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ON VECTOR VARIATIONAL INEQUALITY

GUE MYUNG LEE[1], DO SANG KIM[2],
BYUNG SOO LEE[3] AND SUNG JIN CHO[4]

1. Introduction

Since Giannessi [5] introduced the vector variational inequality in a finite dimensional Euclidean space with further application, Chang *et al.* [17], Chen *et al.* [1-4] and Lee *et al.* [10-16] have considered several kinds of vector variational inequalities in abstract spaces and have obtained existence theorems for their inequalities.

Let X, Y be Banach spaces with dual spaces X^* and Y^* respectively. Let K be a nonempty, closed and convex subset of X, $T : K \to L(X, Y)$ an operator, where $L(X, Y)$ is the space of all linear continuous operators from X into Y, and $H : K \to Y$ an operator. Let P be a convex and pointed cone in Y with $intP \neq \emptyset$, where int denotes the interior of a set, and

$$P^* = \{s \in Y^* : \langle s, x\rangle \geqq 0 \text{ for all } x \in P\}.$$

Consider the following three kinds of vector variational inequalities:

(SVVI) the strong vector variational inequality: Find $x_0 \in K$ such that $\langle T(x_0), x - x_0\rangle + H(x) - H(x_0) \in P$ for all $x \in K$,

(VVI) the vector variational inequality: Find $x_0 \in K$ such that $\langle T(x_0), x - x_0\rangle + H(x) - H(x_0) \notin -P\backslash\{0\}$ for all $x \in K$, and

(WVVI) the weak vector variational inequality: Find $x_0 \in K$ such that $\langle T(x_0), x - x_0\rangle + H(x) - H(x_0) \notin -intP$ for all $x \in K$.

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where $\langle T(x), y\rangle$ denotes the evaluation of the linear operator $T(x)$ at y.

When $H \equiv 0$, (WVVI) becomes the vector variational inequality considered in [1-3,19].

In relation to the three inequalities, we consider the following variational inequality $(\mathrm{VVI})_s$ for a given $s \in P^*$:

$(\mathrm{VVI})_s$ Find $x_0 \in K$ such that $\langle s \circ T(x_0), x - x_0\rangle + s \circ H(x) - s \circ H(x_0) \geqq 0$ for all $x \in K$,

where $s \circ T(x)$ is the composition of s and $T(x)$.

In this paper, we give the relationships among (SVVI), (VVI), (WVVI) and $(\mathrm{VVI})_s$, and obtain the existence theorems for (VVI) and (WVVI) by the scalarization method. Also, we give equivalent relations among the special cases of (VVI) and multiobjective optimization problems.

This paper is composed of four sections. In section 2, we establish relationships among (SVVI), (VVI), (WVVI) and $(\mathrm{VVI})_s$. In section 3, we obtain the existence theorems for (VVI) and (WVVI) by the scalarization method and also extend our results to unbounded sets. In section 4, we consider the special cases of (VVI) formulated in section 1, and give equivalent relations among them and multiobjective optimization problems.

2. Relationships

In this section we give the relationships among (SVVI), (VVI), (WVVI) and $(\mathrm{VVI})_s$.

LEMMA 2.1 [6]. *Let E be a topological vector space with dual space E^* and P a convex cone in E with $intP \neq \emptyset$. Then we have*

$$int\ P = \{x \in E : \langle x^*, x\rangle > 0 \text{ for all } x^* \in P^* \backslash \{0\}\}.$$

Also if E is a reflexive locally convex Hausdorff space with dual space E^* and P is a closed and convex cone in E with $intP^* \neq \emptyset$, then

$$int\ P^* = \{x^* \in E^* : \langle x^*, x\rangle > 0 \text{ for all } x \in P \backslash \{0\}\}.$$

DEFINITION 2.1. Let X, Y be topological vector spaces, K a nonempty, convex subset of X and P a convex cone in Y. Then an operator $F : K \to Y$ is said to be P-convex if for all $\alpha \in (0,1)$ and $x_1, x_2 \in K$,

$$\alpha F(x_1) + (1-\alpha)F(x_2) - F(\alpha x_1 + (1-\alpha)x_2) \in P.$$

DEFINITION 2.2 [7]. Let Y be a topological vector space, P a convex cone in Y with $intP \neq \emptyset$ and K a nonempty set. Then an operator $F : K \to Y$ is said to be P-subconvexlike if there is a $\theta \in intP$ such that for all $\alpha \in (0,1)$, $x_1, x_2 \in K$ and $\varepsilon > 0$, there is an $x_3 \in K$ such that

$$\varepsilon\theta + \alpha F(x_1) + (1-\alpha)F(x_2) - F(x_3) \in P.$$

REMARK. If $F : K \to Y$ is P-convex, then F is P-subconvexlike.

