SER Analysis of QAM with Space Diversity in Rayleigh Fading Channels

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CONTENTS

ABSTRACT

This paper derives the symbol error probability for quadrature amplitude modulation(QAM) with L-fold space diversity in Rayleigh fading channels. Two combining techniques, maximal ratio combining(MRC) and selection combining(SC), are considered. The formula for MRC space diversity is obtained by averaging the symbol error probability of M-ary QAM in an additive white Gaussian noise(AWGN) channel over a chi-square distribution with $2L$ degrees of freedom. The obtained formula overcomes the limitations of the earlier work, which has been limited only to deriving the symbol error rate(SER) of QAM with two branch MRC space diversity. The formula for SC space diversity is obtained by averaging the symbol error probability of M-ary QAM in an AWGN channel over the distribution of the maximum signal-to-noise ratio among all of the diversity channels for SC space diversity. No analysis for QAM with SC space diversity has been reported yet. Analytical results show that the probability of error decreases with the order of diversity. We can also see that the incremental diversity gain per additional branch decreases as the number of branches becomes larger. On the other hand, the performance of 16 QAM with MRC becomes much better than that of SC as the number of branches becomes larger. By giving the order of diversity, L, and the number of signal points, M, we have been able to obtain the SER performance of QAM with general space diversity. These results can be used to determine the order of diversity to achieve the desired SER in land mobile communication system employing QAM modulation.

I. INTRODUCTION

Digital cellular systems have been widely developed to provide mobile communication service. In mobile radio systems, the propagation medium contains several distinguishable paths connecting the transmitter to the receiver. Several signals with different amplitudes, phases and delays corresponding to different transmission paths arrive at a receiver. The different signal components at the receiver add constructively or destructively to give the resultant signal. This multipath propagation phenomenon results in signal fading at the receiver [1], [2]. This nonlinearity of the mobile channel is one of the factors that has led to the use of constant envelope modulation [3] or phase shift keying(PSK) type modulation [4], [5] to the mobile communication.

With the increasing demands of the service, an important topic is to use a spectrally efficient modulation technique to raise the spectrum efficiency in the limited frequency bandwidth. Quadrature amplitude modulation (QAM) [6] is an effective technique to achieve high spectral efficiency in additive white Gaussian noise(AWGN) channels. Also, it is a good candidate for high spectral efficiency in Rayleigh fading channels when the receiver is capable of estimating the channel state information(CSI).

There are two types of channel sounding techniques which transmit some information to the receiver concerning the instantaneous channel state of the mobile radio channel: 1) pilot tone assisted modulation(PTAM), and 2) pilot symbol assisted modulation(PSAM). Among several PTAM methods, transparent tone in band(TTIB) [7] has merits in spectral efficiency and coherency between the true fading value and the estimated fading value. In TTIB, a notch is created in the center of the signal spectrum by the baseband signal processing technique and a pilot tone is inserted in this notch after modulation. At the receiver a pilot tone is extracted to estimate the CSI. But TTIB needs complex signal processing and it increases the peak factor of the modulated signal. On the other hand, PSAM [8]-[12] inserts a known symbol periodically into the data stream. Since the receiver has information as to what this symbol is and when it appears, it can use this information to estimate the CSI. To apply the PSAM sounding technique to the communication, the interpolation technique and the period of pilot insertion have to be determined.

To achieve the desired transmission quality in mobile radio communications, the system performance has to be improved beyond which obtained by channel sounding techniques. Space diversity is a well-known technique to combat multipath fading in mobile radio communications. Diversity gain is achieved without increase in the transmission power but at the expense of increased system complexity. The most prevalent space diversity techniques are maximal ratio combining(MRC), equal gain combining(EGC), and selection combining(SC). In MRC [13],

[14], the signals received from the different branches are weighted in proportion to their signal to noise power ratios, cophased and then summed. In EGC[15], the outputs of the different branches are co-phased, equally weighted and then summed. In SC [16], [17], only one signal which has the highest instantaneous signal to noise ratio(SNR) at a time coming from two or more spatially separated antennas is coupled to a detector. The main advantage of SC consists of its relative simplicity and lower cost, since it requires the use of only one receiver, regardless of the number of antennas employed. On the other hand, MRC requires the use of the same number of receivers as antennas and are more complex. In SC, this scheme is to select signal with the largest instantaneous power from the L branch signals coming from the different antennas.

