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SMOOTH STRUCTURES ON SYMPLECTIC 4–MANIFOLDS WITH FINITE FUNDAMENTAL GROUPS

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1. Introduction

In studying smooth 4-manifolds the Donaldson invariant has played a central role. In [D1] Donaldson showed that non-vanishing Donaldson invariant of a smooth closed oriented 4-manifold X gives rise to the indecomposability of X. For instance, the complex algebraic surface X cannot decompose to a connected sum $X_1 \sharp X_2$ with both $b_2^+(X_i) > 0$.

In [W] Witten has shown that such a connected sum has also vanishing Seiberg-Witten invariants. Recently in [W1] Wang showed that Seiberg-Witten invariants vanish for another class of 4-manifolds which are not diffeomorphic to a connected sum of two manifolds with both $b_2^+ > 0$. In this paper we would like to extend the class and study the manifolds which are homeomorphic but not diffeomorphic to each other.

In [W1] and [W2] Wang studied the simply connected Kähler surface $\overline{X}$ with $K^2_{\overline{X}} > 0$, and free (anti-) holomorphic involutions on $\overline{X}$ where $K_{\overline{X}}$ is the canonical class of $\overline{X}$. Instead we study the closed symplectic 4-manifolds with finite fundamental groups, and free (anti-) symplectic involutions.

We study the Seiberg-Witten invariants for closed symplectic 4-manifolds. In section 2 we will introduce the Seiberg-Witten invariant on 4-manifolds and its basic properties.

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In Section 3 the followings are shown : If (X,ω) is a closed symplectic 4-manifold, then an involution σ on X is symplectic, anti-symplectic if and only if respectively $\sigma_* J = J\sigma_*$, $\sigma_* J = -J\sigma_*$ for some compatible almost complex structure J. If $\overline{X}$ has a finite fundamental group, $b_2^+(\overline{X}) > 1$ and σ a free involution, then the quotient $X = \overline{X}/\sigma$ is not diffeomorphic to any connected sum $X_1 \sharp X_2$ with both $b_2^+(X_i) > 0$. In addition if $b_2^+(\overline{X}) > 3$ and σ a free anti-symplectic involution, then the quotient manifold $X = \overline{X}/\sigma$ has vanishing Seiberg-Witten invariants.

In Section 4 we showed the followings : If $\overline{X}$ is a closed symplectic 4-manifold with a finite fundamential group and η, σ are two free involutions on $\overline{X}$, which are symplectic and anti-symplectic, respectively. Then if $b_2^+(\overline{X}) > 3$, then the quotient manifolds $X = \overline{X}/\eta$, $X' = \overline{X}/\sigma$ are not diffeomorphic to each other. If $\overline{X}$ is simply connected, and X is not spin, then X and X' are homeomorphic to each other.

2. Seiberg-Witten Invariants

Let X be a closed oriented 4-manifold with

$$b_2^+(X) = \frac{1}{2}(\text{rank } H_2(X) + \text{sign } (X)) > 1.$$

There is a one-to-one correspondence between the equivalence classes of complex line bundles on X and the second cohomology classes in $H^2(X, \mathbb{Z})$. For each the first Chern class $c_1(L)$ of a complex line bundle L such that $c_1(L) \equiv w_2(X) \mod 2$, there are a pair of $U(2)$ bundles $W^\pm$ over X. The bundles $W^\pm$ are respectively the (twisted) positive, negative spinor bundles (associated to the line bundle L.). The bundle W^+ (or L) is called a spin^c structure on X.

Let g be a Riemannian metric on X. For each cotangent vector $v \in T^*X$ there is a homomorphism $c(v) : W^+ \to W^-$ and $W^- \to W^+$ given $c(v)^2 = -g(v,v)$.

The map c induces the Clifford multiplication

$$\sigma : W^+ \otimes T^*X \to W^- \quad \text{and} \quad c_+ : \Lambda_+^2 \otimes C \to End(W^+).$$

Let the adjoint of c_+ be

$$\eta : End(W^+)_0 \to \Lambda_+ \otimes C$$

where $End(W^+)_0$ is the set of traceless endomorphisms of W^+.

Then a self-adjoint endmorphism is mapped into an imaginary valued form.

