J. Korean Math. Soc. **33** (1996), No. 4, pp. 879–889

OPERATIONS ON SATURATED FUZZY SYNTOPOGENOUS STRUCTURES

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1. Introduction

In order to describe the nearness between fuzzy sets various structures like the fuzzy neighborhood structure ([7]), the Artico-Moresco fuzzy proximity ([2]) and the Lowen fuzzy uniformity ([8]) have been introduced.

As the syntopogenous structure ([4]) generalizes these structures in the ordinary sets, the concept of fuzzy syntopogenous structure has been introduced by Katsaras to generalize the above mentioned structures ([5]).

In [3], the author has introduced the concept of saturated fuzzy syntopogenous structures and showed that it also generalizes the above three structures and that the category [FSyn] of saturated fuzzy syntopogenous spaces and continuous maps is coreflective in the category of [KFSyn] of fuzzy syntopogenous spaces and continuous maps.

Using the various operations on syntopogenous structures, Császár has characterized perfect, biperfect and symmetric syntopogenous structures ([4]).

The purpose of this paper is to define the counterparts in saturated fuzzy syntopogenous structures to those operations and using these, we investigate relationships between subcategories determined by ordinary operations in [FSyn].

We will recall basic notations related to fuzzy syntopogenous structures.

The unit interval will be denoted by I. For a set X and $A \subseteq X, A$ will also denote the fuzzy set in X which is the characteristic map of

Received October 5, 1995.
1991 AMS Subject Classification: 54A40.
Key words: Saturated fuzzy syntopogenous structure; saturated fuzzy topogenous order; elementary operation; ordinary operation.

A and x the fuzzy set $\{x\}$. For any $a \in I$, the constant map on X into I with the value a is also denoted by a and the fuzzy sets $a \wedge A$ and $a \vee X - A$ in X will be denoted by A_a and A^a respectively.

For the terminology for fuzzy sets, we refer to [6] and to [1] for the category theory.

A *fuzzy semi-topogenous order* on X is a map $\tau : I^X X I^X \to I$ i.e., a fuzzy relation on fuzzy sets on X which satisfies the following:

(T_1) $\tau(0,0) = \tau(1,1) = 1$.

(T_2) $\tau(\alpha,\beta) \leq (1-\alpha(x)) \vee \beta(x)$ for every $x \in X$.

(T_3) $\alpha_1 \leq \alpha$ and $\beta \leq \beta_1$ imply that $\tau(\alpha,\beta) \leq \tau(\alpha_1,\beta_1)$.

(T_4) $|\tau(\alpha,\beta)-\tau(\gamma,\delta)| \leq \|\alpha-\gamma\|+\|\beta-\delta\|$, where $\|\alpha\| = \sup_{x\in X} |\alpha(x)|$.
The fuzzy semi-topogenous order τ is called *topogenous* if it also satisfies

(T_5) $\tau(\alpha_1 \vee \alpha_2, \beta) = \tau(\alpha_1,\beta) \wedge \tau(\alpha_2,\beta); \tau(\alpha,\beta_1 \wedge \beta_2) = \tau(\alpha,\beta_1) \wedge \tau(\alpha,\beta_2)$.

For fuzzy semi-topogenous orders ζ and η on a set X, its pointwise join $\zeta \vee \eta$ is again a fuzzy semi-topogenous order on X and the map $\tau : I^X X I^X \to I$ defined by $\tau(\alpha,\beta) = \vee\{\eta(\alpha,A) \wedge \zeta(A,\beta) | A \subseteq X\}$ is again a fuzzy semi-topogenous order on X which will be denoted by $\zeta \circ \eta$. Moreover, ζ^2 will denote $\zeta \circ \zeta$. If $\eta \leq \zeta$, then we say that η is *coarser* than ζ or ζ is *finer* than η. Let τ be a fuzzy semi-topogenous order on a set X, then the *saturation* $[\tau]$ of τ is defined by $[\tau](\alpha,\beta) = \wedge\{\tau(x_{\alpha(x)}, y^{\beta(y)}) | x, y \in X\}$. Furthermore, τ is said to be *saturated* if $\tau = [\tau]$. It is known [3] that every saturated fuzzy semi-topogenous order is topogenous.

