# MRNS 네트워크에서 특수한 메트릭스를 응용한 병렬 경로배정 알고리즘

최 완 규[†] 정 일 용[††]

## 요 약

MRNS(Mixed Radix Number System) 네트워크는 슈퍼컴퓨터나 MIMD의 모델로 널리 쓰이고 있으며 많은 연구가 진행되고 있는 하이퍼큐브의 일반적인 대수학적 모델이다. 본 논문에서는 MRNS 네트워크상에서 메세지의 전송 알고리즘을 연구하였다. 우리가 이 네트워크상에서 임의의 발신 노드부터 수신 노드까지 n개의 패킷들을 동시에 보내려고 할 때 이들 패킷들이 빠르고, 안전하게 수신 노드까지 도달하기 위해서는 i번째의 경로가 다른 모든 경로들로부터 node-disjoint 되어야 한다. 이를 위해 우리는 특수한 메트릭스인 HCLS(Hamiltonian Circuit Latin Square)[1]를 응용하여 선형 병렬 전송알고리즘을 개발하였다.

# Application of the Special Matrices to the Parallel Routing Algorithm on MRNS Network

Wan Kyoo Choi[†] Il Yong Chung[††]

## ABSTRACT

MRNS network is a general algebraic structure of Hypercube network, which has recently drawn considerable attention to supercomputing and message-passing communications. In this paper, we investigate the routing of a message in an n-dimensional MRNS network, that is a key to the performance of this network. On the n-dimensional MRNS network we would like to transmit packets from a source node to a destination node simultaneously along a fixed number of paths, where the superscript packet will traverse along the superscript path. In order for all packets to arrive at the destination node quickly and securely, the ith path must be node-disjoint from all other paths. By investigating the conditions of node-disjoint paths, we will employ the special matrices called as the Hamiltonian Circuit Latin Square(HCLS) described in [1] to construct a set of node-disjoint paths and suggest a linear-time parallel routing algorithm for the MRNS network.

## 1. Introduction

The rapidly growing and intense interest in interconnection networks used graph-theoretic properties[2] for their investigations and produced various interconnection schemes. Many of these schemes have been derived to optimize important parameters such as degree, diameter, fault-tolerance, hardware cost, and the needs of particular applications. Owing to low degree and diameter,and the relative ease in mapping different graph configurations (ring[3], linear arrays[3], systolic arrays[4], trees[5], and multidimensional meshes[3]) into hypercube,these hypercube multicompu-

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† 준 회 원 : 조선대학교 전자계산학과 석사과정
†† 종신회원 : 조선대학교 전자계산학과 전임강사
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ters have naturally drawn considerable attention to supercomputing[6] and message-passing communications[7]. The MRNS network, a general algebraic structure of hypercube network, consists of N identical processors (nodes) and each processor, provided with its own sizable local memory, is connected through bidirectinal, point-to-point communication channels to l different neighbors to form a communication network.

The routing of message is thus a key to the performance of such networks. There are routing algorithms using well-known methods, such as the Shotrest Path Algorithm(the Forward Algorithm)[8], the Backward Algorithm[9], the Spanning Tree Algorithm[10]. These algorithms provide for only sequential transmission, from the source node to the desired node in a short time. However, we would like to look for algorithms that are capable of handling multiple data items simultaneously transmitted from the starting (sourace) node to the destination node on the n-dimensional hypercube network. There are few algorithms that allow us to locate n disjoint paths such as the Hamithms path Algorithm [10], the Rotation Algorithm using Tree Structure[10], the Disjoint path Algorithm[11], and the Routing Algorithms[7].

Generally speaking, the discovery of the maximum number of node-disjoint paths on a random network is computationally difficult. However, it has been proven that these paths exist in a specific network[6]. From this fact, the Routing Algorithm for finding these paths on the hypercube network has been designed.