In order to achieve high spectral efficiency and improve the system performance in the mobile communication system, Sampei *et al*. [8]-[10] have introduced M-ary QAM with two-branch MRC space diversity and they have employed the PSAM channel sounding technique to obtain the CSI. As a result of computer simulation and laboratory experiments, they have obtained a desirable symbol error rate(SER) performance. However, to obtain the theoretical results, they have employed numerical integration [8], [9] or simplified approximation [10]. In [10], the second term on the right hand side of (1) has been ignored. Furthermore, their analyses have been limited only to two-branch space diversity. For improved performance, it is important to extend the order of diversity to the general case. On the other hand, no analysis has been reported yet for SER of QAM with SC space diversity in Rayleigh fading channels.

In this paper, we derive the symbol error probability for QAM with L-fold MRC or SC space diversity in a Rayleigh fading channel. The derived formula is obtained by averaging the symbol error probability of QAM in AWGN channel over a chi-square distribution with $2L$ degrees of freedom for MRC space diversity. The expression of the symbol error probability for QAM with general SC space diversity in a Rayleigh fading channel is obtained by averaging the symbol error probability of QAM in AWGN channel over the distribution of the maximum signal-to-noise ratio among all of the diversity channels.

II. MAXIMAL RATIO COMBINING

Let us consider only the rectangular signal sets. For such signal structure, the $M = 2^k$ signal points result in a symmetrical form of QAM when k is even. In this case, QAM can be viewed as two separate pulse amplitude modulation signals impressed on phase-quadrature carriers. The probability of symbol error for M-ary QAM signals in the AWGN channel can be expressed as in [18].

$$p_{non} = 2\left(1-\frac{1}{\sqrt{M}}\right)\, erfc\left\{\sqrt{\frac{3k}{2(M-1)}\frac{E_b}{N_o}}\right\}$$

$$\times\left[1-\frac{1}{2}\left(1-\frac{1}{\sqrt{M}}\right)\times erfc\left\{\sqrt{\frac{3k}{2(M-1)}\frac{E_b}{N_o}}\right\}\right], \qquad (1)$$

where

$$erfc(x)=\frac{2}{\sqrt{\pi}}\int_x^{\infty}\exp(-t^2)dt.$$

The frequency nonselective channel results in multiplicative distortion of the transmitted signal. The condition that the channel fades slowly implies that the multiplicative process may be regarded as a constant during at least one signaling interval [19]. For a fixed attenuation α, (1) can be represented as

$$p_{non}(\gamma_b)=2\left(1-\frac{1}{\sqrt{M}}\right)erfc\left\{\sqrt{\frac{3k}{2(M-1)}\gamma_b}\right\}\times\left[1-\frac{1}{2}\left(1-\frac{1}{\sqrt{M}}\right)\times erfc\left\{\sqrt{\frac{3k}{2(M-1)}\gamma_b}\right\}\right], \qquad (2)$$

where

$$\gamma_b=\alpha^2\frac{E_b}{N_0}$$

We view (2) as a conditional symbol error probability, where the condition is that α is fixed. To obtain the error probability when α is random, we average p_{non} over the probability density function (pdf) of γ_b. That is,

$$p_{fad}=\int_0^{\infty}p_{non}(\gamma_b)p(\gamma_b)d\gamma_b, \qquad (3)$$

where $p(\gamma_b)$ is the probability density function of γ_b when α is random.

It is assumed that the received signal has a Rayleigh fading envelope. This is a reasonable assumption for the case in which the transmitted signal is reflected by a multitude of scatters surrounding the received antennas. It is also assumed that the signals coming from the L different antennas are statistically independent and identically distributed (iid) random processes. This assumption will be true if the spatial separation between any two antennas is greater than half a wavelength of the carrier signal. We also assume that the average signal to noise ratio per channel is assumed to be identical for all channels. Under these conditions, the pdf, $p(\gamma_b)$, is given by [18]

$$p(\gamma_b)=\frac{1}{\Gamma(L)\bar{\gamma}_b}\gamma_b^{L-1}e^{-\gamma_b/\bar{\gamma}_b}, \qquad (4)$$

where $\bar{\gamma}_b$ is the average signal to noise ratio per channel and $\Gamma(.)$ denotes the gamma function.