Let $\mathbb{A}$ and $\mathbb{B}$ be sets of fuzzy semi-topogenous orders on a set X. Then we say that $\mathbb{A}$ is *finer* than $\mathbb{B}$ or $\mathbb{B}$ is *coarser* than $\mathbb{A}$ if for any $\tau \in \mathbb{B}$ and $\varepsilon > 0$ there exists $\zeta \in \mathbb{A}$ with $\tau \leq \zeta + \varepsilon$. In this case we write $\mathbb{B} \preceq \mathbb{A}$. Furthermore, we say that $\mathbb{A}$ and $\mathbb{B}$ are *equivalent* and write $\mathbb{A} \cong \mathbb{B}$ if $\mathbb{A} \preceq \mathbb{B}$ and $\mathbb{B} \preceq \mathbb{A}$.

A *fuzzy syntopogenous structure* on a set X is a family $\mathbb{S}$ of fuzzy semi-topogenous orders on X satisfying the following:

(FS_1) $\mathbb{S}$ is directed in the sense that given $\zeta, \eta \in \mathbb{S}$ there exists $\tau \in \mathbb{S}$ with $\zeta, \eta \leq \tau$.

(FS_2) $\mathbb{S} \preceq \mathbb{S}^2$, where $\mathbb{S}^2$ denotes $\{\tau^2 | \tau \in \mathbb{S}\}$.

A fuzzy syntopogenous structure $\mathbb{S}$ on a set X is said to be *saturated* if each member of $\mathbb{S}$ is saturated and $(X, \mathbb{S})$ is called a *saturated fuzzy syntopogenous space.*

For a map $f : X \to Y$ and a fuzzy semi-topogenous order τ on Y, we define the *inverse image* of τ under f by $f^{-1}(\tau)(\alpha, \beta) = \tau(f(\alpha), 1 - f(1 - \beta))$, which is again a fuzzy semi-topogenous order on X.

A map f of a fuzzy syntopogenous space $(X, \mathbb{S})$ into a fuzzy syntopogenous space $(Y, \mathbb{T})$ is said to be *continuous* if $f^{-1}(\mathbb{T}) \preceq \mathbb{S}$, where $f^{-1}(\mathbb{T})$ denotes $\{f^{-1}(\tau) | \tau \in \mathbb{T}\}$.

[FSyn] will denote the category of all saturated fuzzy syntopogenous spaces and continuous maps between them.

For the details of fuzzy syntopogenous structures, we refer to [3, 5].

2. Elementary operations

In what follows, the set of fuzzy semi-topogenous orders on a set X will be denoted by $O(X)$

DEFINITION 2.1. For a set X, a map ^{e} $O(X) \to O(X)$ $(\tau \mapsto \tau^e,\ \tau \in O(X))$ is called an *elementary operation* if it satisfies the following:

(E_1) $\tau \leq \tau^e$,

(E_2) $\tau^{ee} = \tau^e$,

(E_3) $\tau \leq \eta$ implies $\tau^e \leq \eta^e$,

(E_4) $[\tau^e] \leq [\tau]^e$,

(E_5) $\tau^{2e} \leq [\tau]^{e2}$,

(E_6) For a map $f : Y \to X$, $[f^{-1}(\tau^e)] = [f^{-1}(\tau))^e]$ for every $\tau \in O(X)$.

PROPOSITION 2.2. *Suppose that $^e : O(X) \to O(X)$ is an elementary operation, then for any saturated fuzzy semi-topogenous order τ on X, τ^e is also saturated.*

Proof. It is immediate from (E_4) and Proposition 3.2 in [3].

Notation. For a set X, we define the following:

1) for $\tau \in O(X)$, $\tau^c : I^X X I^X \to I$ is the fuzzy semi-topogenous order defined by $\tau^c(\alpha, \beta) = \tau(1 - \beta, 1 - \alpha)$.

2) ^{i} : $O(X) \to O(X)$ is the identity map of $O(X)$.

3) ${}^{[\,]}: O(X) \to O(X)$ is the map given by $\tau^{[\,]} = [\tau]$.

4) ${}^{p}: O(X) \to O(X)$ is the map given by $\tau^{p}(\alpha,\beta) = \wedge\{\tau(\gamma,\beta)|\gamma < \alpha\}$.

5) ${}^{b}: O(X) \to O(X)$ is the map given by $\tau^{b}(\alpha,\beta) = \wedge\{\tau(\gamma,\delta)|\gamma < \alpha, \beta < \delta\}$.

