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Development of A Computer Algorithm For Analysing Freeway Traffic Flow: General Theory

고속도로의 교통류해석을 위한 컴퓨터 알고리즘개발: 이론적 배경을 중심으로

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ABSTRACT

고속도로 및 도시 고속도로는 교통의 단순한 매체로 뿐만 아니라 환경, 에너지, 경제 등등 사회 전반에 걸쳐 그 역할이 다양하며, 영향력이 지대하고, 중요한 비중을 차지함으로써, 이들 도로의 효율적인 운영을 위하여 고속도로 운영체계 수립 및 설계시 교통상황을 예측할 필요성이 있다. 이런 목적을 실현하기 위하여, 기존의 개발된 교통류 모형들을 사용할 수 있으나, 이들의 예측 결과에 대한 낮은 신뢰도, 혹은 모형의 특성(예, 처리용량, 해석방법)에 따른 제약 등등의 이유로 실용화되지 못하고 있는 실정이다. 최근 Newell은 충격파이론을 간편화하여 기존의 다른 교통류 이론들에 비해 많은 장점을 갖은 새로운 교통류 이론은 개발하였다. 하지만, 이 이론도 수작업에 의한 도식적(graphical) 해석방법을 기초로 하고 있기 때문에, 실제 교통운용체계에 사용하기에는 거의 불가능한 비효율적 결함을 지니고 있다. 이 논문의 목적은 Newell의 이론을 추후 실제 현장에서 적용할 수 있도록 Newell의 도식적 해석방법을 체계화(mechanize)한 컴퓨터 알고리즘을 개발하는데 있다.

1. Introduction

In an attempt to solve the multiple link freeway problem, recently G. F. Newell (1993a, 1993b & 1993c) has developed a *"Simplified Theory of Kinematic Waves in Highway Traffic"* which is based on the theory of *"kinematic waves"* described originally by Lighthill and Whitham in 1955. An important simplifying assumption of his theory is that relationship between traffic flow and density on a freeway can be approximated by a triangular flow, *q*, versus density, *k*, relationship. (A detailed description of the simplifying assumption is presented in Section 2.2) Newell's simplified theory avoids the computational difficulties of the Lighthill-Whitham theory by working with cumulative traffic counts instead of flows, and by assuming the triangular *q-k* relationship. Newell's model is capable of predicting the multi-destination traffic flow patterns at any freeway junction or other location of interest. Also, his model gives the analyst the considerable advantages of knowing the cumulative vehicle counts that leave the freeway at each exit ramp junction. Another important feature of Newell's theory is the ability to predict when a queue from a downstream bottleneck backs up to specific upstream location. In fact, such queueing propagation information is very useful for traffic engineers in selecting efficient freeway traffic control schemes. Newell's model promises to be a simple and plausible model as an evaluation and analysis tool for freeway traffic flow.

The objective of this paper is twofold: first, to develope a computer algorithm, which can produce numerical results for traffic flow conditions in reasonable computation times, in order to mechanize the graphical procedures of Newell's theory, and second, to facilitate the application of his theory for freeway traffic operation/control purposes. The computer algorithm includes the procedures for extracting information about freeway traffic flow such as the total travel time, total delay, traffic density, and the location of the tail of a queue. These procedures are based on Newell's simplified theory. In describing the theory, equation from Newell's three papers (1993a, 1993b & 1993c) are referred to by their number in those papers, preceded by the letter N and a hyphen, e. g., N-II-3 refers to equation 3 in Newell's second paper.

2. The Theory

Based on the analogy between traffic flow and a real fluid, Lighthill and Whitham (1955) and Richards (1956) developed a hydrodynamic (Kinematic Waves) theory of traffic flow. The key postulate of this theory is that there is a functional relation between traffic flow and density (concentration), where traffic flow is defined as the rate at which vehicles pass some point, and traffic density is defined as the number of vehicles per unit length of the roadway. At a macroscopic level, the traffic flow is the average vehicle speed (space mean speed) multiplied by the traffic density, i. e., $q = kv$; this relationship is often called the fundamental law of traffic flow.

2.1 Wave Pace

Newell's simplified theory is based on the concept of waves in traffic. A *traffic wave* (hereafter referred to as a wave) is defined as a curve joining points on the time-space plane which have the same flow, q, and density, k. When changes in flow and density occur, the wave can be thought of as carrying such changes through the stream of vehicles.

The "*pace*" (the reciprocal of the "*velocity*") of the wave relative to the road can be determined as follows. In Newell's theory, it is assumed that the q-k relationship might vary with location, x, but not with time, t, so

$$k(x, t) = k^*(q(x, t), x)$$

or (N-I.1)

$$q(x, t) = q^*(k(x, t), x)$$

for some given relations k^* or q^* From equation (N-I.1) and the conservation equation (equation of continuity) for traffic flow,

$$\partial k(x, t)/\partial t + \partial q(x, t)/\partial x = 0, \qquad \text{(N-I.2)}$$

the following partial differential equation for $q(x, t)$ can be derived:

$$w(q, x)\partial q(x, t)/\partial t + \partial q(x, t)/\partial x = 0 \qquad \text{(N-I.3)}$$

where $w(q, x)$ is defined as

$$w(q, x) \equiv \partial k^*(q, x)/\partial q.$$

For the reason outlined in the next paragraph, $w(q, x)$ — which will be very important in the analysis that follows — is called the "*wave pace*" See Haberman(1977) for additional details about the derivation of equation (N-I.3).

To see why $w(q, x)$ is called the wave pace, suppose that the flow is measured by a moving observer and the position of the observer is described by a function $x(t)$. The flow measured by the observer presumably depends on both location and time, so rewrite $q(x(t), t)$. The rate of change of this flow as the observer moves along the road is

$$dq(x(t), t)/dx = [dt/dx][\partial q(x(t), t)/\partial t] + \partial q(x(t), t)/\partial x \qquad (1)$$

Comparing equation (1) with equation (N-I.3), it can be seen that an observer moving at the pace

$$dt/dx = w(q, x) \qquad \text{(N-I.4)}$$

will not see any change in $q(x(t), t)$, so is — by definition — moving along a wave. It follows that the wave's pace must also be $w(q, x) = \partial k^*(q, x)/\partial q$. This is the basis for the notion of a wave and its velocity or pace.

It is very important to notice that the pace of a wave carrying a flow, q, at location, x,

$$dt/dx = w(q, x) = \partial k^*(q, x)/\partial q, \qquad (2)$$

can be interpreted as the slope of the tangent to the q-k relationship at x, as illustrated in Figure 1 (Haberman, 1977). The figure shows that when the density, k, is less than the critical density, k_c (corresponding to the capacity, μ), the pace of a wave is positive and when k is greater than kc the pace of a wave is negative. The negative wave pace indicates that the wave moves in a direction opposite to the direction the vehicles are moving. Waves can therefore be propagated either forward or backward at the pace given by equation (N-I.4)

It is common on a congested highway to

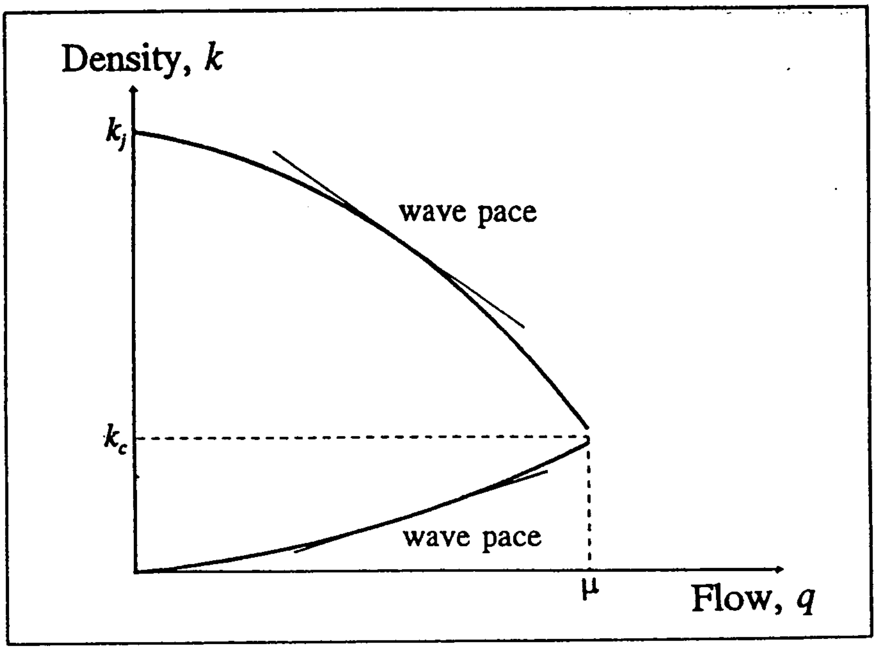

Figure 1: The wave paces

observe a chain reaction being triggered by a downstream driver applying his/her brakes, thus causing the drivers upstream to apply their brakes. The brake lights, which indicate disturbances (changes) in traffic, appear to move backward, whereas the vehicles are moving forward. This example demonstrates that waves can move backward while the vehicles are moving forward.