PROPOSITION 2.1. *Let X, Y be Banach spaces, K a nonempty, closed and convex subset of X, $T : K \to L(X,Y)$ an operator and $H : K \to Y$ an operator. Let P be a closed, convex and pointed cone in Y with $intP \neq \emptyset$ and $intP^* \neq \emptyset$.*

Let

$$A = \{x_0 \in K : \langle T(x_0), x - x_0\rangle + H(x) - H(x_0) \in P \quad \text{for all} \quad x \in K\},$$

$$B = \bigcup_{s \in intP^*} \{x_0 \in K : \langle s \circ T(x_0), x - x_0\rangle + s \circ H(x) - s \circ H(x_0) \geqq 0, \quad \text{for all} \quad x \in K\}$$

$$C = \{x_0 \in K : \langle T(x_0), x - x_0\rangle + H(x) - H(x_0) \notin -P\backslash\{0\} \quad \text{for all} \quad x \in K\},$$

$$D = \{x_0 \in K : \langle T(x_0), x - x_0\rangle + H(x) - H(x_0) \notin -int\ P \quad \text{for all} \quad x \in K\}$$

and

$$E = \bigcup_{s \in P^*\backslash\{0\}} \{x_0 \in K : \langle s \circ T(x_0), x - x_0\rangle + s \circ H(x) - s \circ H(x_0) \geqq 0 \quad \text{for all} \quad x \in K\}.$$

Then the following hold.

(1) $A \subset C \subset D$ and $A \subset B$.

(2) If Y is reflexive, then $B \subset C$.

(3) $E \subset D$.

(4) Furthermore, if for each $z \in K$, $\langle T(z), \cdot\rangle + H(\cdot)$ is P-subconvexlike, then $D \subset E$.

Proof. (1) Since P is pointed, $A \subset C \subset D$ holds. $A \subset B$ is trivial.

(2) Let $x_0 \in B$. Then there exists $s \in intP^*$ such that

$$(*) \qquad \langle s \circ T(x_0), x - x_0\rangle + s \circ H(x) - s \circ H(x_0) \geqq 0 \text{ for all } x \in K.$$

Suppose that there exists a $z \in K$ such that

$$\langle T(x_0), z - x_0\rangle + H(z) - H(x_0) \in -P\backslash\{0\}.$$

Since $s \in intP^*$, by Lemma 2.1,

$$\langle s \circ T(x_0), z - x_0\rangle + s \circ H(z) - s \circ H(x_0) < 0,$$

which contradicts $(*)$. Hence $x_0 \in C$.

(3) By using Lemma 2.1, (3) can be proved similarly to the case of (2).

(4) Let $x_0 \in D$.

$$\langle T(x_0), x_0\rangle + H(x_0) \notin \langle T(x_0), x\rangle + H(x) + intP \text{ for all } x \in K.$$

Let $G(x) = \langle T(x_0), x\rangle + H(x)$. Then $G(x_0) \notin G(K) + intP$. Since G is P-subconvexlike, $G(K) + intP$ is convex (see Lemma 3.2 in [8]). By the separation theorem, there exists $s \in Y^*\backslash\{0\}$ such that $\langle s, G(x_0)\rangle \leqq \langle s, G(x)\rangle + \langle s, c\rangle$ for all $x \in K$ and $c \in intP$. Since the closure of $intP$ is P, we have

$$\langle s, G(x_0)\rangle \leqq \langle s, G(x)\rangle + \langle s, c\rangle \text{ for all } x \in K \text{ and } c \in P.$$

We can easily check that $s \in P^*\backslash\{0\}$. Thus we have

$$\begin{aligned} 0 &\leqq \langle s, G(x)\rangle - \langle s, G(x_0)\rangle \\ &= \langle s \circ T(x_0), x - x_0\rangle + s \circ H(x) - s \circ H(x_0) \text{ for all } x \in K. \end{aligned}$$

Hence $x_0 \in E$.

REMARK. If $H : K \to Y$ is P-convex, then for each $z \in K$, $\langle T(z), \cdot \rangle + H(\cdot)$ is P-subconvexlike.

3. Existence Theorems

First we give definitions and lemmas for existence theorems of solutions for vector variational inequalities (VVI) and (WVVI).