For a spin^C structure $W^+ \to X$, let A be a connection on the complex line bundle $L = \det(W^+)$. For a fixed metric on X, the Levi-Civita connection on T^*X with the connection A induces a covariant derivative on W^+. The covariant derivative is donoted by

$$\nabla_A : \Gamma(W^+) \to \Gamma(W^+ \otimes T^*X).$$

The composition of ∇_A and σ defines a Dirac operator

$$\mathcal{D}_A : \Gamma(W^+) \to \Gamma(W^-).$$

To define the Seiberg-Witten monopole equations, we consider a map from the product space of the space $\mathcal{A}(L)$ of connections on L and the space $\Gamma(W^+)$ of sections of W^+ into the product space of the space of self-dual 2-forms and $\Gamma(W^-)$:

$$P : \mathcal{A}(L) \times \Gamma(W^+) \to i\Omega^{2,+}(X) \times \Gamma(W^-)$$

defined by $P(A, \varphi) = \left(F_A^+ - \frac{1}{4}\eta(\varphi \otimes \varphi^*), \mathcal{D}_A\varphi\right)$.

The group $C^\infty(X, S^1)$ of gauge transformations on L acts on the domain of P by $(A, \varphi) \cdot g = (A - 2g^{-1}dg, \varphi g^{-1})$.

The differential dP together with the differential of the action of gauge group $C^\infty(X, S^1)$ is a first order elliptic differential operator and hence a Fredholm operator.

This operator dP is decomposed into the operators

$$(d^* + d^+, \mathcal{D}_A) : i\Omega^1(X) \oplus \Gamma(W^+) \to i(\Omega^0(X) \oplus \Omega^{2,+}(X)) \oplus \Gamma(W^-)$$

up to a compact perturbation.

We call the equations

$$(*) \qquad \begin{cases} \mathcal{D}_A\varphi & = 0 \\ F_A^+ & = \frac{1}{4}\eta(\varphi \otimes \varphi^*) \end{cases}$$

the Seiberg-Witten monopole equations.

The group of gauge transformations acts on the space of solutions of the monopole equations. The quotient $\mathcal{M}(L) = P^+(O)/C^\infty(X, S^1)$ is called the moduli space associated with the spin^C structure L. If $c_1(L) \neq 0$ and $b_2^+ \geq 1$, then the moduli space $\mathcal{M}(L)$ does not contain reducible solutions for a genetric metric on X or compact perturbation of $(*)$. If $b_2^+(X) \geq 2$, then any two choices of generic metrics give rise to a cobodism between two moduli spaces corresponded by the metrics. By the Atiyah-Singer index theorem, the dimension of $\mathcal{M}(L)$ is $d = -\frac{1}{4}(2\chi(X) + 3sign(X)) + \frac{1}{4}c_1(L)^2$.

Using the Weitzenböck formula for the Dirac operator $\mathcal{D}_A$, in [KM] Kronheimer and Mrowka showed that the moduli space $\mathfrak{M}(L)$ is compact and oriented. If the manifold X is symplectic, then the only non-empty moduli spaces have zero dimension. In this paper we only consider closed symplectic 4-manifolds and so we have only zero dimensional moduli spaces. In this case the Seiberg-Witten invariant for a spin^C structure L on X is defined by $SW(L) =$ the number of the points of moduli space $\mathfrak{M}(L)$ counted with sign. The basic properties of the Seiberg-Witten invariant are the followings.

THEOREM 2.1. *Let X be a compact, oriented 4-manifold with $b_2^+ \geq 2$. If $c_1(L) \equiv w_2(X) \mod 2$, then*

(1) *the invariant $SW(L)$ is independent of the generic choice of the metrics on X.*
(2) *$SW(L)$ depends only on the cohomology class $c_1(L)$.*
(3) *If h is a self-diffeomorphism of X, then $SW(h^*L) = \pm SW(L)$.*

3. Vanishing Theorem

Let X be a closed symplectic 4-manifold. The tangent bundle TX of X admits an almost complex structure which is an endomorphism $J : TX \to TX$ with $J^2 = -Identity$. The almost complex structure J defines a splitting

$$T^*X \otimes \mathbb{C} = T^{1,0} \oplus T^{0,1}$$

where J acts on $T^{1,0}$ and $T^{0,1}$ as multiplication by $-i$ and i, respectively. The canonical bundle K of the almost complex structure J is defined by $K = \Lambda^2 T^{1,0}$.

A symplectic structure ω on X is defined a closed two-form with $\omega \wedge \omega \neq 0$ everywhere. An almost complex structure J is said to be compatible with the symplectic structure ω if

(1) $\omega(Jv_1, Jv_2) = \omega(v_1, v_2)$ and
(2) $\omega(v, Jv) > 0$ for non-zero tangent vector v.