2.2 An Important Simplifying Assumption of the Theory

Recent research (Banks, 1989; Hall et al, 1992; Koshi et al, 1983) has suggested a q-k relationship which consists of two more or less linear branches at low and high density values corresponding to the uncongested and congested flow regimes, respectively. Newell (1993b) simplifies this idea to an assumption that the relation between traffic flow and density on a freeway can be approximated by a triangular q-k relationship as illustrated in Figure 2; this is an important simplifying assumption of his theory.

The triangular q-k relationship shown in Figure 2 leads to only two constant wave paces,

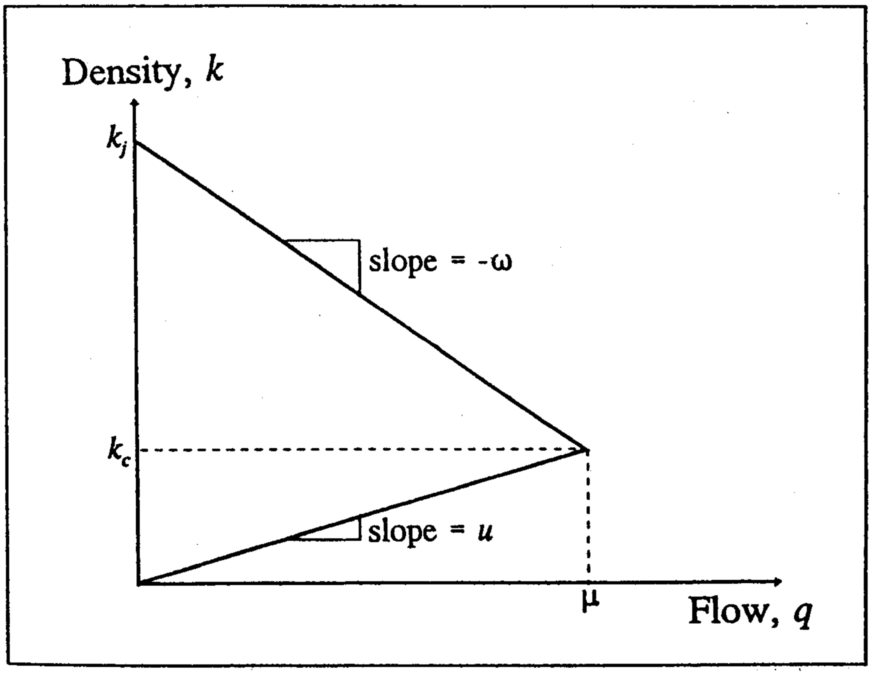

Figure 2: A triangular flow and density rela tionship

u 〉 0 for the uncongested regime where the density is less than the critical density, k_c, and -ω (where ω 〉 0) for the congested regime of density between the critical density and jam density, k_j. (Newell uses uo as the forward wave pace and -ω_0 as the backward wave pace: the subscript has been dropped to simplify some of the expression that will appear later.) It is important to know that having two wave pace does not mean that Newell's theory allows only two speeds on freeways. The forward wave pace equals the critical density divided by the capacity (i. e., u = k_c / μ). This wave pace is equal to the pace of vehicles moving at the free-flow speed, since the slope of the triangular q-k curve for the uncongested flow regime of Figure 2 is identical to the reciprocal of the free-flow speed. The backward wave pace is equal to $-(k_j - k_c)/\mu$. In the remainder of this paper, it is always assumed that k^* and q^* in equation (N-I.1)

2.3 Other Basic Definitions, Assumptions and Key Equations

2.3.1 The Numbering of Station

Each junction of an entrance or exit ramp with a freeway is defined as a "*station*". In addition to the ramp junctions, any point between successive ramps where the capacity is reduced due to a lane drop can also be defined as a station. Once all the stations have been defined, number i ∈ {0, 1, 2, . . .} are assigned to them in consecutive order, beginning with station 0 at the upstream end of the freeway. The location of station i is represented as point x_i, measured from station 0, with $x_{i-1} < x_i$. We do not distinguish between the beginning and end of a ramp taper, so both merges and diverges are modelled as occurring at a point, that is at x_i.

2.3.2 Assumptions

To make Subsection 2.3.4 through 2.3.5 easier to understand, three other assumptions made in Newell's theory are presented here. First, Entrance ramp vehicles have priority at the merge point, so they do not experience delay when entering a freeway. Secondly, the travel time of any vehicle in the roadway section (x_i, x_{i+1}) at time t is independent of its origin or destination; thus, vehicles exiting at x_{i+1} have the same travel time from x_i to x_{i+1} as through vehicles passing x_{i+1} at the same time. Thirdly, in any section (x_i, x_{i+1}), the critical density, $kc(i)$, the jam density, $k_c(i)$, and the capacity, μ_i, remain constant and consequently, the forward wave pace, u_i, and the backward wave pace, $-\omega_i$, in the section are also constant.

The first assumption seems to be an oversimplification of the traffic situation at the junction. However, the assumption is usually correct if there is an adequate merging lane for the ramp vehicles entering the freeway, though there may be some traffic friction. It would be easy to modify the model to allow more complex queue disciplines if the analyst knew how to describe them. With respect to the second assumption, macroscopic models use the average speed of vehicles in the traffic stream in order to simulate the movement of the vehicles. This simulation is usually reasonable for freeway traffic flow because the travel time of vehicles travelling in the section between two successive ramp junctions is mainly dependent upon traffic conditions on the section rather than mutual interference (e. g., lane changing) between the vehicles. As outlined in Subsection 2.3.1, the freeway must be divided into homogeneous sections in terms of geometry conditions and capacity; this is necessary to ensure the validity of the third assumption.

2.3.3 Cumulative Traffic Counts

In determining the cumulative traffic count at any location, it is convenient to pretend that an observer at each freeway location numbers vehicles consecutively as they pass the observation location, starting from the passage of the same reference vehicle at every location. It is temporarily assumed that no vehicles enter or leave the freeway, so that the imaginary observers at every location assign the same number to each car and define $N(x, t)$ as the cumulative number of vehicles which pass location x by time t. The cumulative count $N(x, t)$ is then equal to the number of the last vehicle to pass the count location x before time t and the vehicle's number does not change as it moves.

If we draw the curve $N(x, t)$ in a three-dimensional (N, x, t) space, $N(x, t)$ will actually be a step function since a vehicle count is an integer, but in hydrodynamic traffic flow models, the curve is "smoothened" in order to define a flow $q(x, t)$ as the derivative $dN(x, t)/\mathrm{d}t$ of the curve. From the $N(x, t)$-curve, the traffic density and flow can also be evaluated (Makigami et al, 1971; Newell, 1982) as

$$k(x, t) = -\partial N(x, t)/\partial x$$

and (N-I.6)

$$q(x, t) = \partial N(x, t)/\partial t.$$

The change in $N(x, t)$ along a wave with pace dt/dx = w can be determined by

$$dN = (\partial N/\partial x)dx + (\partial N/\partial t)dt = -kdx + qw = (-k + qw)dx \quad \text{(N-I.8)}$$

2.3.4 Propagation of a Forward Moving Wave

If there is no congestion, the cumulative count $N(x, t)$ at location x at time t can be determined from the cumulative count $N(x_{i-1}, t)$ at location x_{i-1}. According to the assumption that the free flow speed is not a function of flow and time, $N(x_i, t) = N(x_{i-1}, t-(x_i - x_{i-1})u_{i-1})$, since $(x_i - x_{i-1})u_{i-1}$ is the free-flow travel time from x_{i-1} to x_i. Figure 3 graphically explains the determination of N(x_i, t). In the figure, the jth vehicle passes x_{i-1} at time t_1 and arrives at downstream location x_i at time $t_2 = t_1 + (x_i - x_{i-1})u_{i-1}$. Since this is true for every i, the $N(x_i, t)$-curve can be determined by horizontally translating the $N(x_{i-1})$-curve by a time displacement equal to the free-flow travel time between x_{i-1} and x_i. This graphical procedure (hereafter referred to as the "forward wave propagation rule") will be used in Section 4 to evaluate the cumulative number of vehicles which would arrive at x_i if there was no congestion in the roadway section (x_{i-1}, x_i).