DEFINITION 3.1 [9]. Let X, Y be normed spaces, K a nonempty convex subset of X, P a convex cone in Y and $F : K \to Y$ an operator. Then F is said to be P-continuous at $x_0 \in K$ if, for any neighborhood U of $F(x_0)$ in Y, there exists a neighborhood V of x_0 in X such that

$$F(x) \in U + P \text{ for all } x \in V \cap K.$$

We say that F is P-continuous on K if it is P-continuous at any point of K.

REMARK. If $Y = \mathbb{R}$ and $P = \mathbb{R}_+$, then the P-continuity is the same as the lower semicontinuity.

LEMMA 3.1 [9]. *Let X, Y be normed spaces, K a nonempty convex subset of X and P a convex cone in Y. If an operator $F : K \to Y$ is P-convex and P-continuous, then for any $s \in P^*$, $s \circ F$ is lower semicontinuous and convex.*

DEFINITION 3.2. Let X be a Banach space with its dual X^*, K a nonempty convex subset of X and $T : K \to X^*$ an operator.

1. T is said to be monotone if for any $x, y \in K$,

$$\langle T(x) - T(y), x - y \rangle \geqq 0.$$

2. T is called hemicontinuous if for any $x, y, z \in K$ and $t \in (0, 1)$, the mapping $t \to \langle T(x + t(y - x)), z \rangle$ is continuous at 0^+.

DEFINITION 3.3. Let X, Y be Banach spaces, K a nonempty convex subset of X, P a convex cone in Y and $T : K \to L(X, Y)$ an operator

1. T is said be P-monotone if for any $x, y \in K$,

$$\langle T(x) - T(y), x - y \rangle \in P.$$

2. T is called V-hemicontinuous if for any $x, y, z \in K$, $t \in (0, 1)$, the mapping $t \to \langle T(x + t(y - x)), z \rangle$ is continuous at 0^+.

REMARK. Our definition of V-hemicontinuity is slightly different from that in [3].

Modifying the proof of Theorem 3 in [20], we can obtain the following lemma.

PROPOSITION 3.2. *Let X be a reflexive Banach space with its dual X^* and K a nonempty, closed, bounded and convex subset of X. If $F : K \to X^*$ is a monotone and hemicontinuous operator and $h : K \to \mathbb{R}$ is a lower semicontinuous and convex function, then there is an $x_0 \in K$ such that*

$$\langle F(x_0), x - x_0 \rangle + h(x) - h(x_0) \geqq 0 \text{ for all } x \in K.$$

Now we prove the existence theorems for (VVI) and (WVVI) by the scalarization method.

THEOREM 3.1. *Let X, Y be reflexive Banach spaces. Let K be a nonempty, closed, bounded and convex subset of X and P a closed, pointed and convex cone in Y with $intP \neq \emptyset$ and $intP^* \neq \emptyset$. If $T : K \to L(X, Y)$ is a P-monotone and V-hemicontinuous operator and $H : K \to Y$ is a P-continuous and P-convex operator, then there is an $x_0 \in K$ such that x_0 is a solution of (VVI).*

Proof. Let $s \in intP^*$. Define $(s \circ T)(x) = s \circ T(x)$ for any $x \in K$. Then $s \circ T : K \to X^*$, where X^* is the dual space of X. Since T is P-monotone, for any $x, y \in K$,

$$\langle T(x) - T(y), x - y \rangle \in P.$$

Thus we have, for any $x, y \in K$,

$$\langle (s \circ T)(x) - (s \circ T)(y), x - y \rangle = s[\langle T(x) - T(y), x - y \rangle] \geqq 0.$$

Hence $s \circ T$ is monotone. Since T is V-hemicontinuous, for any $x, y, z \in K$ and $t \in (0,1)$,

$$\lim_{t \to 0+} \langle T(x + t(y - x)), z \rangle = \langle T(x), z \rangle.$$

Thus we have, for any $x, y, z \in K$ and $t \in (0,1)$,

$$\lim_{t \to 0+} \langle (s \circ T)(x + t(y - x)), z \rangle = \langle (s \circ T)(x), z \rangle.$$

Hence $s \circ T$ is hemicontinuous. Since H is P-continuous and P-convex, by Lemma 3.1, $s \circ H$ is a lower semicontinuous and convex function. By Lemma 3.2, there exists an $x_0 \in K$ such that x_0 is a solution of $(\mathrm{VVI})_s$. By Proposition 2.1, x_0 is a solution of (VVI).

By choosing $s \in P^* \backslash \{0\}$ and using the method similar to the proof of Theorem 3.1, we can prove the following theorem without the reflexivity of Y and $intP^* \neq \emptyset$ in Theorem 3.1.