The space of compatible almost complex structures of a given symplectic structure on X is non-empty and contractible. If an almost complex structure J is compatible with ω, then for any $v, \omega \in TX$ $g(v,w) = \omega(v, Jw)$ defines a Riemannian metric on X. For such a metric on X, the symplectic structure ω is self-dual and gives the oriention on X. Conversely, any metric on X for which ω is self-dual can define an almost complex structure J which is compatible with the symplectic structure ω.

Let σ be an involution on the symplectic manifold (X, ω), the involution σ is said symplectic, anti-sympletic if respectively $\sigma_*\omega = \omega$, $-\omega$.

LEMMA 3.1. *Let (X, ω) be a closed symplectic 4-manifold. Then an involution σ on X is symplectic, anti-symplectic, if and only if respectively $\sigma_* J = J\sigma_*$, $-J\sigma_*$ for some compatible almost complex structure J with ω.*

Proof. By averaging we may assume that g is a σ-invariant metric on X. There is an almost complex structure J on X which is compatible with the symplectic structure ω.

For any $v, w \in TX$, $g(v,w) = \omega(v, Jw)$. Suppose that σ is anti-symplectic. Then

$$\begin{aligned} \sigma^*\omega = -\omega \text{ iff for any } v, w \in T_pX \text{ at any point } p \in X \\ \omega(\sigma_* v, \sigma_* Jw) = \sigma^*\omega(v, Jw) = -\omega(v, Jw) \\ = -g(v,w) = -g(\sigma_* v, \sigma_* w) = -\omega(\sigma_* v, J\sigma_* w) \end{aligned}$$

$$= \omega(\sigma_* v, -J\sigma_* w),$$
$$\text{iff } \sigma_* Jw = -J\sigma_* w \text{ for any } w \in T_pX,$$
$$\text{iff } \sigma_* J = -J\sigma_*.$$

Similarly we can prove that σ is symplectic if and only if $\sigma_* J = J\sigma_*$. □

In [W1] Wang showed the following proposition using Witten's vanishing theorem for Seiberg-Witten invariant: If $\overline{X}$ is a simply connected Kähler surface with $b_2^+(\overline{X}) > 1$ and σ is a free involution, then the quotient manifold $X = \overline{X}/\sigma$ cannot be decomposed as $X_1 \sharp X_2$ with $b_2^+(X_i) > 0$, $i = 1, 2$. We would like to study this proposition for the symplectic 4-manifolds with finite fundamental groups.

THEOREM 3.2. *If $\overline{X}$ is a closed symplectic 4-manifold with a finite fundamental group $\pi_1\overline{X}$ and $b_2^+(\overline{X}) > 1$. If $\sigma : \overline{X} \to \overline{X}$ is a free involution, then the quotient manifold $X = \overline{X}/\sigma$ cannot be decomposed as $X_1 \sharp X_2$ with both $b_2^+(X_i) > 0$.*

Proof. Considering the double cover $\overline{X} \to X$, we have a homotopy exact sequence $0 \to \pi_1\overline{X} \to \pi_1 X \to \mathbb{Z}_2 \to 0$. Since $\pi_1\overline{X}$ is finite, the order $|\pi_1 X| \equiv n$ of $\pi_1 X$ is finite and ≥ 2.

Assume that $X = X_1 \sharp X_2$ with $b_2^+(X_i) > 0$, $i = 1, 2$. Since $\pi_1 X$ is finite we may assume that $\pi_1 X \cong \pi_1 X_1$ and $\pi_1 X_2 = \{1\}$. Let $\overline{\overline{X}}_1$ be the universal cover of X_1. Then the universal cover $\overline{\overline{X}}$ of X is decomposed as $\overline{\overline{X}} \cong \overline{\overline{X}}_1 \sharp n X_2$.

Since $n \geq 2$ the Seiberg-Witten invariants on $\overline{\overline{X}}$ vanish for all spin^C structure on $\overline{\overline{X}}$. While $(\overline{X}, \omega)$ is a symplectic 4-manifold, hence $\overline{\overline{X}}$ is a covering space of $\overline{X}$. Let $\pi : \overline{\overline{X}} \to (\overline{X}, \omega)$ be the projection. The pull back $\pi^*\omega$ is also a closed nondegenerate 2-form on $\overline{\overline{X}}$. Therefore $(\overline{\overline{X}}, \pi^*\omega)$ is a symplectic 4-manifold, with $b_2^+(\overline{\overline{X}}) \geq n b_2^+(X_2) \geq 2$, and hence has a non-zero Seiberg-Witten invariant. This contradiction gives the proof of the proposition. □

In [W1] Wang proved a vanishing theorem for Seiberg-Witten invariants on the quotients of Kähler surfaces under free anti-holomorphic involutions.