2.3.5 Propagation of a Backward Moving Wave

A backward wave moves from x_{i+1} to x_i

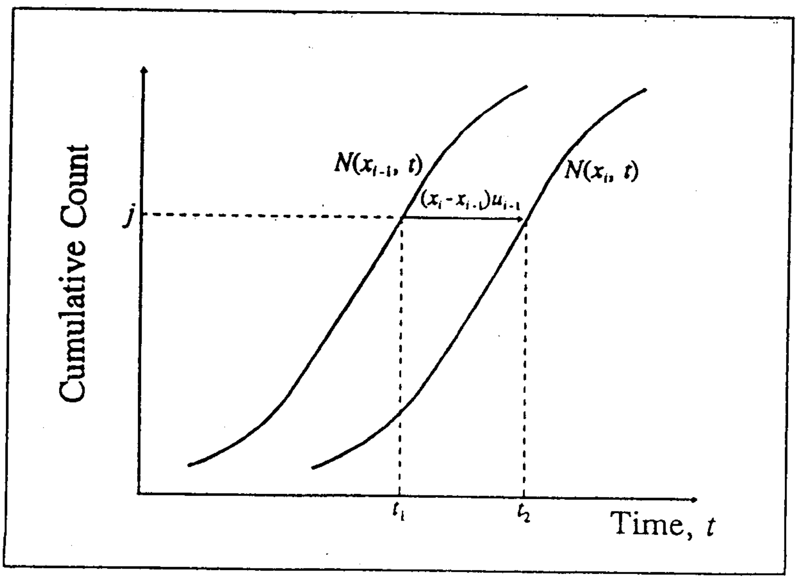

Figure 3: A forward moving wave propaga- tion.

within a congested freeway section (x_i, x_{i+1}) with the wave pace w = $-\omega_i = -(k_j(i) - k)/q$ where $k > kc(i)$. From equation (N-I.8), with this pace,

$$dN/dx = -k + qw = -k - q\omega_i = -k - q(k_j(i)-k)/q = -k - k_j(i) + k = -k_j(i). \quad (3)$$

Figure 4 shows a part of the $N(x, t)$-curve, and graphically explains the change in the cumulative count $N(x, t)$ in the section (x_i, x_{i+1})along a backward wave. The backward wave passes x_{i+1} at time t1, then traverses the distance dx = dt/wi and reaches point x_{i+1} dx at time t_1+dt. In this figure, the dashed curve between points 1 and 3, which is a portion of the trajectory of the backward wave in the three-dimensional (N, x, t) space, connects two cumulative counts $N(x_{i+1}-dx, t_1+dt)$ and $N(x_{i+1}, t_1)$. If the dashed curve between points 1 and 3 is projected onto the $c(x, t)$ plane, the projection will be a straight line. The slope of the straight line is equal to the backward wave pace, $dt/dx = w_i$.

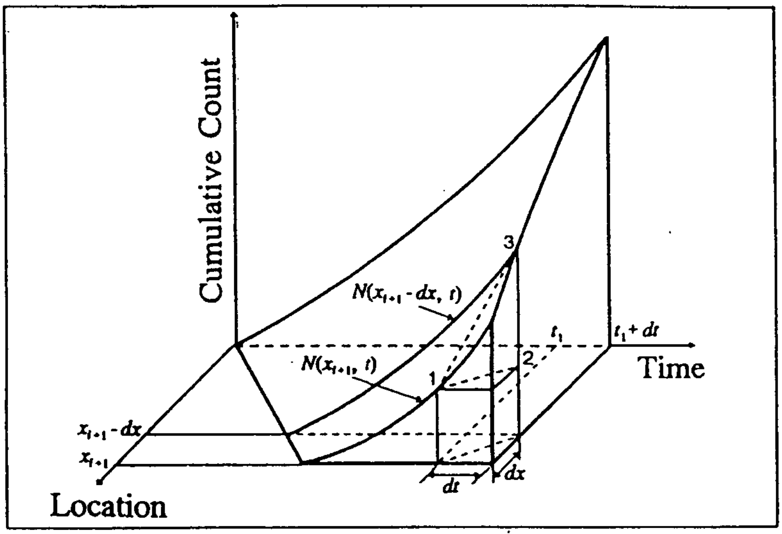

Figure 4: A bockward moving wave propagation

Figure 5 is obtained by projecting the curves $N(x_{i+1}-dx,t)$ and $N(x_{i+1},t)$ in Figure 4 onto the (N, t) plane. These two curves can be approximated by a straight line for an infinitesimal dt. In this figure, the horizontal displacement, $\omega_i dx$, between points 1' and 2' is equal to the backward wave propagation time from x_{i+1} to $x_{i+1}-dx$. The vertical displacement, $dN = N(x_{i+1}-dx, t_1+dt) - N(x_{i+1}, t_1)$, between points 2' and 3' is equal to $k_j(i)dx$, according to equation (3). The $k_j(i)dx$ represents the number of vehicles which can be presented between $x_{i+1}-dx$ and x_{i+1} at jam density. From the figure, it can be seen that the cumulative count at location $x_{i+1}-dx$ at time t_1+dt, $N(x_{i+1}-dx,$

t_1+dt), can be determined by translating point $N(x_{i+1}, t_1)$ horizontally by a displacement $\omega_i$dx and vertically by a displacement $k_j(i)dx$. This transition can be done for every point on the $N(x_{i+1}, t)$-curve. Hence the entire $N(x_i, t)$-curve or at least some piece of it can be constructed by translating the curve $N(x_{i+1}, t_1)$ as follows.

Suppose that the queue from xi+1 backs up to some location x $=x_{i+1}-\Delta x$ between x_i and x_{i+1} at time $t = t_1 + (x_{i+1}-x)\omega_i$. Since it has been assumed that $k_j(i)$ and $-\omega_i$ are constant in the section (x_i, x_{i+1}), dN/dx remains constant along the backward wave moving from x_{i+1} to x_i, according to equation (3). Thus, the difference between the cumulative traffic counts at x and x_{i+1}, $\Delta N = N(x, t) - N(x_{i+1}, t_1)$, can be determined by multiplying $dN/dx = k_j(i)$ by Δx. Hence, $N(x, t) = N(x_{i+1}, t_1) + (x_{i+1}-x)k_j(i)$. Since $-\omega_i$ is constant in the section, the projection of the curve between $N(x_i, t)$ and $N(x_{i+1}, t_1)$ onto the (x, t) plane is a straight line and its slope is $-\omega_i$. Thus, the value of $N(x, t)$ can also be determined by translating the $N(x_{i+1}, t_1)$ horizontally by a displacement $(x_{i+1}-x)\omega_i$ and vertically by a displacement $(x_{i+1}-x)k_j(i)$. This graphical method (hereafter referred to as the "backward wave propagation rule") will be used for the determination of the cumulative number of vehicles at x_i and x_{i+1} if the queue from x_{i+1} reaches x_i in Section 4.

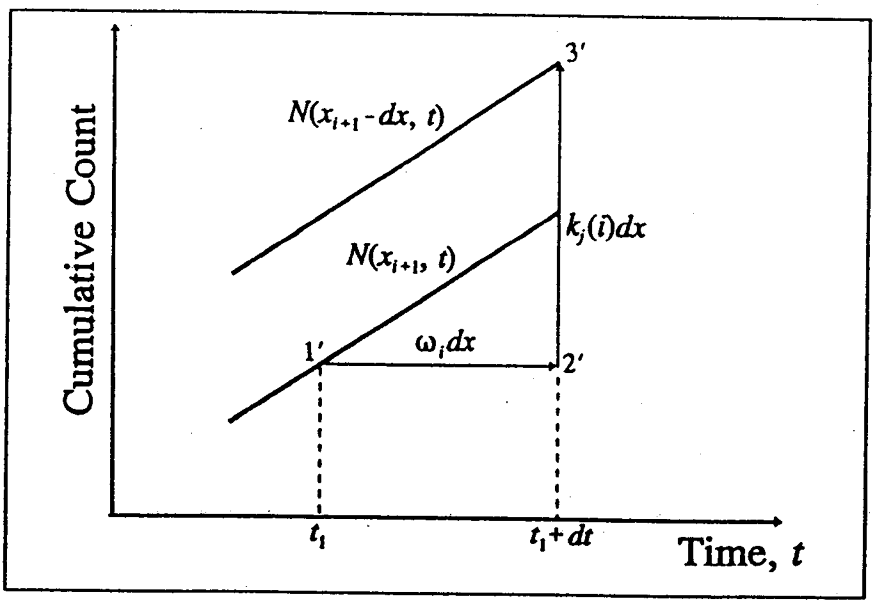

Figure 5: The change in N(x, t) along a backward wave in a three-dimensional (N, x, t) space.

3. Input Requirement and the Results of the Computer Algorithm

In addition to the given time-dependent origin-destination (O-D) flows, $A_{ij}(t)$, the following information are required for the input of the computer algorithm:

(1) the speed for the uncongested flow;
(2) the capacity;
(3) the jam density;
(4) the location of each station (ramp junction); and
(5) the number of lanes on the freeway sec tions.

The first three parameters are important determinants of the triangular q-k relationship. The forward wave and backward wave paces are determined from these three parameters.