THEOREM 3.2. *Let X be a reflexive Banach space, Y a Banach space, K a nonempty, closed, bounded and convex subset of X and P a closed, pointed and convex cone in Y with $intP \neq \emptyset$. If $T : K \to L(X, Y)$ is a P-monotone and V-hemicontinuous operator and $H : K \to Y$ is a P-continuous and P-convex operator, then there is an $x_0 \in K$ such that x_0 is a solution of (WVVI).*

REMARK. The above Theorem 3.2 is a slight generalization of part (i) of Theorem 2.1 in [3]. But the definition of V-hemicontinuity in Theorem 3.2 is slightly different from that in part (i) of Theorem 2.1 in [3].

Now we extend Theorem 3.1 and 3.2 to unbounded sets.

THEOREM 3.3. *Let X, Y be reflexive Banach spaces, K a nonempty, closed and convex subset of X and P a closed, pointed and convex cone*

in Y with $intP \neq \emptyset$ and $intP^* \neq \emptyset$. Let $T : K \to L(X,Y)$ be P-monotone and V-hemicontinuous and $H : K \to Y$ be P-continuous and P-convex. If there exists a nonempty bounded subset U of K such that for each $x \in K\backslash U$, there is a $u \in U$ such that

$$\langle T(x), u - x\rangle + H(u) - H(x) \in -P\backslash\{0\},$$

then there exists $x_0 \in K$ such that x_0 is a solution of (VVI).

Proof. Since U is bounded, there exists $r > 0$ such that for all $x \in U$, $\|x\| < r$. Let $K_r = \{x \in K : \|x\| \leqq r\}$. Then K_r is a nonempty, closed, bounded and convex subset of X. By Theorem 3.1, there exists an $x_r \in K_r$ such that

$$\langle T(x_r), x - x_r\rangle + H(x) - H(x_r) \notin -P\backslash\{0\} \text{ for all } x \in K_r. \quad (*)$$

If $x_r \notin U$, by the assumption, there is a $u \in U$ such that

$$\langle T(x_r), u - x_r\rangle + H(u) - H(x_r) \in -P\backslash\{0\},$$

which contradicts $(*)$. Hence $x_r \in U$. Let x be any fixed point of K. Since K is convex and $\|x_r\| < r$, there exists $\alpha \in (0,1)$ such that $w \equiv \alpha x + (1-\alpha)x_r \in K_r$. By $(*)$, we have

$$\langle T(x_r), w - x_r\rangle + H(w) - H(x_r) \notin -P\backslash\{0\}.$$

Since H is P-convex, $H(w) - \alpha H(x) - (1-\alpha)H(x_r) \in -P$. Thus we have

$$\langle T(x_r), w - x_r\rangle + \alpha H(x) + (1-\alpha)H(x_r) - H(x_r) \notin -P\backslash\{0\}.$$

Hence we have

$$\langle T(x_r), \alpha(x - x_r)\rangle + \alpha H(x) - \alpha H(x_r) \notin -P\backslash\{0\}.$$

Dividing by α, we have

$$\langle T(x_r), x - x_r\rangle + H(x) - H(x_r) \notin -P\backslash\{0\}.$$

Hence x_r is a solution of (VVI).

By the method similar to the proof of Theorem 3.3, we can obtain the following theorem from Theorem 3.2.

THEOREM 3.4. *Let X be a reflexive Banach space and Y a Banach space. Let K be a nonempty, closed and convex subset of X and P a closed, pointed and convex cone in Y with int $P \neq \emptyset$. Let $T : K \to L(X, Y)$ be P-monotone and V-hemicontinuous and $H : K \to Y$ be P-continuous and P-convex. If there exists a nonempty bounded subset U of K such that for each $x \in K \backslash U$, there is a $u \in U$ such that*

$$\langle T(x), u - x \rangle + H(u) - H(x) \in -intP,$$

then there exists an $x_0 \in K$ such that x_0 is a solution of (WVVI).

4. Multiobjective Optimization Problems

In this section, we consider the special cases of the vector variational inequality (VVI) formulated in section 1, and give relationships among those inequalities and multiobjective optimization problems.

First we give the concept of solution of multiobjective optimization problems.

Let $f = (f_1, \cdots, f_p)$ be an operator from $\mathbb{R}^n$ to $\mathbb{R}^p$ and K be a nonempty subset of $\mathbb{R}^n$.