In this paper, we propose the algebraic approach to the routing of message on the n-dimensinal MRNS network. As described above,we would like to simultaneously transmit $\ell$ packets from the starting(source) node to the destination node. In this case,the superscrimt packet is sent along the ith path from the starting node to the destination node. In order for all packets to arrive at the destination node quickiy and securely, the ith path must be node-disjoint from all other paths. Chung[1] investigates the conditions that make a set of n node-disjoint paths from a arbitrary starting node to the destination node on n-dimensional hypercube. From the above conditions so developed, a special matrix, called the Matrix for Generating Node-Disjoint Paths(MGNDP), is constructed. Designing the MGNDP is, however, computatio-nally a difficult problem. In order to decrease the difficulty, the subclass of the MGNDP is called the Matrix for Generating the Shortest Node-Disjoint Paths(MGSNDP), is generated. Later, using the CSnLS(Cyclic Subsets of order n Latin Square[12]) and the HCLS(Hamiltonian Circuit Latin Square), the MCSnM(Modified Cyclic Subsets of order n Matrix) and the MHCM(Modified Hamilt onian Circuit Matrix), which belong to the MGSNDP, will be presented and constructed.

Our objective in the paper is to design a set of $\ell$ shortest and node-disjoint paths on n-dimensional MRNS network[11]. To accomplish this, employing these special latin squares mentioned above and considering the structure of tne MRNS network, we construct a linear-time parallel routing algirithm on this network.

This paper is organized of the following three sections. Section 2 describes what MRNS network is. In Section 3, the special latin square is applied to construct a set of node-disjoint paths on the MRNS network. Finally, the paper concludes with Section 4.

## 2. Description of The MRNS Network

The MRNS network is constructed from the mixed radix number system(MRNS). The routing algorithms of the MRNS network are similar to those of the hypercube network[9]-[11]. Each algorithm is composed of two phases. The first phase is to transmit the packet to a randomly chosen intermediate node through the secret route. The second phase is to send the packet from this intermediate node to the destination node along the secret path. This section provides the definition of the MRNS, gives a description of the MRNS network, and presents two routing algorithms of the MRNS network.

### 2.1 A Mixed Radix Number System (MRNS)

Let N be the total number of nodes of the MRNS network and be represented as a product of $m_i$'s, where $m_i$ is the number of vertices on the $i^{th}$ dimension, $m_i > 1$ for $0 \le i \le n-1$.

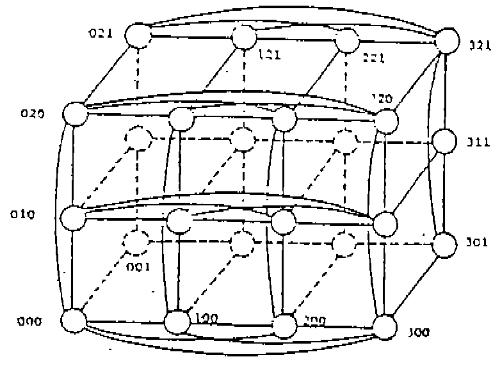
$$N = m_{m-1} * m_{n-2} * \cdots * m_1 * m_0$$

Then, each node u between 0 and N-1 can be represented as an n-tuple $(u_{n-1}u_{n-2}\cdots u_1 u_-)$ for $0 \le ui \le (m_i - 1)$. Associated with each ui is a weight wi, such that

$$u = \sum_{i=0}^{n-1} u_i * w_i \text{ and } w_i = \prod_{j=0}^{i-1} = m_{i-1} * m_{i-2} \cdots m_0.\ w_0 = 1.$$

Example 1: For N = 24, $m_i$ and $w_i$ can be computed as follows.

24 = 4 * 3 * 2.
$m_0 = 2,\ m_1 = 3,\ m_2 = 4$
$w_0 = 1,\ w_1 = 2,\ w_2 = 6.$




(Fig. 1) 4*3*2 MRNS Network

Then, $u = (u_2 u_1 u_0)$, $0 \le u_0 \le 1$, $0 \le u_1 \le 2$, $0 \le u_2 \le 3$ for any u in the range 0 – 23. $0_{10} = (000)$, $23_{10} = (321)$ in this mixed number system. Any node can be desribed in this system between (000) and (321). Node (000) is directly connected to nodes (001), (010), (020), (100), (200) and (300) as shown in Fig. 1. For the sake of clarity, connection is not completed in this figure shown by dotted lines.

### 2.2 Structure of the MRNS Network

Each node $u = (u_{n-1}u_{n-2}\cdots u_i \cdots u_0)$ is connected to nodes $(u_{n-1}u_{n-2}\cdots u'_i \cdots u_0)$ for all $0 \le i \le n-1$, where $u'_i$ can be any integer from $\{0, 1, \cdots, m_i - 1\}$ except ui itself. Given n-dimensions with $m_i$ number of nodes in the $i^{th}$ dimension, the following facts are described.