The results of the computer algorithm include the following information about the traffic flow patterns and queueing on the freeway during a specific time period:

- The cumulative number of vehicles which would arrive at each station in the absence ofcongestion.
- The cumulative number of vehicles which can pass each station.
- The cumulative number of vehicles which leave the freeway at each exit ramp.
- The total travel time and total delay incurred by all vehicles that travelled on each freeway section between two successive ramp junctions or on the entire freeway sections.
- The outline of the congested region, queue length or the location of the tail of a queue, on the freeway system.

Beside the above information, the average traffic density on each freeway section at any particular time can be written to the output file if the analyst desires it.

4. Evaluation of Traffic Flow Patterns

Newell(1993b) uses a "moving time coordinate system" in his graphical method. In the moving time coordinate system, an imaginary observer at each location not only measures the cumulative traffic count from the passage of some reference vehicle travelling at the forward wave pace, but also the time relative to the passage of this vehicle. However, in this paper, Newell's graphical procedures and the computer algorithm are described by using the "real time coordinate system" instead of the moving time coordinate system. The reason for this is that it seems convenient for understanding the algorithm to deal with the problem as taking place in the real time coordinate system.

This section consists of three subsections: Subsection 4.1 illustrates the graphical and computer procedures for evaluating traffic flow patterns at each "station" on a deliberately simple system with no ramps. In Subsection 4.2, we consider a system which has an entrance ramp at one of the stations and Subsection 4.3 deals with a system having an exit ramp.

4.1 A System With No Ramps

To begin the analysis, the following notation is defined:

$A(x_i, t)$ = The cumulative number of vehicles which pass upstream location x_{i-1} by time $t-(x_i-x_{i-1})u_{i-1}$, so will arrive at x_i by time t if they are not delayed by congestion in section (x_{i-1}, x_i);

and

$D(x_i, t)$ = The cumulative number of vehicles which passed x_i by time t.

It should be noted that $A(x_i, t)$ does not include vehicles that would have reached x_i by time t if there were no queueing anywhere, but fail to do so because they lost time in a queue upstream from x_{i-1}. Furthermore, if there is congestion in section (x_{i-1}, x_i), $A(x_i, t)$ does not indicate the number of actual arrivals at x_i, but the number that would have arrived if there were no congestion in the section (x_{i-1}, x_i). It is also important to notice here that if there is no congestion in the section (x_{i-1}, x_i), then $D(x_i, t)$ is equal to the value of $N(x_i, t)$ which is determined by the forward wave pace propagation rule: however, if there is congestion in the section (x_{i-1}, x_i), $D(x_i, t)$ is equal to the value of $N(x_i, t)$ which is determined by the backward wave propagation rule.

Both the graphical and computer procedures start at station 1, and work downstream. The traffic flow pattern $D(x_0, t)$ at station 0, the upstream end of a system, must be known. Practically speaking, this means that station 0 must be a place that never becomes congested, though it might run at capacity. The last station n could be one that runs at the capacity, so there can be congestion in section (x_{n-1}, x_n). However, there must be no congestion downstream of station n. It is convenient to assume that the freeway is empty at the initial time t_0 and that vehicles start entering the freeway at every on-ramp at t_0. The above noted conditions and assumptions are applied in the analysis that follows and throughout the remainder of this paper.

4.1.1 The Graphical Procedure

To begin, suppose that the flow passing each station is not constrained by the capacity of the station. This is a temporary assumption to make the graphical procedure for determining the cumulative departure count easier to understand; it will be relaxed in the next paragraph. In the first step, the cumulative arrival curve $A(x_1, t)$ at station 1 is determined by moving the known $D(x_0, t)$-curve horizontally by a time displacement equal to the free-flow travel time from station 0 to 1, $(x_1 - x_0)u_0$ according to the forward wave propagation rule. In the absence of congestion, the cumulative departure curve, $D(x_1, t)$ at station 1 is equal to the curve $A(x_1, t)$. After evaluating the curves $A(x_1, t)$ and $D(x_1, t)$, the graphical method evaluates the $A(x_i, t)$ and $D(x_i, t)$ curves in the same manner for every station $i > 1$.

Consider now a queueing situation where the flow passing some $x_i > x_0$ is constrained by the capacity at x_i, μ_i. Figure 6 shows a graphical construction of the curve $D(x_i, t)$ at x_i. In the figure, the flow reaching x_i is equal to the capacity μ_i for the first time at time t_1, so until that time the curve D(x_i, t) is the same as the curve A(x_i, t). However, after time t_1, D(x_i, t) is determined by drawing a straight line of slope of μ_i tangent to the curve A(x_i, t) at time t_1.

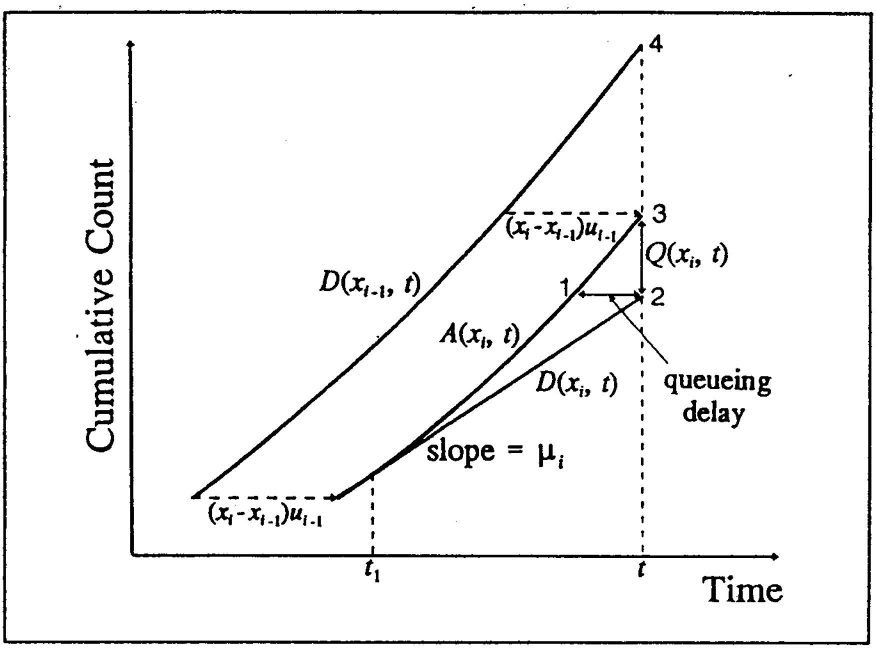

Figure 6: A graphical construction of the curve D(x_i, t).

It is important to notice here that in the conventional deterministic queueing analysis, the vertical distance,

$$Q(x_i, t) = A(x_i, t) - D(x_i, t), \qquad (4)$$

between points 2 and 3 in Figure 6 is thought of as the number of vehicles in the queue in section (x_{i-1}, x_i) at time t. However, Q(x_i, t) simply represents the difference between the cumulative number of vehicles which have already passed x_{i-1} by time t - (x_i - x_{i-1})u_{i-1}, but have not yet passed x_i by time t due to congestion in the section (x_{i-1}, x_i), and the cumulative number of vehicles that have actually passed xi by time t. Thus, Q(x_i, t) is not the "real physical queue", which is the number of vehicles between the tail of the queue and location x_i, but a "point queue" (or "vertical stack queue" or "standing queue"), the queue which would exist if all queued vehicles were stored in a vertical stack at x_i rather than lined up along the roadway.

In Figure 6, it is assumed that vehicles do not pass each other, so the horizontal distance between points 1 and 2 is the queueing delay for a vehicle which arrives at x_i at time t. The vertical difference, D(x_{i-1}, t) - D(x_i, t), between points 2 and 4 represents the number of vehicles which have passed x_{i-1}, but have not yet passed x_i ; it is the total number of vehicles on the roadway section (x_{i-1}, x_i) at time t, including vehicles in the queue as well as those that have not yet joined the queue.

The queue formed at x_i will grow as long as the oncoming flow to x_i exceeds the capacity μ_i at x_i. In this situation, one must check if or when the queue from xi backs up to x_{i-1} in order to determine the actual departure curve $D(x_{i-1}, t)$ at x_{i-1}. This check can be accomplished by a translation of the $D(x_i, t)$-curve, according to the propagation rule of the backward wave. Figure 7 shows a graphical construction of the $D(x_i, t)$-curve. In the figure, the $D(x_i, t)$-curve between points 1 and 2 is first translated horizontally by a time displacement equal to the backward wave propagation time from x_i to x_{i-1}, $(x_i - x_{i-1})\omega_{i-1}$; the translated $D(x_i, t)$ curve is represented by the dashed curve between points 3 and 4. That curve is now translated vertically by $(x_i - x_{i-1})k_j(i-1)$ to give the dashed curve between points 5 and 6 (hereafter referred to as the final translated $D(x_i, t)$ curve).

