The Derivation of a New Blind Equalization Algorithm

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CONTENTS

ABSTRACT

Blind equalization is a technique for adaptive equalization of a communication channel without the aid of training sequences. This paper proposes a new blind equalization algorithm. The advantage of the new algorithm is that it has the lower residual error than the GA (proposed by Godard) and Sign_GA (proposed by Weerackody *et al.*). The superior performance of the proposed algorithm is illustrated for the 16-QAM signal constellation. A Rummler channel model is assumed as a transmission medium. The performance of the proposed algorithm is compared to the GA, Sign_GA and Stop & Go Algorithm (SGA). The simulation results demonstrate that an improvement in performance is achieved with the proposed equalization algorithm.

I. INTRODUCTION

There has been an increasing interest in the blind equalization problem in recent years. The main feature of blind equalizer is that it does not require training sequences to start-up and retrain an adaptive equalizer.

There are several applications in digital communications using blind equalizer. For instance, considering a multipoint network, a blind retrain is needed if a channel from the master to one of the tributary stations goes down at any time following the initial training period, and it is desired to retrain only the corresponding tributary.

Apart from the multipoint system, blind equalization is often required due to severe fading in digital microwave links where a reverse channel is not available and in transmission monitoring, where a training sequence is not supplied for the benefit of the monitoring receiver [1].

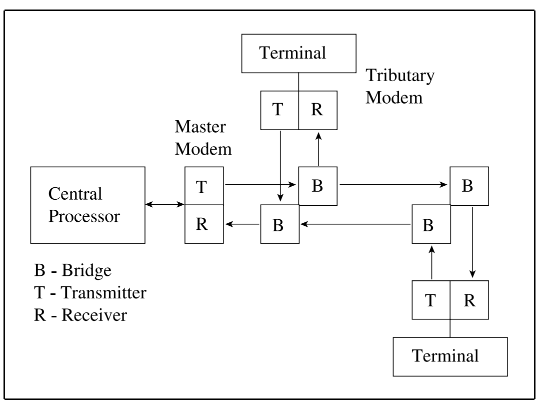

Fig. 1. Multipoint network.

The first known blind equalization algorithm is the decision directed (DD) algorithm which is usually run in a tracking mode when an initial convergence has already been obtained during a training period. In 1975, Sato proposed a simple blind equalization algorithm that was generalized afterwards to the complex signal case by Benveniste and Goursat. The Sato algorithm has a small computational load. But, its convergence property was relatively poor [2], [3].

In 1980, Godard proposed a new blind equalization algorithm using a newly defined cost function. This algorithm has a large amount of residual error after convergence. Therefore this algorithm needs "gear shifting[1]" to reduce the residual error, and then to switchover to the decision directed algorithm (DDA) after convergence [4].

In 1987, Prati & Picchi proposed a new algorithm which has a good convergence performance with a relatively small amount of residual error after convergence. This algorithm uses a "stop-and-go" adaptation rule. Stop and go algorithm (SGA) updates the filter taps using DD error[2] if both the DD error and Sato-like error[3] have the same sign, otherwise stop updating [5].

In 1992, the sign godard algorithm (Sign_GA), a signed version of Godard Algorithm, was proposed by Weerakody, Kassam

[1] A method for reducing the convergence parameter gradually

[2] DD error = equalizer output - estimated symbol

[3] Sato-like error = equalizer output-$\gamma *$sign(estimated symbol)

and Laker. The advantage of Sign_GA is the absence of multiplications in its tap update process and has a relatively good initial convergence property. But Sign_GA has a large amount of residual error after convergence as does the GA has [6]. In this paper, we present a new adaptive algorithm for blind equalization that has better performance characteristics than conventional algorithms based upon a novel combination of the Sign_GA and the modified stop-and-go adaptation rule.

The paper is organized as follows, In Section II, the GA and Sign_GA are presented, and the proposed algorithm is derivated in section III. In Section IV, simulation results are presented, and the concluding remarks are contained in Section V.

II. THE CONVENTIONAL BLIND EQUALIZATION ALGORITHMS

1. Godard Algorithm [4]

The baseband model of a digital communication channel is considerd to be characterized by a finite impusle response filter and an additive white noise source.

The equalizer has a FIR structure defined as

$$z_n = \sum_{i=0}^{m-1} c_i \cdot x_{n-i}, \tag{1}$$

where m is the order of the equalizer, and are the complex equalizer weights.

Godard proposed a whole new class of blind equalization algorithms for two dimensional modulation schemes that minimize a non-convex cost function independent of carrier phase and signal constellation.

A new non-convex cost function is given by

$$E(|e_n|^2) = E(\||z_n|^2 - R_G|^2) \tag{2}$$
$$R_G = \frac{E(|a_n|^4)}{E(|a_n|^2)},$$

where z_n is equalizer output, a_n is input data to the channel, and $E(\cdot)$ denotes expected value. The minimization of the mean squared error leads to the following stochastic algorithm

$$c_{n+1} = c_n + \alpha z_n (|z_n|^2 - R_G) x_n^*, \tag{3}$$

where α is a small step size parameter, x_n is the vector of input values to the equalizer at time instant n, z_n is the equalizer output, c_n is the vector of the complex equalizer weights, and * denotes the complex conjugation.