6) ${}^{s}: O(X) \to O(X)$ is the map given by $\tau^{s}(\alpha,\beta) = [\tau \vee \tau^{c}](\alpha,\beta)$.

Using the above notation, we have the following

LEMMA 2.3. *The operations* ^{i}, ${}^{[\,]}$, ^{p}, ^{b} *and* ^{s} *on* $O(X)$ *are elementary operations.*

Proof. Clearly ^{i} is an elementary operation and ${}^{[\,]}$ is also an elementary operation by Proposition 3.2 and Proposition 3.13 in [3]. Regarding ^{p}, we first show that for any $\tau \in O(X), \tau^{p} \in O(X)$. To see this, suppose that $\tau^{p}(\alpha,\gamma) < \tau^{p}(\alpha,\beta)$. Then we have

$$\begin{aligned} &\tau^{p}(\alpha,\beta) - \tau^{p}(\alpha,\gamma) \\ &= (\wedge\{\tau(\alpha',\beta)|\alpha' < \alpha\}) - (\wedge\{\tau(a'',\gamma)|a'' < \alpha\}) \\ &= \vee\{(\wedge\{\tau(\alpha',\beta)|\alpha' < \alpha\}) - \tau(\delta,\gamma) \vee \delta < \alpha\} \\ &\le \vee\{\tau(\delta,\beta) - \tau(\delta,\gamma)|\delta < \alpha\}. \end{aligned}$$

Since for each fuzzy set δ $|\tau(\delta,\beta) - \tau(\delta,\gamma)| \le \|\beta - \gamma\|$, $\tau^{p}(\alpha,\beta) - \tau^{p}(\alpha,\gamma) \le \|\beta - \gamma\|$. Thus $|\tau^{p}(\alpha,\beta) - \tau^{p}(\alpha,\gamma)| \le \|\beta - \gamma\|$. Similarly, we have $|\tau^{p}(\alpha,\beta) - \tau^{p}(\delta,\beta)| \le \|\alpha - \delta\|$. The remaining axioms are clear. Clearly ^{p} satisfies (E_1), (E_2) and (E_3). For each $\delta < \alpha, [\tau^{p}](\alpha,\beta) = \wedge\{\tau^{p}(x_{\alpha(x)}, y^{\beta(y)})|x,y \in X\} \le \wedge\{\tau(x_{\delta(x)}, y^{\beta(y)})|x,y \in X\} = [\tau](\delta,\beta)$. This proves that $[\tau^{p}] \le [\tau]^{p}$. Take any $a \in I$ with $a < \tau^{2p}(\alpha,\beta)$. For each $\gamma < \alpha$, there exists a subset B_γ of X such that $a < \tau(\gamma, B_\gamma) \wedge \tau(B_\gamma,\beta)$. Putting

$$d = \wedge\{\tau(\gamma,B_\gamma) \wedge \tau(B_\gamma,\beta)|\gamma < \alpha\},$$

we have $a \le d$. Let $B = \cup\{B_\gamma|\gamma < \alpha\}$. Since $B_\gamma \subset B$ for each $\gamma < \alpha, \tau(\gamma,B_\gamma) \le \tau(\gamma,B)$; hence $d \le \wedge\{\tau(\gamma,B) \wedge \tau(B_\gamma,\beta)|\gamma < \alpha\}$. Since $[\tau^{p}](B,\beta) = \wedge\{[\tau^{p}](B_\gamma,\beta)|\gamma < \alpha\}$,

$$\begin{aligned} d &\le [\tau^{p}](\alpha,B) \wedge [\tau^{p}](B,\beta) \\ &\le \vee\{[\tau^{p}](\alpha,A) \wedge [\tau^{p}](A,\beta)|A \subseteq X\} \\ &= [\tau^{p}]^{2}(\alpha,\beta). \end{aligned}$$

Thus $a \leq [\tau^p]^2(\alpha, \beta)$; hence by (E_4) $a \leq [\tau]^{p2}(\alpha, \beta)$. This proves that $\tau^{2p} \leq [\tau^p]^2$. Finally, let $x, y \in X$ and $u, v \in I$. Then since $f(x_u) = f(x)_u$ and $1 - y^v = y_{1-v}$ for any map $f : X \to Y$, we have

$$\begin{aligned} f^{-1}(\tau^p)(x_u, y^v) &= \tau^p(f(x)_u, f(y)^v) \\ &= \wedge \{\tau(f(x)_w, f(y)^v) | w < u\} \\ &= \wedge \{f^{-1}(\tau)(x_w, y^v) | w < u\} \\ &= (f^{-1}(\tau))^p(x_u, y^v), \end{aligned}$$

so that $[f^{-1}(\tau^p)] = [(f^{-1}(\tau))^p]$. Therefore ^{p} is an elementary operation.