By conservation of the number of cars, the

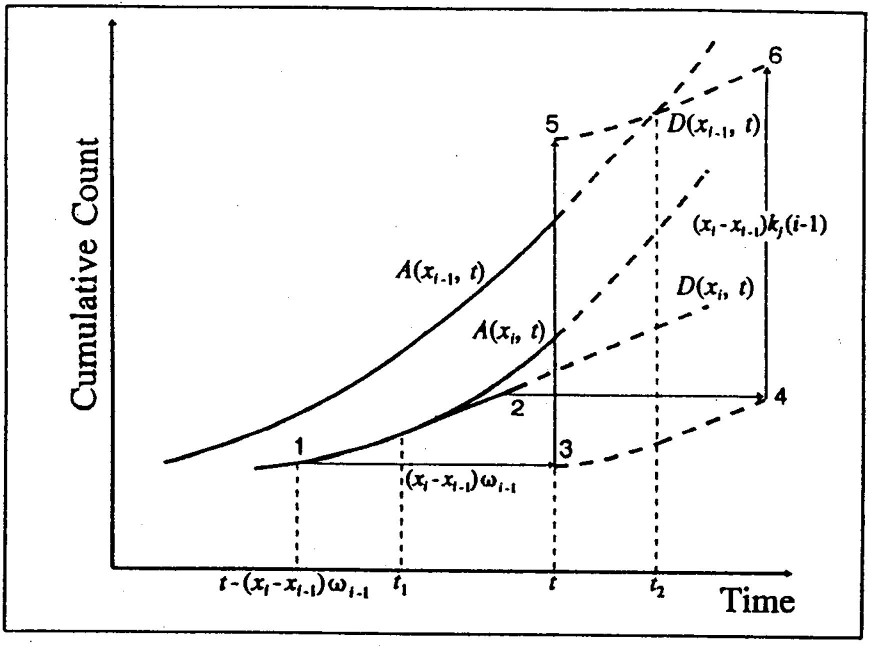

Figure 7: A graphical representation of the backward wave propagation.

cumulative number of vehicles which passed at any location should be the same when viewed from either side. Thus, if the tail of the queue from x_i is located exactly at point x_{i-1} at time t, two cumulative number of vehicles — one from the forward wave moving from x_{i-2} to x_{i-1} and the other from the backward wave moving from x_i to x_{i-1} — should be equal. In Figure 7, the final translated $D(x_i, t)$ curve is above the arrival curve $A(x_{i-1}, t)$ at time t. This indicates that the queue from x_i has not yet reached x_{i-1} at time t, so the flow passing x_{i-1} at time t is not constrained by the queue from x_i. The end of an existing queue passes x_{i-1} at time t_2 when the curve A(x_{i-1}, t) intersects the final translated D(x_i, t) curve. In fact, the queue at x_{i-1} is the continuation of the queue from x_i. One unique feature of Newell's traffic flow model is that the model divides a big queue which forms at a bottleneck into little point queues at each station within the congested region. In this manner, Newell's model takes into account the effect of a physical queue on the determination of traffic flow patterns upstream at the bottleneck regardless of the length of the queue. From time t_2, the curve A(x_{i-1}, t) is above the final translated D(x_i, t) curve. This indicates that the flow passing x_{i-1} at time t_2 is constrained by the queue extending upstream from x_i, though the flow is less than the capacity at x_{i-1}. Thus, the actual departure curve at x_{i-1}, D(x_{i-1}, t), from t_2 is determined by the final translated D(x_i, t) curve, which is the smaller of the candidate values implied by the final translated D(x_i, t) curve and the curve A(x_{i-1}, t). Note that to the left of time t_2 the actual departure curve D(x_{i-1} t) at x_{i-1} is equal to the arrival curve A(x_{i-1}, t).

When the curve A(x_{i-1}, t) again drops below the final translated D(x_i, t) curve, it indicates that the queue from x_i no longer extends past x_{i-1}, so the flow passing x_{i-1} is no longer constrained by the queue. After that time, the curve A(x_{i-1}, t) will determine the actual departure curve D(x_{i-1}, t) at x_{i-1}. This is not illustrated in Figure 7.

4.1.2 The Computer Procedure

Since it has been assumed that the freeway is empty at the initial time t = 0, the cumulative counts A(x_i t) and D(x_i, t) at the initial time t_0 for every i are zero. The departure counts D(x_0, t) at x_0 for all discrete times t ∈ {t_0 + kτ, k = 0, 1, 2, . . .}, where τ is a time increment chosen by the user, are known. From the "given" D(x_0, t), the computer algorithm evaluates the cumulative counts A(x_i t) and D(x_i, t) at time t = t_0 +τ for stations i = 1 to n (the last station) where A(x_i t) 〉 0, then A(x_i, t) and D(x_i, t) at t = t_0 + 2τ for i = 1 to n where A(x_i, t) 〉 0, etc. by the procedure that follows.

The computer procedure for determining A(x_i t) is perhaps best understood by looking at Figure 8 which shows the cumulative vehicle counts at x_{i-1} and x_i. In the figure, dark filled squares indicate which are known and the empty square indicates the target point which is to be determined. Since the algorithm employs an assumption that traffic flow within each time increment is constant at each "station", the cumulative count curves are piece-wise linear. The arrival count, A(x_i, t), at x_i is equal to the number of vehicles which passed upstream location x_{i-1} by time t - (x_i- x_{i-1})u_{i-1} : A(x_i, t) = D(x_{i-1}, t-(x_i- x_{i-1})u_{i-1}). However, the free-flow travel time (x_i- x_{i-1})u_{i-1} is not generally an integer multiple of the time increment, τ. Therefore, the count D(x_{i-1}, t-(x_i- x_{i-1})u_{i-1}) identified by point 1 in Figure 8 is an intermediate value between the cumulative counts at x_{i-1} at two discrete times ta and t_a +τ, where t_a is the last time point t - kτ before time t-(x_i- x_{i-1})u_{i-1}.

In order to evaluate such increment values, the computer algorithm employs a linear interpolation:

$$A(x_i, t) = D(x_{i-1}, t_a) + [D(x_{i-1}, t_a+\tau) - D(x_{i-1}, ta)][\Delta\tau/\tau] \quad (5)$$

where

$\Delta\tau = (n + 1)\tau - (x_i - x_{i-1})u_{i-1}$

as shown in Figure 8 and

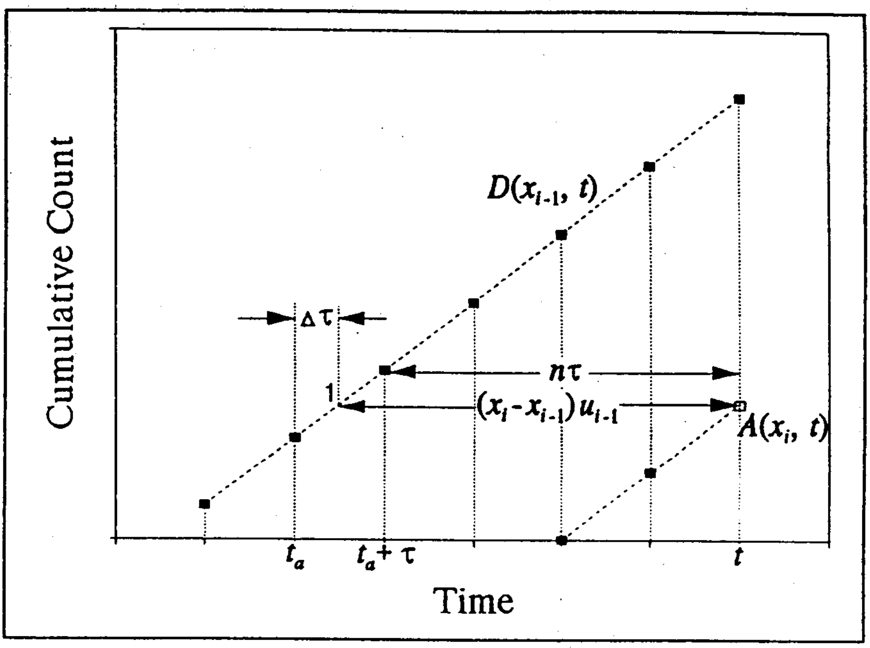

Figure 8: The cumulative traffic counts at x_{i-1} and x_i.

n = the integer part of $[(x_i - x_{i-1})u_{i-1}/\tau]$

which is the number of time increments within the free-flow travel time from x_{i-1} to x_i. It is important to notice here that n has memory implications for the computer program: in order to determine $A(x_i, t)$, the computer program must store temporarily at least n + 2 data points of $D(x_{i-1}, t_a)$ within the time period from $t_a = t - (n + 1)$ to t. In Figure 8, for example n = 4 and there are six data points of $D(x_{i-1}, t)$ within the time interval [ta, t]. The time increment τ is most likely to be less than or equal to one minute. It can be expected that the accuracy of the computer algorithm would be dependent on the value of τ. If τ is small (e. g., 15 seconds), the accuracy of the estimated results from the computer output should be high. However, smaller values require more computations and, since n is larger for a smaller value of τ, more memory space. It is, therefore, necessary to identify an appropriate τ in terms of balancing the accuracy of the output and the computational efficiency of the computer algorithm. In essence, τ should be chosen in order to capture any possible surges in the arrivals and departures at any time at any station. To identify an appropriate τ, the accuracy of the algorithm's results and computational efficiencies for different τ's must be compared.