Let us transform (2) into another form as follows:

$$p_{non}=\frac{\sqrt{M}-1}{M}\left\{(\sqrt{M}-1)+4\frac{1}{2}erfc(\sqrt{\beta\gamma_b})-(\sqrt{M}-1)erf^2(\sqrt{\beta\gamma_b})\right\}, \qquad (5)$$

where

$$\beta=\frac{3k}{2(M-1)}.$$

Then, (3) becomes

$$p_{fad}=\frac{\sqrt{M}-1}{M}\{(\sqrt{M}-1)+4I_1-(\sqrt{M}-1)I_2\}, \qquad (6)$$

where

$$I_1=\int_0^{\infty}\frac{1}{2}erfc(\sqrt{\beta\gamma_b})p(\gamma_b)d\gamma_b \qquad (7)$$

and

$$I_2=\int_0^{\infty}erf^2(\sqrt{\beta\gamma_b})p(\gamma_b)d\gamma_b. \qquad (8)$$

To evaluate I_1, we make use of the result in [18], where a closed form solution for an I_1

type integral has been given. With some modification, I_1 is expressed as

$$I_1 = \left(\frac{1-\mu}{2}\right)^L \sum_{m=0}^{L-1} \binom{L-1+m}{m} \left(\frac{1+\mu}{2}\right)^m, \tag{9}$$

where

$$\mu = \sqrt{\frac{\beta\bar{\gamma}_b}{1+\beta\bar{\gamma}_b}}.$$

Finally, to solve I_2, we apply the integration by parts to (8). In the Appendix, I_2 is derived as

$$I_2 = \begin{cases} \dfrac{4}{\pi}\sqrt{\dfrac{\beta\bar{\gamma}_b}{1+\beta\bar{\gamma}_b}} \tan^{-1}\sqrt{\dfrac{\beta\bar{\gamma}_b}{1+\beta\bar{\gamma}_b}} & for\ L=1 \\[2ex] \dfrac{4}{\pi}\displaystyle\sum_{n=0}^{L-1} \frac{(2n)!}{2^{2n}(n!)^2} \left\{ \left(\frac{1}{1+\beta\bar{\gamma}_b}\right)^n \times \sqrt{\frac{\beta\bar{\gamma}_b}{1+\beta\bar{\gamma}_b}} \tan^{-1}\sqrt{\frac{\beta\bar{\gamma}_b}{1+\beta\bar{\gamma}_b}} \right\} & \\ +\dfrac{2}{\pi}\displaystyle\sum_{n=1}^{L-1} \frac{(2n)!}{2^{2n}(n!)^2} & for\ L \geq 2. \\ \times \left\{ \displaystyle\sum_{m=1}^{n} \frac{2^{2m} m!}{(2m)!} \left(\frac{1}{1+\beta\bar{\gamma}_b}\right)^{n-m+1} \times \left(\frac{\beta\bar{\gamma}_b}{1+2\beta\bar{\gamma}_b}\right) \cdot (m-1)! \times \left(\frac{1}{1+2\beta\bar{\gamma}_b}\right)^{m-1} \right\} & \end{cases} \tag{10}$$

III. SELECTION COMBINING

In this section, we evaluate the symbol error probability of QAM with SC diversity in a traditional way by averaging the result for a time invariant channel over the distribution of the maximum SNR among the diversity receptions. The probability density function of the maximum selection SNR over L iid diversity paths is well-known. Let z be the maximum SNR. The mean of each branch signal, $\bar{\gamma}_b$, is also assumed to be equal. Under these conditions, the pdf is given by [16]

$$\begin{aligned} p_z(z) &= \frac{L}{\bar{\gamma}_b}(1-e^{-z/\bar{\gamma}_b})^{L-1} e^{-z/\bar{\gamma}_b} \\ &= \frac{L}{\bar{\gamma}_b} \sum_{k=0}^{L-1} (-1)^k \binom{L-1}{k} e^{-(k+1)z/\bar{\gamma}_b}. \end{aligned} \tag{11}$$

Let us transform (2) into another form as follows:

$$\begin{aligned} p_{non} = \frac{\sqrt{M}-1}{M} \times \{ &(\sqrt{M}+1) - 2\,erf(\sqrt{\beta\gamma_b}) \\ &-(\sqrt{M}-1)erf^2(\sqrt{\beta\gamma_b})\}. \end{aligned} \tag{12}$$