Using the same argument as above, one can show that ^{b} and ^{s} are elementary operation. We left the detail to the readers.

DEFINITION 2.4([5]). A fuzzy semi-topogenous order τ on a set X is called:

1) *perfect* if $\tau(\vee\{\alpha_j | \ j \in J\}, \beta) = \wedge\{\tau(\alpha_j, \beta) | \ j \in J\}$.

2) *biperfect* if it is perfect and $\tau(\alpha, \wedge\{\beta_j | \ j \in J\}) = \wedge\{\tau(\alpha, \beta_j) | \ j \in J\}$.

3) *symmetric* if $\tau = \tau^c$.

It is easy to see that τ is biperfect if and only if τ and τ^c are both perfect.

THEOREM 2.5. *Let τ be a saturated fuzzy topogenous order on X. Then one has the following:*

1) *τ^p is the coarsest perfect saturated fuzzy topogenous order finer than τ.*
2) *τ^b is the coarsest biperfect saturated fuzzy topogenous order finer than τ.*
3) *τ^s is the coarsest symmetric saturated fuzzy topogenous order finer than τ.*

Proof. 1) It is immediate from (E_5) and Proposition 3.2.3 in [3] that τ^p is a saturated topogenous order. Let $a = \vee\{a_j | \ j \in J\}$. If $c < a$, then there exists $j \in J$ with $c < a_j$. Thus $\tau^p(x_{a_j}, y^b) \leq \tau(x_c, y^b)$, for $\tau^p(x_{a_j}, y^b) = \wedge\{\tau(x_c, y^b) | c < a_j\} \leq \tau(x_c, y^b)$. Hence $\wedge\{\tau^p(x_{a_j}, y^b) | j \in J\} \leq \tau(x_c, y^b)$ for all $c < a$; hence $\wedge\{\tau^p(x_{a_j}, y^b) | j \in$

$J\} \leq \{\tau(x_c, y^b) | c < a\} = \tau^p(x_a, y^b)$. This proves that $\wedge\{\tau^p(x_{a_j}, y^b) | j \in J\} = \tau^p(x_a, y^b)$. Thus by Theorem 3.8 in [3], τ^p is a perfect saturated fuzzy topogenous order. For the proof of 2) and 3), we use an argument analogous to the one used in 1) and omit the details.

The following is immediate from (E_2).

PROPOSITION 2.6. *If ^{e} and ^{d} are elementary operations such that $\tau^{eded} = \tau^{ed}$ holds for any fuzzy semi-topogenous order τ, then ^{ed} is also an elementary operation.*

COROLLARY 2.7. *1) If ^{e} and ^{d} are elementary operations such that $\tau^{ed} = \tau^{de}$ holds for any fuzzy semi-topogenous order τ, then $^{ed=de}$ is also an elementary operation.*

2) If ^{e} is an elementary operation, $^{[\,]e}$ is an elementary operation.

Proof. The first half is immediate from Proposition 2.6 and (E_2). The second half follows from (E_4) and Proposition 3.2 in [3] that $[\tau]^e = [[\tau]^e]$ for any fuzzy semi-topogenous order τ. Thus we have $[\tau]^e = [[\tau]^e]^e$; hence by Proposition 2.6, $^{[\,]e}$ is an elementary operation.

Using $(E_1), (E_2), (E_3)$, one has the following:

PROPOSITION 2.8. *If ^{e} is an elementary operation, then for any family $\{\tau_j | j \in J\}$ of fuzzy semi-topogenous orders, $(\vee\{\tau_j | j \in J\})^e = (\vee\{\tau_j^e | j \in J\})^e$.*

DEFINITION 2.9. An elementary operation ^{e} is said to be *symmetric* if it is permutable with the operation ^{c}, i.e., $\tau^{ec} = \tau^{ce}$ for any fuzzy semi-topogenous order τ.