(1) The total number of links per node is

$$l = \sum_{i=0}^{n-1} (m_i - 1)$$

(2) The total number of links in the MRNS network is $N/2 * \ell$ where N is the total number of nodes.

(3) Each dimension is constructed as a complete graph. This means that for

the $i^{th}$ dimension, the total number of vertices is $m_i$ and the total number of links is $m_i * (m_i-1)/2$. Then, the link(p,q) is represented as (i,j), where

$$j = \sum_{k=0}^{p-1} (m_i-k-1)+(q-p), \quad p<q, \quad p,q \in Z_{mi}.$$

(4) The n-dimensional MRNS is a connected graph of diameter n.

Now, we examine the flexibility of designing the MRNS network. Given N nodes(N= prime number), more than one kind of MRNS network can be designed based on considerations such as the dynamic security, the volume of data to be transmitted, and the cost of the hardware. If the network is more secure and has a large volume of data, then the network can be constructed with more links. However, the cost for constructing the network is a primary consideration, so the network should be designed with as few links as possible.

Example 2: Given 24 nodes, four kinds of MRNS networks, $NK_1$, $NK_2$, $NK_3$, and $NK_4$, can be designed.

$$NK_1 = Z_2 \times Z_{12}$$
$$NK_2 = Z_3 \times Z_8$$
$$NK_3 = Z_4 \times Z_6$$
$$NK_4 = Z_2 \times Z_3 \times Z_4$$

Employing (2) above, the total number of links is computed as 144, 108, 96, 72 for $NK_1$, $NK_2$, $NK_3$, $NK_4$, respectively.

## 3. The Application of the Special Matrice for the Parallel Routing Algorithm on the MRNS Network

The routing algorithm of the MRNS network are similar to those of the hypercube network. For the MRNS network, the number of channels is determined by the modular number for each dimension, while the modular number of each dimension in the hypercube network is always 2. Considering the structure of the MRNS network, the following two propositions are described.

Proposition 1: Let A and B be any two nodes in the MRNS network and assume that H(A,B)[12] < n. Then there are H(A,B) parallel paths of length H(A,B) between the nodes A and B.

Proof : Let H(A,B) = k. Then the bit positions that differ between A and B are $\{a_1, a_2, \ldots, a_k\}$. We can write k permutations of this set, indicating the different bit positions for k parallel paths of length k. These k permutations are used to design the special latin square matrix. Using this special latin square matrix, k parallel paths are obtained automatically. [q.e.d.]

Proposition 2: Let A and B be any two nodes of an n-dimensional MRNS network and assume that H(A,B) < n. Then there are $\ell$ parallel paths between A and B, where $\ell = \sum_{i=0}^{n-1} (m_i-1)$. The length of each path is at most H(A,B) + 2.

Proof : In addition to the k parallel paths mentioned in Proposition 1, we consider the other ($\ell$-k) different paths. There are two types of paths. The length of the path for the first type is k+1. The length of the path for the other type is k+2. For the first type, each of the additional paths starts at some bit position d at which A and B differ. The packet is sent initially along an *incorrect link* in some dimension *d*; that is, the address of the node where the packet arrives differs from the address of the destination node by exactly the same number of bit positions as the original node does. Then, the packet is sent along the *correct links* in all dimensions other than *d*, as in Proposition 1. Finally, we send the packet along the link in dimension *d* that takes the packet to the destination node. This link can be found easily from the fact that each dimension of the MRNS network is constructed as a complete graph.

For the latter type, each of the additional paths starts at a bit position at which A and B do not differ. Then, correct bits a1 through ak by choosing one of the k paths from the previous proposition. Finally, change the starting bit back to what it was originally. The additional paths will never cross each other, since these paths select different links at the first and the last steps, and select the links for the remaining steps by Proposition 1. These $\ell$ paths are disjoint because each set of paths has a different length. [q.e.d.]

For the design of parallel routing algorithm, we describe the special matrices called the HCLS(Hamitonian Circuit Latin Square) mentioned on Reference [1].