This subsection now focuses on the determination of the cumulative counts of vehicles which passed x_i by time t, $D(x_i, t)$. Before proceeding any further, it should be noted that the computer algorithm takes into account the following three possible conditions under which the actual departure count $D(x_i, t)$ at station i can be determined:

(1) the flow passing x_i is constrained by the capacity at x_i, μ_i;
(2) the flow passing x_i is constrained by a queue which backs up to xi from x_{i-1}; and
(3) the flow passing x_i is constrained by neither the capacity at x_i nor a queue extend from x_{i-1}.

Under condition (1), $D(x_i, t)$ is the cumulative departure count evaluated at the immediate previous time t $-\tau$ plus the number of vehicles that can pass x_i at capacity during the time period (t $-\tau$, t):

$$D(x_i, t) = D(x_{i-1}, t - \tau) + \mu_i \tau . \qquad (6)$$

This cumulative departure count will be called the first departure count. The second departure count, under condition (2), can be evaluated by the propagation rule for backward waves as

$$D(x_i, t) = D(x_{i-1}, t - (x_{i-1}-x_i)\omega_i) + (x_{i-1} - x_i)k_j(i), \qquad (7)$$

where $(x_{i-1}-x_i)\omega_i$ is the backward wave propagation time from x_{i-1} to x_i. This time is not generally an integer multiple of the time increment, τ, so $D(x_{i-1}, t - (x_{i-1} - x_i)\omega_i)$ must be determined by a linear interpolation between the cumulative departure counts at x_{i-1} just before and after time t - $(x_{i-1} - x_i)\omega_i$. The first term of the right side of equation (7) is related to the horizontal translation of the curve $D(x_{i-1}, t)$ in the graphical construction of the backward wave propagation rule and the second term is related to the vertical translation. Equation (7) gives the same value for the cumulative traffic count as the one obtained by the graphical procedure except for minor inaccuracy due to linear interpolation. However, in order to use equation (7), the time increment τ must satisfy the following condition:

$$\tau \le \min[(x_{i-1}-x_i)\omega_i, \ i = 0, 1, \ldots, n - 1]. \qquad (8)$$

Under condition (3), the cumulative departure count at x_i by time t, $D(x_i, t)$, is equal to the cumulative arrival count $A(x_i, t)$. This departure count will be called the third departure count.

In order to determine the actual departure count at each station, the computer algorithm always evaluates the possible departure counts under conditions (1) and (3), but not always the departure count under condition (2). If there was no congestion on the roadway section (x_i, x_{i-1}) at time t - τ, the actual cumulative departure count is

$$D(x_i, t) = \min[A(x_i, t), D(x_i, t - \tau) + \mu_i \tau]. \qquad (9)$$

since $D(x_i, t)$ will be subject to only the constraints, $D(x_i, t) \le A(x_i, t)$ and $dD(x_i, t)/dt \le$

μ_i. However, if there was congestion in the section (x_i, x_{i+1}) at time t - τ, the algorithm evaluates all three possible departure counts and then determines the actual cumulative count as

$$D(x_i, t) = \min[A(x_i, t), D(x_i, t-\tau) + \mu\tau, D(x_{i-1}, t-(x_{i-1}-x_i)\omega)+(x_{i-1}-x_i)k_j(i)]. \quad (10)$$

4.2 A System With an Entrance Ramp

Traffic flow patterns at time t should be different just upstream and downstream from a ramp junction located at x_i if there are vehicles entering or exiting the freeway at station i. To consider a system which has an entrance ramp at station i, the following additional notation is defined:

$A_i(t)$ = the cumulative number of ramp vehicles which enter the freeway at station i by time t ;

x_i^- = the location just upstream of the ramp at x_i ;

and

x_i^+ = the location just downstream of the ramp x_i.

As pointed out earlier, however, merges and diverges are modelled as occurring at a point, so x_i^- and x_i^+ do not have values different from x_i: the purpose of - and + is to indicate whether the associated cumulative traffic counts are upstream or downstream of the ramp. Clearly the downstream arrival count can be obtained by simply adding the ramp count to the upstream count:

$$A(x_{i+1}, t) = A(x_{i-1}, t) + A_i(t). \quad (11)$$

Since it has been assumed that the ramp vehicles pass x_i with no delay, a similar relationship also holds for the departure flows,

$$D(x_i^+, t) = D(x_i^-, t) + A_i(t), \quad (12)$$

and the point queue at station x_i is the "queue" of through traffic:

$$Q(x_i, t) = A(x_i^+, t) - D(x_i^+, t) = A(x_i^-, t) - D(x_i^-, t). \quad (13)$$

From equation (12), it can be noted that the service rate for through flow, $D(x_i^-, t)$, varies with time, though the capacity of the bottleneck is assumed to be constant over time. Thus, in the model, the effect on through traffic of time-dependent ramp flows at a bottleneck is similar to having a time-dependent capacity at the bottleneck.

The graphical and computer procedures for determining $A(x_i^-, t)$ and $D(x_i^+, t)$ are the same as those for $A(x_i, t)$ and $D(x_i, t)$ described in Subsection 4.1. After $A(x_i^-, t)$ and $D(x_i^+, t)$ have been evaluated by either the graphical or computer procedure, $A(x_i^+, t)$ and $D(x_i^-, t)$ can be obtained from equations (11) and (12), respectively.

If there is congestion in the roadway section (x_{i-1}, x_i), the on-ramp flow at x_i will cause a decrease in the flow passing x_i^- which, in turn, affects the time when a queue from x_i might back up to x_{i-1} as well as the travel time from x_{i-1} to x_i. Therefore, a check whether or not the queue has back up to x_{i-1} must be conducted using the departure curve, $D(x_i^-, t)$, at x_i^-, according to the backward wave propagation rule. If the final translated $D(x_i^-, t)$ curve is below the arrival curve, $A(x_{i-1}^+ t)$ at time t at x_{i-1}^+, the actual departure value, $D(x_{i-1}^+, t)$ will be determined by the final translated $D(x_i^-, t)$ curve. The computer algorithm evaluates all three possible departure counts, $A(x_{i-1}^+, t)$, $D(x_{i-1}^+, t-\tau) + \mu_{i-1}\tau$ and $D(x_i^-, t-(x_i-x_{i-1})\omega_{i-1}+(x_i-x_{i-1})k_j(i-1))$. The actual departure count $D(x_{i-1}^+, t)$ is determined as the smallest among them.

Determination of Multi-destination Flow Pattern

"Given" time-dependent origin-destination (O-D) entry flows can be stratified by destination as

$A_{ij}(t)$ = the cumulative number of vehicles which enter the freeway at station i by time t destined for an exit at $x_{j'}$ (j 〉 i) or beyond.

$A_{ij}(t)$ - $A_{ij+1}(t)$ is the number of vehicles-entering the freeway at station i by time t destined for an exit at x_j.

The follwoing analysis focuses on the determination of the cumulative counts of vehicles that have passed every junction, also stratified according to destination. For each i,

$A_j(x_i, t)$ = the cumulative number of vehicles which pass x_{i-1} by time t - $(x_i-x_{i-1})u_{i-1}$ destined for x_j 〉 x_i or beyond, so will arrive at x_i by time t if they are not delayed by congestion in section (x_{i-1}, x_i),

and

$D_j(x_i, t)$ = the cumulative number of vehicles destined for $x_{j'}$ 〉 x_i or beyond, which passed x_i by time t.

It is assumed that the collective flow of all vehicles that enter the freeway at xi with destination $x_{j'}$ 〉 x_i or beyond satisfies all the equations previously described in this manner, independent of the distribution of the vehicles. Thus, equations (11) and (12) can be rewritten as

$A_j(x_i^+, t) = A_j(x_{i-1}, t) + A_{ij}(t)$ for all j 〉 i (14)

and

$D_j(x_i^+, t) = D_j(x_i^-, t) + A_{ij}(t)$ for all j 〉 i. (15)

To make the following description easier to understand, the procedure for evaluating the arrival curve $A_j(x_i^-, t)$ for i = 1 is first described. From the given $D_j(x_0^+, t)$ = $A_{0j}(t)$ for each j ≥ 1, the arrival curve at station 1, $A_j(x_i^-, t)$ for any j ≥ 1, can be determined by the forward wave propagation rule. The computer algorithm evaluates $A_j(x_i^-, t)$ = $D_j(x_0^+, t - (x_i-x_0)u_0)$ for any j 〉 1 from equation (5) with appropriate substitutions. Then $A_j(x_i^+, t)$ for any j 〉 1 can be easily determined from $A_j(x_i^-, t)$ and $A_{1j}(t)$, according to equation (14). The same type of graphical and computer procedures can be used to construct curve $A_j(x_i^-, t)$ for any i 〉 1 from the departure curve $D_j(x_{i-1}^+, t)$ at the next upstream location x_{i-1}. Thus, the remainder of this section focuses on the determination of the departure curve at any i.