PROPOSITION 2.10. *^{i}, $^{[\,]}$, ^{s}, and ^{b} are symmetric operations.*

Proof. For any fuzzy semi-topogenous order τ and any $\alpha, \beta \in I^X$, we have

$$\begin{aligned}
[\tau^c](\alpha, \beta) &= \wedge\{\tau^c(x_{\alpha(x)}, y^{\beta(y)}) | x, y \in X\} \\
&= \wedge\{\tau(1 - y^{\beta(y)}, 1 - x_{\alpha(x)}) | x, y \in X\} \\
&= \wedge\{\tau(y_{1-\beta(y)}, x^{1-\alpha(x)} | x, y \in X\} \\
&= [\tau](1 - \beta, 1 - \alpha) \\
&= [\tau]^c(\alpha, \beta),
\end{aligned}$$

and therefore $^{[\,]}$ is symmetric. Similarly we have the remaining cases.

COROLLARY 2.11. 1) *If* ^{e} *is a symmetric elementary operation and* τ *is a symmetric fuzzy semi-topogenous order, then* τ^e *is also symmetric.*

2) *If* ^{e} *is a symmetric elementary operation, then* ^{se} *is also a symmetric elementary operation.*

Proof. 1) It is immediate from the definitions.

2) We note that for any fuzzy semi-topogenous order τ, $\tau^{eec} = \tau^{ece} = \tau^{cee}$, for ^{e} and ^{s} are symmetric. On the other hand, $\tau^{se} = \tau^{sse} = [\tau^s \wedge \tau^{sc}]^e = [[\tau^s]^e \wedge [\tau^{sc}]^e]^e = [[\tau^s]^e \wedge [\tau^s]^{ec}]^e = [\tau^s]^{ese} = \tau^{sese}$ from Lemma 2.3, Proposition 2.8 and Proposition 2.10, which completes the proof by Proposition 2.6.

3. Ordinary operations

In what follows, by $TO(X)$ we mean the set of all saturated fuzzy topogenous orders on a set X of which the power set will be denoted by $P(TO(X))$. A member $\mathbb{A}$ of $P(TO(X))$ will be called a *fuzzy order family* on X.

For an fuzzy order family $\mathbb{A} = \{\tau_j | j \in J\}$ on a set X, let $\mathbb{A}^g = \{[\vee\{\tau_k | k \in F\}] | F$ is a non-empty finite subset of $J\}$. Then $\mathbb{A}^g$ is clearly the coarsest directed fuzzy order family finer than $\mathbb{A}$.

For a fuzzy order family $\mathbb{A}$ on a set X, let $[\mathbb{A}] = \{[\tau] | \tau \in \mathbb{A}\}$.

DEFINITION 3.1. For a set X, a map $^o : P(TO(X)) \to P(TO(X))$ $(\mathbb{A} \mapsto \mathbb{A}^o, \mathbb{A} \in P(TO(X)))$ is called an *ordinary operation* if it satisfies the following:

(O_1) $\mathbb{A} \preceq \mathbb{A}^o$,

(O_2) $\mathbb{A}^{oo} = \mathbb{A}^o$,

(O_3) $\mathbb{A} \preceq \mathbb{B}$ implies $\mathbb{A}^o \preceq \mathbb{B}^o$,

(O_4) $\mathbb{A}^{2o} \preceq \mathbb{A}^{o2}$,

(O_5) $\mathbb{A}^{og} \preceq \mathbb{A}^{go}$,

(O_6) For a map $f : Y \to X, [f^{-1}(\mathbb{A}^o)] \preceq [(f^{-1}(\mathbb{A}))^o]$, where $f^{-1}(\mathbb{A})$ denotes $\{f^{-1}(\tau) | \tau \in \mathbb{A}\}$.

REMARK 3.2. 1) ^{g} is an ordinary operation.

2) For an elementary operation $^o : O(X) \to O(X)$, it follows from Proposition 2.2 that $^o(TO(X)) \subseteq TO(X)$, and hence one has an operation on $P(TO(X))$ given by the images, which is also denoted by $^e : P(TO(X)) \to P(TO(X))$. Then ^{e} is an ordinary operation.