Then, (3) becomes

$$p_{fad} = \frac{\sqrt{M}-1}{M} \cdot \left\{ (\sqrt{M}+1) - 2I_3 - (\sqrt{M}-1)I_4 \right\}, \tag{13}$$

where

$$\begin{aligned} I_3 &= \frac{L}{\bar{\gamma}_b} \sum_{k=0}^{L-1} (-1)^k \binom{L-1}{k} \\ &\quad \times \int_0^\infty erf(\sqrt{\beta\gamma_b}) e^{-(k+1)\gamma_b/\bar{\gamma}_b} d\gamma_b \\ &= L \sum_{k=0}^{L-1} (-1)^k \frac{1}{k+1} \binom{L-1}{k} \\ &\quad \times \sqrt{\frac{3k\bar{\gamma}_b}{2(M-1)(k+1)+3k\bar{\gamma}_b}} \end{aligned} \tag{14}$$

and

$$\begin{aligned} I_4 &= \frac{L}{\bar{\gamma}_b} \sum_{k=0}^{L-1} (-1)^k \binom{L-1}{k} \\ &\quad \times \int_0^\infty erf^2(\sqrt{\beta\gamma_b}) e^{-(k+1)\gamma_b/\bar{\gamma}_b} d\gamma_b \\ &= \frac{4L}{\pi} \sum_{k=0}^{L-1} (-1)^k \frac{1}{k+1} \binom{L-1}{k} \end{aligned}$$

$$\times \frac{1}{\sqrt{1+\frac{\bar{\gamma}_b}{(k+1)\beta}}} \tan^{-1}\left(\frac{1}{\sqrt{1+\frac{\bar{\gamma}_b}{(k+1)\beta}}}\right). \tag{15}$$

IV. RESULTS AND DISCUSSION

The SER performance of QAM with MRC space diversity is obtained from (6). Fig. 1 shows the SER performance of 16 QAM with MRC space diversity in Rayleigh fading environments for $L = 1 \sim 8$.

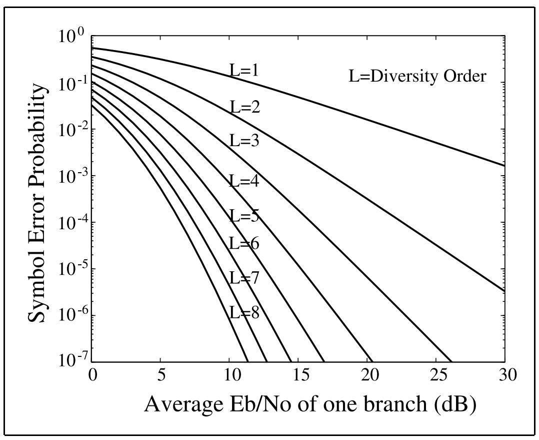

Fig. 1. Performance of 16QAM with MRC space diversity in Rayleigh fading channels.

Fig. 2 shows the SER performance of 64 QAM with MRC space diversity in Rayleigh fading environments for $L = 1 \sim 8$. From figures 1 and 2, we can see that the probability of error decreases with the order of diversity. The results illustrate the advantage of diversity as a means for combating the fading phenomena. We can also see that the incremental diversity gain per additional branch decreases as the number of branches becomes larger.

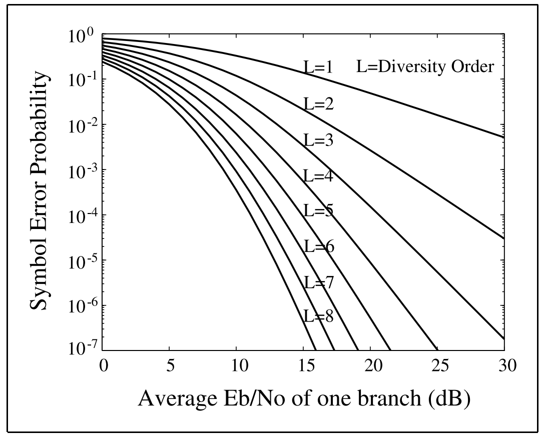

Fig. 2. Performance of 64QAM with MRC space diversity in Rayleigh fading channels.