Definition 1: The HCLS is constructed as follows: Given distinct n points,a Hamiltonian circuit $a_0\ a_1 \cdots a_{n-2}\ a_{n-1}\ a_0$ is randomly selected. On the circuit each row of the matrix obtained from the Hamiltonian path, starting at any position $a_j(0 \le j \le n-1)$, under the condition that no two rows begin at the same position. If a Hamiltonian path is $a_k\ a_{k+1}, \ldots, a_{k-1}$, then the row obtained from it is $[a_k\ a_{k+1} \cdots a_{k-1}]$

Let us first find a set of $\ell$ node−disjoint paths from a starting node A to a desired node B on n−dimensional MRNS network, having d(A,B)=k. Then, the special latin square of order k is applied to the construction of k node−disjoint paths. Then, the remainin g ($\ell$−k) paths are constructed by Proposition 2 above. The following algorithm (MRNS—Routing) expressed in a psudocode form, generates n position sets, the i^th^ positions through which the i^th^ packet is transmitted from the starting node to the destination node ($0 \le i \le \ell-1$).

*MRNS_Routing*

N←{0,1,...,n−1}/∗N=the set of bit positions enumeration∗/

A←$\{a_{n-1}a_{n-2},\ldots,a_0\}$/∗A=the address of the starting node A∗/

B←$\{b_{n-1}b_{n-2},\ldots,b_0\}$∗B=the address of the destination node B∗/

(1) Compute the set C of the links($C=\{(p_0,q_0),(p_1,q_1),\ldots,(p_{k-1},q_{k-1})\}$), where pi is a bit position that differs between the two nodes A and B, and $q_i$ is a specific link on $p_i^{th}$ dimension(see the structure of this network mentioned on Section II.2).

(2) Using k distinct bit positions defined in C, design the special matrix L for constructing k parallel paths.
/∗Subscript= the superscript position set consisting of links along which the data traverses from node A to node B ∗/

(3) $S_i$ ← the i^th^ row of the matrix L, $0 \le i \le k-1$.
m ← 0

(4) while (m < n)
begin (∗ while ∗)
In order to obtain the remaining ($\ell$−k) parallel paths,
we apply the idea of Proposition 2.

(4.1) if($m \in p_i$ in C, $0 \leq i \leq k-1$) then
construct the $(w_m-1)x(k+2)$ matrix M,
$M_{i,0} = M_{i,k+1}$, $M_{i,0} \in C1$, $C1 = \{(m,q_0),(m,q_1),...\}$, $|C1| = (w_m - 1)$,
$M_{i,0} \neq M_{j,0} (i \neq j)$, $\{M_{i,1}, M_{i,2}, ..., M_{i,k}\} = C$, $0 \leq i, j \leq w_m - 1$.

(4.2)else
construct the $(w_i-2)*(k+1)$ matrix M,
$M_{i,0} \in C1$, $C1 = \{(m,q_0),(m,q_1),...\} - (m,q_x)$, $(m,q_x) \in C$, $|C1| = (w_m - 2)$,
$M_{i,0} \neq M_{j,0}(i \neq j)$, $\{M_{i,1}, M_{i,2}, ..., M_{i,k-1}\} = C-(m,qx_x$, $0 \leq i, j \leq w_m - 2$.
Using the method to obtain the link connected to a desired node (see Section II.2), $M_{i,k}$ can be easily computed.

(5) $S_j$=the $j^{th}$ row of the matrix M;
m ← m+1
end(* while *)
(* end of *MRNS—Routing* *)

The following example will provide a better understading of the Algorithm given above.

Example 3: Let the MRNS network M = ($Z_2$ x $Z_3$ x $Z_3$ x $Z_4$), the starting node A = (0000) and the desired node B = (0223). Then, eight parallel paths from A to B are designed as follows:

(1) Following the first step of the Algorithm, we locate the bit positions(dimensions) that differ between nodes A and B, and the specific links on these dimensions.
C={(0,3),(1,2),(2,2)}

(2) Step(2) of the Algorithm requires the design of a (3x3) HCLS, which is described as follows: According to Definition 1, the Hamiltonian circuit (1→2→0→1) is randomly selected among 3! Hamiltonian circuit. Then, the first, second and third rows are obtained from four hamiltonian paths, starting at the third, first, second positions, respectively.