It is important to notice that $D_j(x_i^+, t)$ for j = i + 1 is the same as the aforementioned $D(x_i^+, t)$. Thus, $D_{i+1}(x_i^+, t)$ can be determined in the same manner just described. After evaluating curve $D_{i+1}(x_i^+, t)$, curve $D_{i+1}(x_i^-, t)$ can be determined by subtracting $A_{ij}(t)$ from the curve $D_{i+1}(x_i^+, t)$. The computer algorithm evaluates $D_{i+1}(x_i^-, t)$ from equation (15).

Figure 9 shows a graphical construction of curve $D_j(x_i^-, t)$ for some j 〉 i + 1. In the figure, traveller m who passed x_{i-1} at time t_l passes x_i at time t, regardless of his or her destination. The horizontal separation, $t - t_l$, between points 1 and 2 represents the travel time of traveller m from x_{i-1} to x_i, $T_i(t)$. The computer algorithm evaluates the travel time $T_i(t)$ as the time difference between time t and the time at which the departure count, $D_{i+1}(x_{i-1}^+, t)$, at x_{i-1} is equal to $D_{i+1}(x_i^-, t)$ at x_i. To identify the value of $D_{i+1}(x_{i-1}^+, t)$, the algorithm compares the departure counts $D_{i+1}(x_i^-, t)$ and $D_{i+1}(x_{i-1}^+, t - k\tau)$. The count $D_{i+1}(x_i^-, t)$ identified by point 1 in Figure 9 is perhaps equal to an intermediate value between the counts $D_{i+1}(x_{i-1}^+, t)$ at x_{i-1} at two earlier discrete times t_a and $t_a + \tau$. Here ta is the last time $t - k\tau$ when $D_{i+1}(x_{i-1}^+, t) \leq D_{i+1}(x_i^-, t)$ before time t. After identifying two counts $D_{i+1}(x_{i-1}^+, t_a)$ and $D_{i+1}(x_{i-1}^+, t_a+\tau)$, the algorithm evaluates $T_i(t)$ by the following linear interpolation formula for any $i \geq 1$,

$$Ti(t) = (t - t_a) - \tau[D_{i+1}(x_i^-, t) - D_{i+1}(x_{i-1}^+, t_a)]/[D_{i+1}(x_{i-1}^+, t_a+\tau) - D_{i+1}(x_{i-1}^+, t_a)]. \quad (16)$$

Since it has been assumed that all vehicles passing x_i at time t have the same travel time $T_i(t)$ (including delay), regardless of their destination, the travel time $T_i(t)$ plays the important role of determining the departure curves at x_i, $D_j(x_i^+, t)$ and $D_j(x_i^-, t)$ for destinations j 〉 i + 1, as follows.

In the analysis of a queueing problem behind a bottleneck on a freeway, it is convenient to assume that the queue discipline of the system is FIFO (First In First Out), which amounts to assuming that there is no passing. In Figure 9, a vehicle destined for x_j or beyond which passed x_{i-1} at time t_l is identified by point 3 on curve $D_j(x_{i-1}^+, t)$. Since it passes x_{i-1} at time t_l, this is the same vehicle as the one represented by point 1 — vehicle m. Thus, we already know that it reaches x_i at time t, hence is the vehicle at point 4. Therefore, point 4 must be on the curve $D_j(x_i^-, t)$, which is the curve we are trying to construct. Clearly, we can do so by the process just described, though interpolation is required because the count at point 3 is an intermediate value between the cumulative counts at x_{i-1}^+ at two discrete times t_a and $t_a+\tau$. The computer algorithm evaluates it by linear interpolation:

$$D_j(x_i^-, t) = D_j(x_{i-1}^+, t_l) = D_j(x_{i-1}^+, t_a) + [D_j(x_{i-1}^+, t_a+\tau) - D_j(x_{i-1}^+, t_a)]/[\Delta\tau/\tau], \quad (17)$$

where $\Delta\tau = (t - t_a) - T_i(t)$.

4.3 A System With an Exit ramp

Consider now a system which has an exit

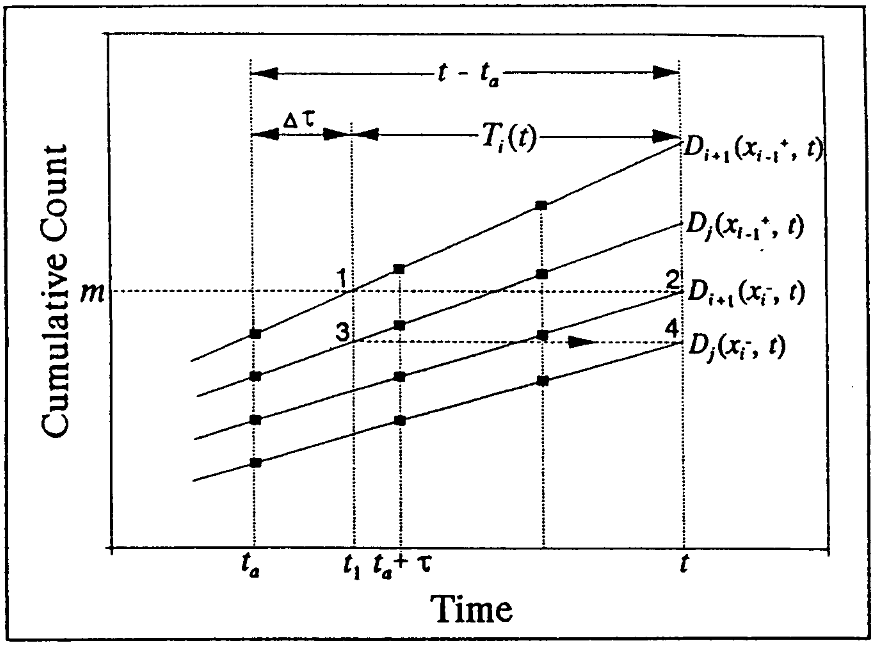

Figure 9: A graphical determination of $T_i(t)$ and $D_j(x_i^-, t)$-curve.

ramp at station i. Since no vehicle can enter the freeway at station i, $A_j(x_i^+, t)$, $A_j(x_i^-, t)$, $D_j(x_i^+, t)$, and $D_j(x_i^-, t)$ for any destination j > i, and $D_i(x_i^-, t)$ can be determined by the graphical or computer procedures described in Section 4.2. Therefore, this section treats only how to evaluate the cumulative number of exit ramp vehicles which leave the freeway at station i by time t, $E_i(t)$, which is not known. By definition, $D_i(x_i^-, t)$ represents the cumulative number of vehicles that passed x_i^- destined for x_i or beyond by time t, including $E_i(t)$. Thus, if the departure counts $D_i(x_i^-, t)$ and $D_{i+1}(x_i^-, t)$ at x_i have been evaluated, $E_i(t)$ can be determined by

$$E_i(t) = D_i(x_i^-, t) - D_{i+1}(x_i^+, t). \quad (18)$$

5. Extracting Information About Freeway Traffic Flow

5.1 The Total travel Time and Total Delay

Figure 10 shows a geometrical interpretation of the total travel time and total delay of all vehicles that travelled on section (x_{i-1}, x_i) during the time interval $(t-\tau, t)$. The vertical distance between points 2 and 6, $N_i(t) = D_i(x_{i-1}^+, t) - D_i(x_{i-1}, t)$, represents the number of vehicles in the section (x_{i-1}, x_i) at time t and the vertical distance between points 1 and 5, $N_i(t-\tau)$, equals the number of vehicles in the same section at time $t-\tau$. Area 1-2-4-6-5-3-1 is equal to the total travel time incurred by all vehicles travelling on the section during the time interval $(t-\tau,$

t). Since traffic flow passing each station within each time increment, τ, is assumed to be constant, this area is parallelogram and the computer algorithm estimates the cumulative total travel time on the section by time t, $T(x_i, t)$, as

$$T(x_i, t) = T(x_i, t-\tau) + [N_i(t) - N_i(t-\tau)][\tau/2], \quad (19)$$

where $T(x_i, t-\tau)$ is the cumulative total travel time on the same section by time $t-\tau$ and $N_i(t) = D_i(x_{i-1}^+, t) - D_i(x_i^-, t)$.