He used the condition $K^2 > 0$ to eliminate the reducible solutions of the Seiberg-Witten equations for all metrics. We would like to go around this condition by using σ-invariant generic perturbation of metrics on $\overline{X}$. In [C3] we used the similar method to get an equivariant moduli space of instantons.

THEOREM 3.3. *Let $\overline{X}$ be a closed symplectic 4-manifold with a finite fundamental group and $b_2^+(\overline{X}) > 3$. Suppose that $\sigma : \overline{X} \to \overline{X}$ is a free anti-symplectic involution. Then the quotient manifold $X = \overline{X}/\sigma$ has vanishing Seiberg-Witten invariants.*

Proof. Let $\pi : \overline{X} \to X$ be the double covering projection. The Euler characteristics and the signatures are related by

$$\chi(\overline{X}) = 2\chi(X) \quad \text{and} \quad \text{sign}(\overline{X}) = 2 \cdot \text{sign}(X).$$

Since the first Betti numbers $b_1(\overline{X}) = b_1(X)$, we have $b_2^+(X) = \frac{1}{2}[b_2^+(\overline{X}) - 1] > 1$.

Assume that the manifold X has a non-vanishing Seiberg-Witten invariant. We may choose a generic metric g on X. where the genericity means that the self-dual part of the curvature for the solution of the Seiberg-Witten equations is non-zero.

Let L be a spin^C structure on X. It pulls back to a spin^C structure $\overline{L}$ on $\overline{X}$ through the projection π. Let $W^\pm$ be the spinor bundles on X associated to the complex line bundle L. Then the pull back $\overline{W}^\pm = \pi^* W^\pm$ of $W^\pm$ is the associated spinor bundles of the line bundle $\overline{L}$ on $\overline{X}$. The pull back $\overline{g} = \pi^* g$ is a σ-invariant metric on $\overline{X}$. Let $\overline{\omega}$ be a self-dual symplectic 2-form on $\overline{X}$. Then we may have an almost complex structure J on $\overline{X}$ such that

$$\overline{g}(v, w) = \overline{\omega}(v, Jw) \quad \text{for all} \quad v, w \in T\overline{X}.$$

Suppose that (A, ϕ) is an irreducible solution to the Seiberg-Witten equations for the spin^C structure L and the generic metric g on X, where A is a connection on L and ϕ a section on W^+. We can easily check that the pull-back $(\overline{A}, \overline{\phi})$ through the projection π is also a solution to the Seiberg-Witten equations on $\overline{X}$, because essentially everything is locally the same, for details see [W1]. Moreover $(\overline{A}, \overline{\phi})$ is also irreducible. Indeed, if (A, ϕ) is irreducible, then $F_A^+ \not\equiv 0$ is not identically zero. There is a point $x \in X$ such that $F_A^+(x) \neq 0$. If $\pi(\overline{x}) = x$, then $F_{\overline{A}}^+(\overline{x}) \neq 0$. Since $(\overline{A}, \overline{\phi})$ is an irreducible solution. the self-dual part $F_{\overline{A}}^+$ of $F_{\overline{A}}$ is not identically zero.

The first Chern class $c_1(\overline{L}) = \frac{i}{2\pi} F_{\overline{A}}$ is not anti-self-dual. Since the symplectic structure $\overline{\omega}$ is self-dual, we have $c_1(\overline{L}) \cdot \overline{\omega} \gneqq 0$ by (4.7) of Witten [W]. While the anti-symplectic involution σ preserves the orientation of $\overline{X}$ because $\sigma^*(\overline{\omega} \wedge \overline{\omega}) = \overline{\omega} \wedge \overline{\omega}$. However $c_1(\overline{L}) \cdot \overline{\omega} = \sigma^* c_1(\overline{L}) \cdot \sigma^* \overline{\omega} = -(c_1(\overline{L}) \cdot \overline{\omega})$ and hence $c_1(\overline{L}) \cdot \overline{\omega} = 0$. The contradiction proves the vanishing of Seiberg-Witten invariants on X.

REMARK. 1. In the Theorem 3.3 the quotient space $X = \overline{X}/\sigma$ is a smooth 4-manifold which does not have any symplectic structure.

2. By (3.2) and (3.3) the quotient $X = \overline{X}/\sigma$ cannot be decomposed as $X = X_1 \sharp X_2$ with both $b_2^+(X_i) > 0$ and has vanishing Seiberg-Witten invariant.