3) ^{t} is an ordinary operation, where $\mathbb{A}^t = [\vee\{\tau|\tau \in \mathbb{A}\}]$ for any fuzzy order family $\mathbb{A}$.

Proof. 1) Clearly ^{g} satisfies $(O_1) - (O_6)$ except (O_5). For (O_5), we note that for any fuzzy order family $\{\tau_j|j \in J\}$ on a set $X, [\vee\{\tau_1^2|j \in J\}] \leq [\vee\{\tau_1|j \in J\}]^2$ (See Proposition 3.2 in [3]) and hence $\mathbb{A}^{og} \preceq \mathbb{A}^{go}$ for $\mathbb{A} \in P(TO(X))$.

2) We note that for any $\mathbb{A} \in P(TO(X)), \mathbb{A}^e = \{\tau^e|\tau \in \mathbb{A}\}$. Thus ^{e} satisfies $(O_1) - (O_6)$ except (O_5) by the corresponding $(E_1) - (E_6)$. For $\mathbb{A} \in P(TO(X))$ and $\zeta, \eta \in \mathbb{A}$, one has $\zeta^e \vee \eta^e \leq (\zeta \vee \eta)^e$ by (E_3) and hence $\mathbb{A}^{ge}$ is directed and $\mathbb{A}^e \preceq \mathbb{A}^{ge}$. Thus one has $\mathbb{A}^{eg} \preceq \mathbb{A}^{ge}$. Thus ^{e} satisfies (E_5).

3) It follows from the same argument as that in 1).

The following is immediate from the definition and we omit the proof.

REMARK 3.3. 1) If ^{o} and ^{k} are ordinary operations such that $\mathbb{A}^{okok} = \mathbb{A}^{ok}$ holds for any fuzzy order family $\mathbb{A}$, then ^{ok} is also an ordinary operation.

2) If ^{e} is an elementary operation, then ^{ge} and ^{te} are ordinary operations.

3) A saturated fuzzy syntopogenous structure $\mathbb{S}$ on a set is precisely fuzzy order family such that $\mathbb{S}^g \preceq \mathbb{S} \preceq \mathbb{S}^2$.

4) A fuzzy order family $\mathbb{A}$ is directed if and only if, $\mathbb{A}^g \cong \mathbb{A}$, or equivalently $\mathbb{A}^g \preceq \mathbb{A}$.

Noting that a fuzzy order family $\mathbb{A}$ is directed if and only if, $\mathbb{A}^g \cong \mathbb{A}$, or equivalently $\mathbb{A}^g \preceq \mathbb{A}$, one has the following from (O_3) and (O_5).

LEMMA 3.4. *If $\mathbb{A}$ is a directed fuzzy order family and ^{o} an ordinary operation, then the fuzzy order family $\mathbb{A}^o$ is also directed.*

PROPOSITION 3.5. *If $\mathbb{S}$ is a saturated fuzzy syntopogenous structure on a set X and ^{o} is an ordinary operation, then $\mathbb{S}^o$ is also a saturated fuzzy syntopogenous structure on X.*

Proof. Since $\mathbb{S}$ is directed, $\mathbb{S}^{og} \preceq \mathbb{S}^o$ by Lemma 3.4. On the other hand, $\mathbb{S}^o \preceq \mathbb{S}^{2o} \preceq \mathbb{S}^{o2}$ from Remark 3.3.3, (O_3) and (O_4). Thus $\mathbb{S}^o$ is again a saturated fuzzy syntopogenous structure on X.

DEFINITION 3.6. Let ^{o} be an ordinary operation. Then a saturated fuzzy syntopogenous structure $\mathbb{S}$ on a set is said to be an ^{o}-*syntopogenous structure* if $\mathbb{S} \cong \mathbb{S}^o$.

Notation. For an ordinary operation ^{o}, we will denote the full subcategory of [FSyn] determined by all saturated fuzzy ^{o}-syntopogenous spaces by [^{o}-FSyn].