$$\begin{bmatrix} (0,3) & (1,2) & (2,2) \\ (1,2) & (2,2) & (0,3) \\ (2,2) & (0,3) & (1,2) \end{bmatrix}$$

(3) In step(3), we were required to determine the $i^{th}$ position set Subscript for the $i^{th}$ path($0 \leq i \leq 2$).

$$\begin{bmatrix} S_0 = \{(0,3),\ (1,2),\ (2,2)\} \\ S_1 = \{(1,2),\ (2,2),\ (0,3)\} \\ S_2 = \{(2,2),\ (0,3),\ (1,2)\} \end{bmatrix}$$

(4) Using Propostion 2, the remaining five parallel paths are now designed.
In the case of m=0, step (4.2) is adopted, and the (2x4) matrix is constructed. Then, C1 = {(0,1), (0,2)}, $\{M_{i,1}, M_{i,2}\}$ = {(1,2), (2,2)}. $M_{0,0}$ and $M_{1,0}$ can be (0,1) and (0,2), respectively. Given (0,1) and (0,2), $M_{0,3}$ and $M_{1,3}$ are obtained as (0,5) and (0,6).

$$\begin{bmatrix} (0,1) & (1,2) & (2,2) & (0,5) \\ (0,2) & (1,2) & (2,2) & (0,6) \end{bmatrix}$$

For m=1,2 step (4.2) is also adopted. However, step (4.1) is applied when m =3, and the (1x5) matrix is constructed. Then, C1 = {(3,1)}, $\{M_{i,1}, M_{i,2}, M_{i,3}\}$ = {(0,3), (1,2), (2,2)}. $M_{0,0}$ = (3,1) and $M_{0,4}$ = (3,1).
[(3,1) (0,3) (1,2) (2,2) (3,1)]

(5) The $i^{th}$ position set Subscript for the i

ʰ path($3 \leq i \leq 7$) is determined.

$S_3 = \{(0,1), (1,2), (2,2), (0,5)\}$

$S_4 = \{(0,2), (1,2), (2,2), (0,6)\}$

$S_5 = \{(1,1), (2,2), (0,3), (1,3)\}$

$S_6 = \{(2,1), (0,3), (1,2), (2,3)\}$

$S_7 = \{(3,1), (0,3), (1,2), (2,2), (3,1)\}$

From the above design, there are 8 parallel paths—each of three paths has length 3, each of four paths has length 4, and the remaining path has length 5. The path $P_i$ has the sequence of distinct nodes as follows:

$P_0$ : A→(0003)→(0023)→B

$P_1$ : A→(0020)→(0220)→B

$P_2$ : A→(0200)→(0203)→B

$P_3$ : A→(0001)→(0021)→(0221)→B

$P_4$ : A→(0002)→(0022)→(0222)→B

$P_5$ : A→(0010)→(0210)→(0213)→B

$P_6$ : A→(0100)→(0103)→(0123)→B

$P_7$ : A→(1000)→(1003)→(1023)→(1223)→B

## 4. Condusion

This paper presents parallel routing algorithm on an n-dimensinal MRNS network by using the special Latin square matrices. Given the starting node A and the destination node B on this network, we generates a set of shortest and node-disjoint paths from node A to node B. The main cost of the routing algorithm is the time complexity required in designing the special matrices used. Since the time complexity for the design of the HCLS(see Definition 1) is O(n), we can create a linear-time, parallel routing algorithm for constructing a set of shortest and node-disjoint paths.

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**최 완 규**

1988년 서울대학교 종교학과 졸업(학사)
1994년~현재 조선대학교 전자계산학과 석사과정
1992년~93년 (주)공성통신 전산실 근무
1993년~94년 (주)한양시스템 전산실 근무
관심분야: 소프트웨어 재사용, 분산 객체지향 시스템

**정 일 용**

1983년 한양대학교 공과대학 졸업(학사)
1987년 미국 City University of New York 전산학과(석사)
1991년 미국 City University of New York 전산학과(박사)
1991년~94년 한국전자통신연구소 선임연구원
1994년~현재 조선대학교 전자계산학과 전임강사
관심분야: 컴퓨터 네트워크, 분산 및 병렬 시스템, 망 보안시스템