4. Smooth Structures on Quotient Manifolds

Donaldson proved that the Dolgachev surface $D_{2,3}$ and $\mathbb{CP}^2 \sharp 9\overline{\mathbb{CP}^2}$ are homeomorphic but not diffeomorphic. This implies that the h-cobordism conjecture in 4-manifolds does not hold. After that many people have gotten many good results on smooth structures of 4-manifolds using the Donaldson's invariant. Recently in [W2] Wang showed that the quotients of a complex surface under free holomorphic, anti-holomo- rphic involutios are homeomorphic but not difeomorphic to each other using the Seiberg-Witten invariants. A smooth map $\sigma : (M_1, J_1) \to (M_2, J_2)$ between complex manifolds is called anti-holomorphic if $\sigma_* J_1 = -J_2 \sigma_*$ on the tangent bundles, where J_1 and J_2 are the complex structures on M_1 and M_2, respectively. We denote K_M the canonical bundle of an almost complex manifold M.

THEOREM 4.1 [W2]. *Let $\overline{X}$ be a simply connected Kähler surface, and suppose that η, σ are two free involutions on $\overline{X}$, which are respectively holomorphic, anti-holomorphic.*

(1) *If $K_{\overline{X}}^2 > 0$ and $b_2^+(\overline{X}) > 3$, then the quotient manifolds $X =$*

$\overline{X}/\eta$, $X' = \overline{X}/\sigma$ are not diffeomorphic to each other.

(2) *If X is not spin, then X and X' are homeomorphic to each other.*

In this section we would like to study the Theorem 4.1 for the symplectic 4-manifolds with finite fundamental groups. We will use the Theorem 3.3 to avoid the condition $K^2 > 0$.

THEOREM 4.2. *Let $\overline{X}$ be a closed symplectic 4-manifold with a finite fundamental group. Suppose that η, σ are two free involutions on $\overline{X}$, which are respectively symplectic, anti-symplectic.*

(a) *If $b_2^+(\overline{X}) > 3$, then the quotient manifolds $X = \overline{X}/\eta$, $X' = \overline{X}/\sigma$ are not diffeomorphic to each other.*

(b) *If $\overline{X}$ is simply connected and X is not spin, then X and X' are homeomorphic to each other.*

Proof. (a) Considering the double covering $\pi : \overline{X} \to X$ we have the Euler characteristics and signature formulae $\chi(\overline{X}) = 2\chi(X)$, sign $(\overline{X}) = 2\,\mathrm{sign}(X)$, and $b_2^+(X) = \frac{1}{2}[b_2^+(\overline{X}) - 1] > 1$.

By averaging $\overline{g} = \frac{1}{2}(h + \eta^* h)$ for any metric h on $\overline{X}$ and a self-dual symplectic structure $\overline{\omega}$ on $\overline{X}$ we have an almost complex structure J on $\overline{X}$ which is compatible with $\overline{\omega}$. This symplectic structure $\overline{\omega}$ is preserved by η. The projection $\pi : \overline{X} \to X$ pushes down the symplectic structure $\overline{\omega}$ and the metric $\overline{g}$ to the symplectic structure ω and the metric g on X. Therefore the quotient manifold (X, ω) is a symplectic manifold.

By Taubes [T], the Seiberg-Witten invariants on X are defined generically, and the Seiberg-Witten invariant for the canonical bundle K_X on X is non-trivial. While by Theorem 3.3 all Seiberg-Witten invariants on X' vanish. Since the Seiberg-Witten invariants are diffeomorphic invariants, the quotient manifolds X and X' are not diffeomorphic to each other.

(b) The proof is the same as the proof of (2) of Theorem 4.1. For details see [W2].

REMARK 4.3. 1. If $\overline{X}$ is a simply connected closed symplectic 4-manifold with $b_2^+(\overline{X}) > 3$, then the quotient manifold by symplectic (anti-symplectic) involutions as in Theorem 4.2 are homeomorphic but not diffeomorphic to each other.

2. In (b) of Theorem 4.2, we assume that $\overline{X}$ is simply connected because the fundamental groups of the quotients will be $\mathbb{Z}_2$, and spin structures determine the topological structures of the quotients. If we know the classification of 4-manifolds with finite fundamental groups, then we may extend the Theorem 4.2 (b).

3. There are many (simply connected) closed non-Kähler, symplectic 4-manifolds. In fact Gompf [G] constructed infinite families of simply connected symplectic 4-manifolds which are non-Kähler. For instance, let M_1, M_2 be simply connected Dolgachev surfaces. By Dehn twisting fiber sum for the fibers of the relative prime multiplicities we have the symplectic fiber sum $M = M_1 \sharp M_2$ which is not Kähler, but symplectic.

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