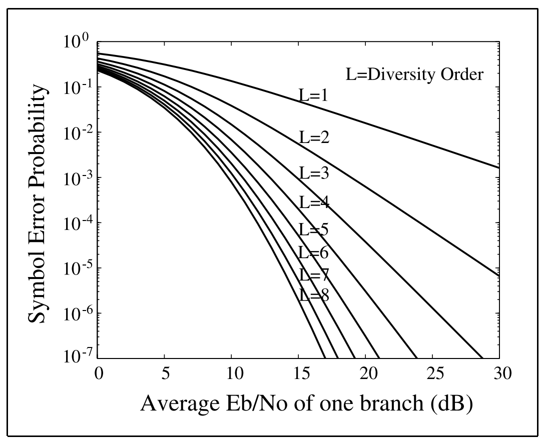

Fig. 3. Performance of 16QAM with SC space diversity in Rayleigh fading channels.

The SER performance of QAM with SC space diversity is obtained from (13). Fig. 3 shows the SER performance of 16 QAM with SC space diversity in Rayleigh fading environments for $L = 1 \sim 8$. Fig. 4 shows the SER per-

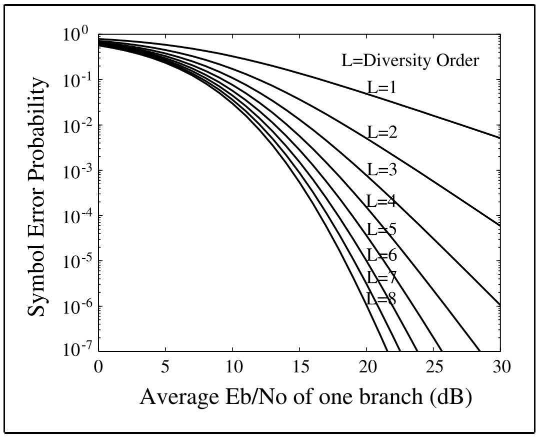

Fig. 4. Performance of 64QAM with SC space diversity in Rayleigh fading channels.

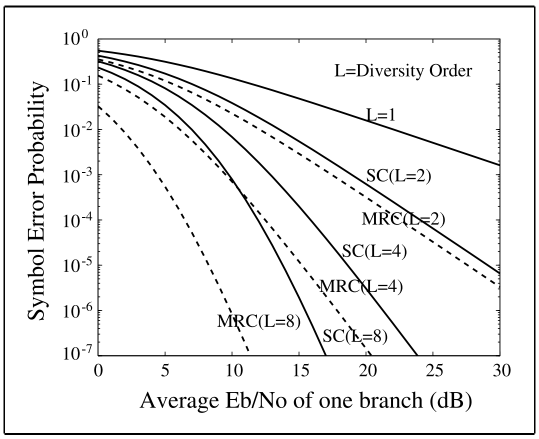

Fig. 5. Comparison of 16QAM with SC vs 16QAM with MRC in Rayleigh fading channels.

formance of 64 QAM with SC space diversity in Rayleigh fading environments for $L = 1 \sim 8$. From figures 3 and 4, we can see the similar phenomena as MRC space diversity. Figure 5 shows the performance comparison between 16 QAM with MRC and 16 QAM with SC as a function of E_b/N_o as the number of branches becomes larger for $L = 1,\ 2,\ 4$ and 8. The difference between the SER of 16 QAM with MRC and 16 QAM with SC increases with the number of diversity branch. This is the expected result because the former makes use of all branch signals simultaneously while the latter uses only one branch at a time.

V. CONCLUSIONS

In this paper, we derive closed form expressions for the symbol error probability of QAM with general MRC or SC space diversity in Rayleigh fading channels. By choosing the order of diversity and the number of signal points from the derived formula, we can obtain the SER performance of the QAM with MRC or SC diversity in Rayleigh fading environments. Analytical results show that the probability of error decreases with the order of diversity. The results indicate the advantage of diversity as a means for combating the fading phenomena. We can also see that the incremental diversity gain per additional branch decreases as the number of branches becomes larger. On the other hand, the difference between the SER of 16 QAM with MRC and 16 QAM with SC increases with the number of diversity branch. We can obtain the SER performance of quadrature phase shift keying with MRC or SC diversity from (6) or (13) by setting $M = 4$, respectively. These results can be used to determine the order of diversity to achieve the desired SER in land mobile communication systems employing QAM modulation.