In Figure 10, the vertical distance 4-6, $Q(x_i^-, t) = A_i(x_i^-, t) - D_i(x_i^-, t)$, is the queue at time t and the vertical distance between points 3 and 5, $Q(x_i^-, t-\tau)$, is the queue at time $t-\tau$. Therefore, area 3-4-6-5-3 between the two curves is equal to the total delay experienced by all queued vehicles in the section (x_{i-1}, x_i) during the time interval $(t-\tau, t)$. The cumulative total delay occurring on this section by time t, $d_i(t)$, can be obtained by

$$d_i(t) = d_i(t-\tau) + [Q(x_i, t) + Q(x_i, t-\tau)][\tau/2]. \quad (20)$$

In the same figure, area 1-2-4-3-1 would be the total free-flow travel time of all vehicles travelling on the same section during the same time period if there were no queue at x_i. The delay calculated by the second term of the right side of equation (20) is equal to the total travel time minus this free-flow travel time, so confirms to the usual definition of delay as the difference between the actual travel time and the

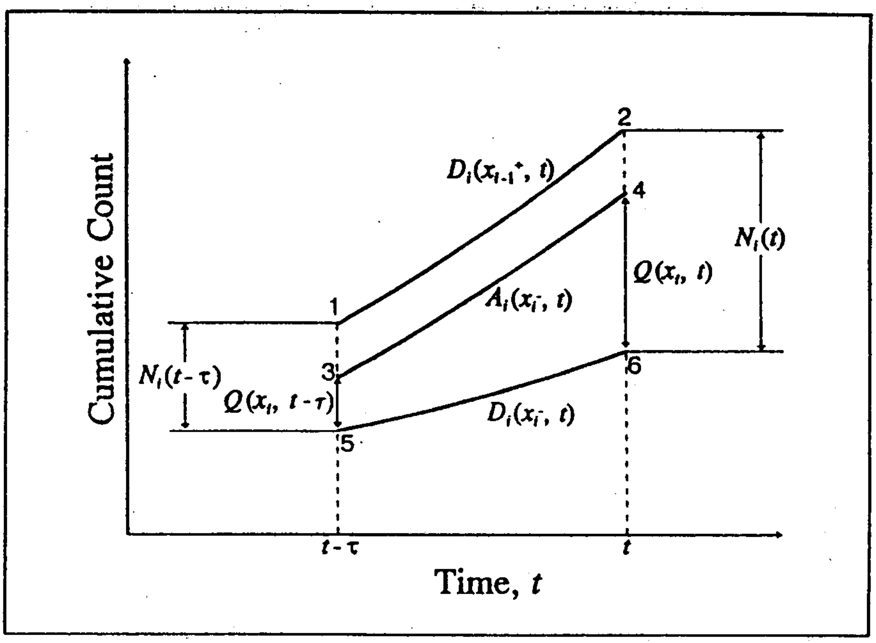

Figure 10: A geometrical determination of the total travel time and total delay.

travel time in the absence of congestion.

5.2 The Location of the Tail of a Queue

The rationale of the procedure for locating the tail of a queue is a "shock condition". Suppose that a road section between x_{i-1} and x_i contains the tail of a queue that extends upstream from x_i. In this situation, as outlined in Section 2, there are two ways to evaluate the cumulative number of vehicles, N(x, t), that can pass some point x in the section (x_{i-1}, x_i) by time t : one from the forward wave moving from x_{i-1} to x_i and the other from the backward wave moving from x_i to x_{i-1}. As described in Section 4.1.1, by conservation of the number of cars, the cumulative number of vehicles that passed any location should be the same when viewed from either side. Thus, if the tail of the queue from x_i is located exactly at point x at time t, the two values of N(x, t) obtained from the forward wave and backward should be equal. However, at the tail of the queue, the first derivatives of N(x, t) are not continuous, since the flow and density upstream of point x are different from those downstream. This discontinuity in flow and density is called a shock wave in the terminology of traffic flow theory. If the flow-density relationship is not triangular, there may be other, less dramatic shock waves as well.

Figure 11 depicts two curves of the cumulative vehicle count versus location between x_{i-1} and x_i at some time t. The labels on the upper horizontal x-axis show the locations downstream from x_{i-1} which are related to the forward wave moving from x_{i-1} to x_i. The labels on the lower horizontal x-axis show the locations upstream from x_i which are related to the backward wave moving from x_i to x_{i-1}. As an example, point $x_{i-1}+\tau/u_{i-1}$ on the upper horizontal x-axis indicates the location downstream from x_{i-1} where the forward wave leaving would reach within one time increment of length τ, and point $x_i-\tau/\omega_{i-1}$ indicates the location upstream from x_i where the backward wave leaving x_i would reach within τ time period. The heavy dashed curve EFGHIJ represents the cumulative vehicle counts at different locations between x_{i-1} and x_i which are evaluated by the rule of the backward wave propagation rule, and the solid curve ABB'C'CD represents the cumulative vehicle counts determined by the rule of the forward wave propagation. Since it has been assumed that traffic flow passing each station for each τ time period is constant, the two curves are piece-wise linear curves. However, it is important to know that the slope of the curve would not be constant between x_{i-1} and x_i, because traffic flows recorded at τ time periods vary with time. The two curves intersect at point K. Thus, the location of the tail of the queue at time t, $l_i(t)$, can be determined from data B', C', F, and G.

5.3 The Traffic Density

The average traffic density is defined as the number of vehicles occupying a unit length of roadway at an instant time and is generally expressed in vehicles per kilometer or vehicles per kilometer per lane. Thus, the average traffic density in section (x_{i-1}, x_i) at time t, k(i), can be calculated as the number of vehicles on the section at time t, $D_i(x_{i-1}^+, t)$ - $D_i(x_i^-, t)$, divided by the section length, $x_i^- x_{i-1}$, and the number of lanes on the section, L_i:

$$k(i) = [D_i(x_{i-1}^+,t) - D_i(x_i^-,t)]/[(x_i - x_{i-1})L_i]. \quad (21)$$

However, if the tail of a queue is located within the section (x_{i-1}, x_i), this average density is not what one would find anywhere on the

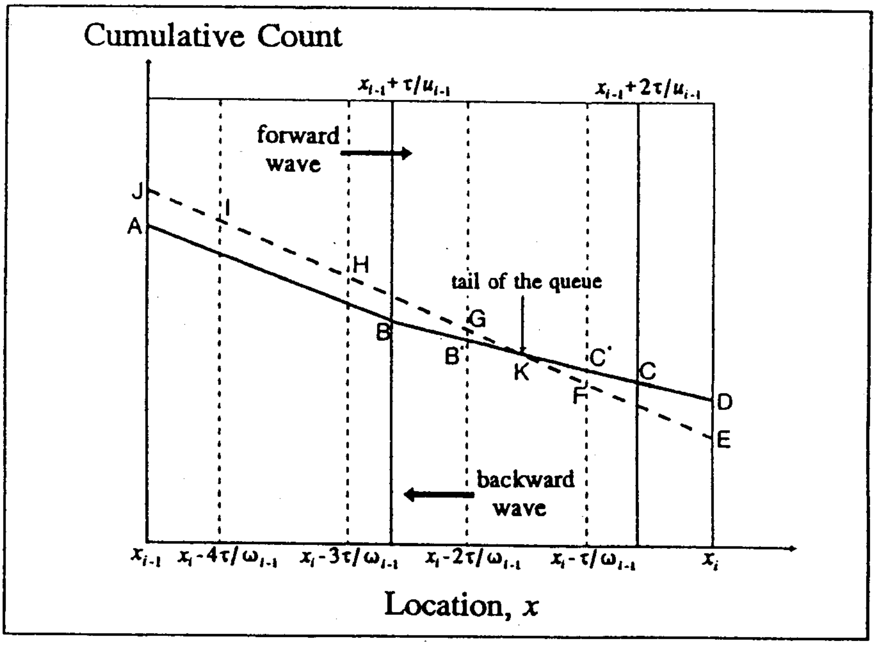

Figure 11: The location of the tail of a queue.

section, so the section will be divided into two segments: the uncongested segment (x_{i-1}, $l_i(t)$) and the congested segment ($l_i(t)$, x_i). The traffic densities in the two segments can be computed in the same manner as equation (21) using the counts $D_i(x_{i-1}^+, t)$, $D_i(l_i(t), t)$, and $D_i(x_i^-, t)$.

6. Summary

In order to assess the effects of physical (geometric) improvements to a freeway system and to devise efficient traffic control strategies, the magnitude of the resulting change in traffic measurements (e.g., the total travel time, the total delay) on the system must be estimated. The great appeal of reliable freeway traffic flow models is that they would enable analysts to accomplish this task.

A computer algorithm which can produce numerical results for traffic flow conditions in reasonable computation times was developed in this paper. The computer algorithm mechanizes the graphical procedures of Newell's simplified kinematic wave theory and then facilitates the application of his theory for freeway traffic operation/control purposes. The algorithm includes the procedures for extracting information about freeway traffic flow such as the total travel time, total delay, traffic density, and the location of the tail of a queue. The numerical test of the accuracy of the computer algorithm has been performed through three numerical examples (Son, 1996).

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