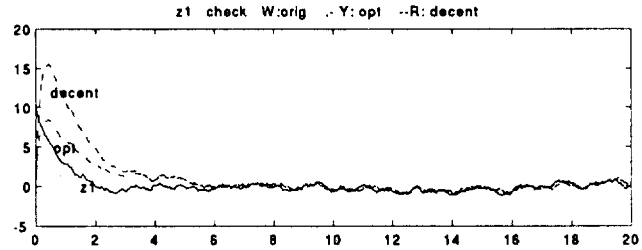

Fig. 2 Z_1 estimate

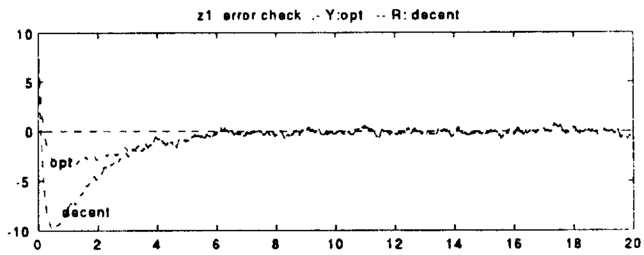

Fig. 3 Z_1. error

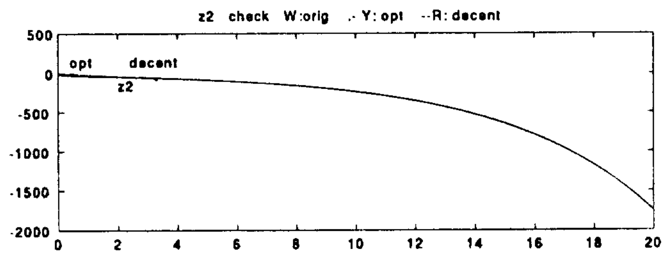

Fig. 4 Z_2 estimate

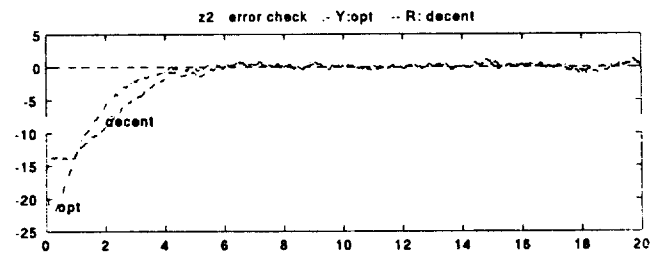

Fig. 5 Z_2 error

$$x(0) = \begin{bmatrix} 10 \\ -10 \\ 10 \end{bmatrix} \qquad \hat{x}(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

where $\hat{x}(0)$ is an initial state of the filter. The optimal estimate of z_1 ($\hat{z}_{1opt}$) and the decentralized estimate of z_1 ($\hat{z}_{1dec}$) are shown in Fig.2. The Fig.3 shows the optimal error ($z_1 - \hat{z}_{1opt}$) and the decentralized error ($z_1 - \hat{z}_{1dec}$). The estimates of z_2 and the errors in estimate of z_2 are shown in Fig.4 and Fig.5, respectively. In these comparisons, we can see that the decentralized estimation errors decay to zero, although the H_2-norm of the decentralized estimation error is larger than that of optimal estimation error.

5. Conclusion

Decentralized suboptimal H_2 filtering has been examined. The sufficient and necessary condition has been derived to guarantee the existence of proposed decentralized suboptimal H_2 filter. We also show that we can minimize the H_2 norm of decentralized filtering error by means of minimizing that of $Q(s)$. A practical procedure for decentralized filter design based on the exact model matching problem has been proposed.

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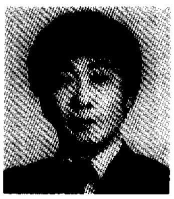

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