APPENDIX

To derive (10), we apply integration by parts to (8). First consider the case $L=1$,

$$I_{21}=\int_0^\infty erf^2(\sqrt{\beta\gamma_b})\frac{1}{\bar{\gamma}_b}e^{-\gamma_b/\bar{\gamma}_b}d\gamma_b, \tag{A1}$$

where the notation I_{2L} means that L is the number of diversity stage. (A1) is obtained from [20].

$$I_{21}=\frac{4}{\pi}\sqrt{\frac{\beta\bar{\gamma}_b}{1+\beta\bar{\gamma}_b}}\tan^{-1}\sqrt{\frac{\beta\bar{\gamma}_b}{1+\beta\bar{\gamma}_b}}. \tag{A2}$$

We can obtain I_{22} as

$$\begin{aligned}I_{22}&=\int_0^\infty erf^2(\sqrt{\beta\gamma_b})\frac{\gamma_b}{1!\bar{\gamma}_b^2}\exp\left(-\frac{\gamma_b}{\bar{\gamma}_b}\right)d\gamma_b\\&=I_{21}+2\sqrt{\frac{\beta}{\pi}}\frac{1}{\bar{\gamma}_b}\int_0^\infty erf(\sqrt{\beta\gamma_b})\\&\qquad\times e^{-(\beta+1/\bar{\gamma}_b)\gamma_b}\gamma_b^{1/2}d\gamma_b.\end{aligned} \tag{A3}$$

Let $\gamma_b=\lambda^2$. Then,

$$\begin{aligned}I_{22}&=I_{21}+2\sqrt{\frac{\beta}{\pi}}\frac{1}{\bar{\gamma}_b}\int_0^\infty erf(\sqrt{\beta}\lambda)\\&\qquad\times e^{-(\beta+1/\bar{\gamma}_b)\lambda^2}2\lambda^2d\gamma_b\\&=\frac{4}{\pi}\left(1+\frac{1}{1+\beta\bar{\gamma}_b}\right)\sqrt{\frac{\beta\bar{\gamma}_b}{1+\beta\bar{\gamma}_b}}\\&\qquad\times\tan^{-1}\sqrt{\frac{\beta\bar{\gamma}_b}{1+\beta\bar{\gamma}_b}}\\&\quad+\frac{2}{\pi}\frac{1}{1+\beta\bar{\gamma}_b}\frac{\beta\bar{\gamma}_b}{1+2\beta\bar{\gamma}_b}.\end{aligned} \tag{A4}$$

Likewise, we can find I_{23} and I_{24} as

$$\begin{aligned}I_{23}=&\frac{4}{\pi}\left\{1+\left(\frac{1}{2}\right)\frac{1}{1+\beta\bar{\gamma}_b}\right.\\&\left.+\frac{1}{2!}\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{1+\beta\bar{\gamma}_b}\right)^2\right\}\\&\times\sqrt{\frac{\beta\bar{\gamma}_b}{1+\beta\bar{\gamma}_b}}\tan^{-1}\sqrt{\frac{\beta\bar{\gamma}_b}{1+\beta\bar{\gamma}_b}}\\&+\frac{2}{\pi}\left\{\frac{1}{1!}\left(\frac{1}{2}\right)\left(\frac{2}{1}\right)\frac{1}{1+\beta\bar{\gamma}_b}\frac{\beta\bar{\gamma}_b}{1+2\beta\bar{\gamma}_b}\right\}\\&+\frac{2}{\pi}\left\{\begin{array}{l}\frac{1}{2!}\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)\left(\frac{2}{1}\right)\left(\frac{1}{1+\beta\bar{\gamma}_b}\right)^2\\\quad\times\frac{\beta\bar{\gamma}_b}{1+2\beta\bar{\gamma}_b}\\+\frac{1}{2!}\left(\frac{1}{2}\right)\left(\frac{2}{1}\right)\\\quad\times\left(\frac{1}{1+\beta\bar{\gamma}_b}\right)^1\frac{\beta\bar{\gamma}_b}{1+2\beta\bar{\gamma}_b}\frac{1}{1+2\beta\bar{\gamma}_b}\end{array}\right\}\end{aligned} \tag{A5}$$