Consider the following multiobjective optimization problem (P);

$$\text{(P) Minimize } (f_1(x), \cdots, f_p(x)) \text{ subject to } x \in K.$$

Optimization in (P) means obtaining efficient solutions of (P) defined as follows;

DEFINITION 4.1. $x_0 \in K$ is said to be an efficient solution of (P) if

$$f(x) - f(x_0) \notin -\mathbb{R}^p_+ \backslash \{0\} \text{for any } x \in K,$$

where $\mathbb{R}^p_+ = \{(z_1, \cdots, z_p) \in \mathbb{R}^p : z_i \geq 0, i = 1, \cdots \ p\}$.

Now we give some special cases of (VVI) formulated in section 1;

(1) Let K be a closed convex subset of $\mathbb{R}^n$ and $T : \mathbb{R}^n \to \mathbb{R}^{p \times n}$ an operator, where $T(x) = (T_1(x), \cdots, T_p(x))^t$ and $T_i(x) \in \mathbb{R}^n$.

Consider the following vector variational inequality (VVI)′ introduced by Giannessi [5];

(VVI)′ Find $x_0 \in K$ such that

$$(T_1(x_0)^t(x - x_0), \cdots, T_p(x_0)^t(x - x_0)) \notin -\mathbb{R}^p_+ \backslash \{0\} \text{for all } x \in K.$$

(2) Let $f = (f_1, \cdots, f_p)$ be a differentiable operator from $\mathbb{R}^n$ to $\mathbb{R}^p$ and K a polyhedral convex set in $\mathbb{R}^n$, that is, the intersection of some finite collection of closed halfspaces in $\mathbb{R}^n$, such that $intK \neq \emptyset$.

Consider the following vector variational inequality (VVI)″;

(VVI)″ Find $x_0 \in K$ such that

$$(\nabla f_1(x_0)^t(x - x_0), \cdots, \nabla f_p(x_0)^t(x - x_0)) \notin -\mathbb{R}^p_+ \backslash \{0\} \text{ for all } x \in K.$$

By Definition 4.1, we can obtain easily the following proposition;

PROPOSITION 4.1. *$x_0 \in K$ is a solution of (VVI)′ if and only if $x_0 \in K$ is an efficient solution of a multiobjective linear optimization problem: Minimize $(T_1(x_0)^t x, \cdots, T_p(x_0)^t x)$ subject to $x \in K$.*

THEOREM 4.1. *The followings are equivalent;*

(1) *$x_0 \in K$ is a solution of* (VVI)″.

(2) *$x_0 \in K$ is an efficient solution of a multiobjective linear optimization problem: Minimize $(\nabla f_1(x_0)^t x, \cdots, \nabla f_p(x_0)^t x)$ subject to $x \in K$.*

(3) *There exists $\lambda_i > 0, i = 1, \cdots, p$ such that $x_0 \in K$ is an optimal solution of the ordinary linear optimization problem: minimize $\lambda_1 \nabla f_1(x_0)^t x + \cdots + \lambda_p \nabla f_p(x_0)^t x$ subject to $x \in K$.*

(4) *There exists $\lambda_i > 0, i = 1, \cdots, p$ such that $g(x) := \lambda_1 \nabla f_1(x_0)^t x + \cdots + \lambda_p \nabla f_p(x_0)^t x + \phi_K(x)$ has a global minimum at $x_0 \in K$, where*

$$\phi_K(x) = \begin{cases} 0 & \text{if } x \in K \\ \infty & \text{if } x \notin K. \end{cases}$$

(5) *There exists $\lambda_i > 0, i = 1, \cdots, p$ such that*

$$0 \in \lambda_1 \nabla f_1(x_0) + \cdots + \lambda_p \nabla f_p(x_0) + \partial \varphi_K(x_0),$$

where $\partial \phi_K(x_0)$ is the subdifferential of ϕ_K at $x_0 \in K$.

(6) *$x_0 \in K$ and there exists $\lambda_i > 0, i = 1, \cdots, p$ such that $-\sum_{i=1}^p \lambda_i \nabla f_i(x_0) \in (K - x_0)^-$, where $(K - x_0)^- = \{z \in \mathbb{R}^n :$*

Proof. By Proposition 4.1, (1) is equivalent to (2). By Theorem 3.4.7 in [18], (2) is equivalent to (3). It is easily checked that (3) is equivalent to (4). Also, it is easily checked that (4) is equivalent to (5). Since $\partial\phi_K(x_0) = (K - x_0)^-$, (5) is equivalent to (6).

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GUE MYUNG LEE, DO SANG KIM AND SUNG JIN CHO
DEPARTMENT OF APPLIED MATHEMATICS, PUKYONG NATIONAL UNIVERSITY, PUSAN 608-737, KOREA

BYUNG SOO LEE
DEPARTMENT OF MATHEMATICS, KYUNGSUNG UNIVERSITY, PUSAN 608-736, KOREA