PROPOSITION 3.7. *If ^{o} and ^{k} are ordinary operations such that $\mathbb{A}^o \preceq \mathbb{A}^k$ holds for any fuzzy order family $\mathbb{A}$, then [^{k}-FSyn] is coreflective in [^{o}-FSyn].*

Proof. Take any $(X, \mathbb{S}) \in$ [^{o}-FSyn], then clearly $(X, \mathbb{S}^k) \in$ [^{k}-FSyn] and the identity map $1_X : (X, \mathbb{S}^k) \to (X, \mathbb{S})$ is continuous. Suppose that $(Y, \mathbb{T}) \in$ [^{k}-FSyn] and $f : (Y, \mathbb{T}) \to (X, \mathbb{S})$ is a continuous map. Let $g : (Y, \mathbb{T}) \to (X, \mathbb{S}^k)$ be the map f as set map. Since f is continuous and $(Y, \mathbb{T}) \in$ [^{k}-FSyn], $g^{-1}(\mathbb{S})^k \preceq \mathbb{T}^k = \mathbb{T}$. Thus it follows from (O_6) and $[\mathbb{T}] = \mathbb{T}$ that g is continuous. Therefore $1_X : (X, \mathbb{S}^k) \to (X, \mathbb{S})$ is the [^{k}-FSyn]-coreflection of $(X, \mathbb{S})$.

The following is immediate from Proposition 3.7.

COROLLARY 3.8. *If ^{o} is an ordinary operation and $\mathbb{S}$ is a saturated fuzzy syntopogenous structure, then $\mathbb{S}^o$ is the coarsest saturated fuzzy $^{o-}$syntopogenous structure finer than the syntopogenous structure $\mathbb{S}$.*

DEFINITION 3.9. A saturated fuzzy syntopogenous spaces $(X, \mathbb{S})$ is called:

1) *perfect* if $\mathbb{S} \cong \mathbb{S}^p$.
2) *biperfect* if $\mathbb{S} \cong \mathbb{S}^b$.
3) *symmetric* if $\mathbb{S} \cong \mathbb{S}^s$.
4) *biperfect symmetric* if $\mathbb{S} \cong \mathbb{S}^{bs}$.
5) *topogenous* if $\mathbb{S} = \mathbb{S}^t$.

Noatation. 1) Let [PFSyn] ([BFSyn], [SFSyn], [BSFSyn], resp.) denote the full subcategory of [FSyn] determined by saturated fuzzy perfect (biperfect, symmetric, biperfect symmetric, resp.) syntopogenous spaces.

2) Let [TFSyn] ([PTFSyn], [STFSyn], resp.) denote the full subcategory of [FSyn] determined by all saturated fuzzy (perfect, symmetric, resp.) topogenous spaces.

Noting that [FSyn]=[$^{i-}$FSyn], [PFSyn]=[$^{p-}$FSyn], [BFSyn]=[$^{b-}$FSyn], [BSFSyn]=[$^{bs-}$FSyn], [TFSyn]=[$^{t-}$FSyn], [TSFSyn]=[$^{ts-}$FSyn] and [PTFSyn]=[$^{tp-}$FSyn], we have the following from Proposition 3.8.

THEOREM 3.10. [PFSyn], [BFSyn], [SFSyn], [BSFSyn], [TFSyn], [PTFSyn] *and* [STFSyn] *are all coreflective in* [FSyn].
2) [BFSyn] *is coreflective in* [PFSyn].
3) [BSFSyn] *is coreflective in* [BFSyn].
4) [BSFSyn] *is coreflective in* [SFSyn].
5) [STFSyn] *is coreflective in* [TFSyn].
6) [PTFSyn] *is coreflective in* [TFSyn].

Notation. 1) [QFProx] ([FProx], resp.) denotes the category of all saturated fuzzy proximity spaces (symmetric saturated fuzzy proximity spaces, resp.) and all proximity maps between them.
2) FTop denotes the category of all fuzzy neighborhood spaces and continuous maps between them.
3) QFUnif (FUnif, resp.) denotes the category of fuzzy quasi-uniform spaces (fuzzy uniform spaces, resp.) and uniformly continuous maps between them.

It is known [3] that [BFSyn] ([BSFSyn], [PTFSyn], [STFSyn], [TFSyn], resp.) is isomorphic with QFUnif (FUnif, FTop, [FProx], [QFProx], resp.). Thus we have the following by Theorem 3.10.

COROLLARY 3.11. 1) [FProx] *is coreflective in* [QFProx].
2) FTop *is coreflective in* [QFProx].
3) FUnif *is coreflective in* [QFUnif].

ACKNOWLEDGEMENT. The author is indebted to the referee of the first version of this paper for suggestions which have greatly improved this version.

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