and

$$\begin{aligned}I_{24}=&\frac{4}{\pi}\left\{1+\left(\frac{1}{2}\right)\frac{1}{1+\beta\bar{\gamma}_b}\right.\\&+\frac{1}{2!}\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{1+\beta\bar{\gamma}_b}\right)^2\\&\left.+\frac{1}{3!}\left(\frac{5}{2}\right)\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{1+\beta\bar{\gamma}_b}\right)^3\right\}\\&\cdot\sqrt{\frac{\beta\bar{\gamma}_b}{1+\beta\bar{\gamma}_b}}\tan^{-1}\sqrt{\frac{\beta\bar{\gamma}_b}{1+\beta\bar{\gamma}_b}}\\&+\frac{2}{\pi}\left\{\frac{1}{1!}\left(\frac{1}{2}\right)\left(\frac{2}{1}\right)\frac{1}{1+\beta\bar{\gamma}_b}\frac{\beta\bar{\gamma}_b}{1+2\beta\bar{\gamma}_b}\right\}\\&+\frac{2}{\pi}\left\{\frac{1}{2!}\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)\left(\frac{2}{1}\right)\left(\frac{1}{1+\beta\bar{\gamma}_b}\right)^2\right.\\&\times\frac{\beta\bar{\gamma}_b}{1+2\beta\bar{\gamma}_b}+\frac{1}{2!}\left(\frac{1}{2}\right)\left(\frac{2}{1}\right)\left(\frac{1}{1+\beta\bar{\gamma}_b}\right)^1\\&\left.\times\frac{\beta\bar{\gamma}_b}{1+2\beta\bar{\gamma}_b}\frac{1}{1+2\beta\bar{\gamma}_b}\right\}\end{aligned}$$

$$
+\frac{2}{\pi}\left\{\begin{array}{l}
\frac{1}{3!}\left(\frac{5}{2}\right)\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)\left(\frac{2}{1}\right)\left(\frac{1}{1+\beta\bar{\gamma}_b}\right)^3 \\
\quad\times\frac{\beta\bar{\gamma}_b}{1+2\beta\bar{\gamma}_b} \\
+\frac{1}{3!}\left(\frac{5}{2}\right)\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)\left(\frac{2}{1}\right)\left(\frac{2}{3}\right) \\
\quad\times\left(\frac{1}{1+\beta\bar{\gamma}_b}\right)^2\frac{\beta\bar{\gamma}_b}{1+2\beta\bar{\gamma}_b}\frac{1}{1+2\beta\bar{\gamma}_b} \\
+\frac{1}{3!}\left(\frac{5}{2}\right)\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)\left(\frac{2}{1}\right)\left(\frac{2}{3}\right)\left(\frac{2}{5}\right) \\
\quad\times\left(\frac{1}{1+\beta\bar{\gamma}_b}\right)^1\frac{\beta\bar{\gamma}_b}{1+2\beta\bar{\gamma}_b} \\
\quad\times 2!\left(\frac{1}{1+2\beta\bar{\gamma}_b}\right)^2
\end{array}\right\} \tag{A6}
$$

By repeating the similar operations, we can find the general equation as

$$
I_2=\left\{\begin{array}{ll}
\frac{4}{\pi}\sqrt{\frac{\beta\bar{\gamma}_b}{1+\beta\bar{\gamma}_b}}\tan^{-1}\sqrt{\frac{\beta\bar{\gamma}_b}{1+\beta\bar{\gamma}_b}} & for\ L=1 \\
\frac{4}{\pi}\sum_{n=0}^{L-1}\frac{(2n)!}{2^{2n}(n!)^2}\left\{\left(\frac{1}{1+\beta\bar{\gamma}_b}\right)^n\right. & \\
\quad\left.\times\sqrt{\frac{\beta\bar{\gamma}_b}{1+\beta\bar{\gamma}_b}}\tan^{-1}\sqrt{\frac{\beta\bar{\gamma}_b}{1+\beta\bar{\gamma}_b}}\right\} & \\
+\frac{2}{\pi}\sum_{n=1}^{L-1}\frac{(2n)!}{2^{2n}(n!)^2} & for\ L\geq 2. \\
\times\left\{\begin{array}{l}
\sum_{m=1}^{n}\frac{2^{2m}m!}{(2m)!}\left(\frac{1}{1+\beta\bar{\gamma}_b}\right)^{n-m+1} \\
\times\left(\frac{\beta\bar{\gamma}_b}{1+2\beta\bar{\gamma}_b}\right)\cdot(m-1)! \\
\times\left(\frac{1}{1+2\beta\bar{\gamma}_b}\right)^{m-1}
\end{array}\right\} &
\end{array}\right. \tag{A7}
$$

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