The zero error contour of (2) is depicted in Fig. 2. The Godard's cost function attempts to drive the equalizer output to lie on a circle of radius $\sqrt{R_G}$.

2. Sign Godard Algorithm (Sign_GA) [6]

The cost function of Sign_GA is given by

$$E(|e_n^s|) = E(|z_{r,n}| + |z_{i,n}| - R_s), \tag{4}$$

where R_s is constant, $z_{r,n}$ is the real part of equalizer output, and $z_{i,n}$ is the imaginary part of equalizer output [6].

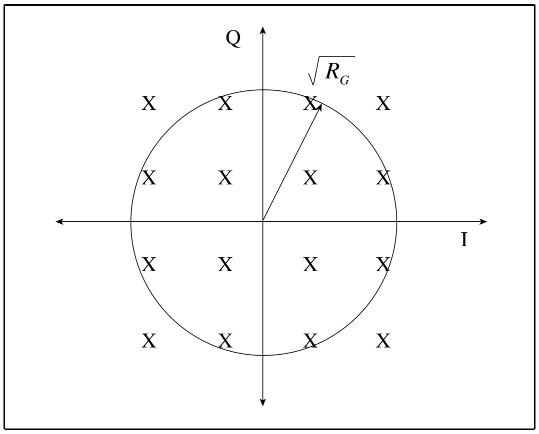

Fig. 2. The zero error contour of Godard Algorithm.

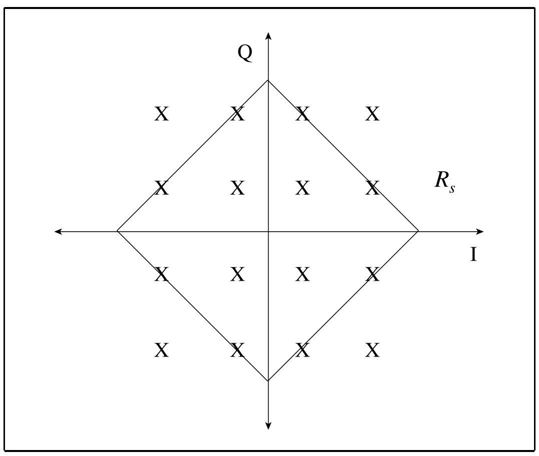

Fig. 3. The zero error contour of Sign Godard Algorithm.

The zero error contour of (4) is depicted in Fig. 3. From Fig. 3 the cost function (4) attempts to force the equalizer output to lie on a square rotated by 45°. It is shown in [6] that for an infinite-length equalizer and a rectangular QAM data set, the cost function (4) has local minima corresponding $z(n) = a(n)$ and $z(n) = a(n)e^{j\pi/4}$. It is also shown that the stationary points corresponding to the 45 degree rotation of the data constellation are the global minima of the cost function. The minimization of the cost function (4) leads to the following stocahstic gradient algorithm

$$c_{n+1} = c_n - \alpha \cdot \mathrm{sgn}(|z_{r,n}| + |z_{i,n}| - R_s) \cdot [\mathrm{sgn}(z_{r,n}) + j\mathrm{sgn}(z_{i,n})] \cdot x_n^*, \qquad (5)$$

where sgn(·) denotes sign function.

In [6], Weeracody shows that GA and Sign_GA converge slowly and approximately at a similar rate.

III. A NEW ALGORITHM FOR BLIND EQUALIZTION

Let us consider the filter tap coefficeint updating equation (5) used in the Sign_GA to derive it for the proposed algorithm.

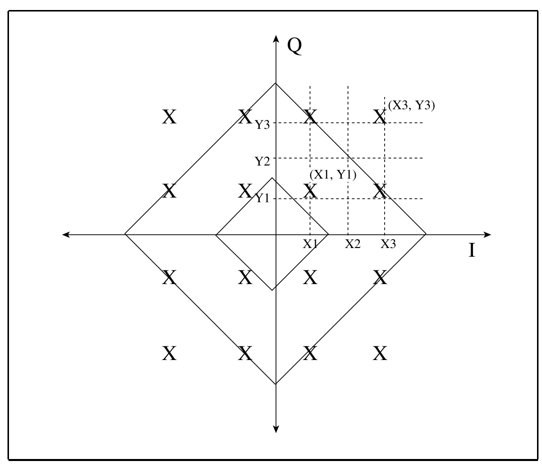

Fig. 4. The zero error contour of the proposed algorithm.

We modify the above equation by replacing the costant value R_s with the estimated level value $\overline{R_c}$ and removing the sign function to minimize the residual error after convergence. The estimated level value $\overline{R}_c$ is

$$\overline{R}_c = \begin{cases} \sqrt{X1^2+Y1^2} \\ \text{or} \\ \sqrt{X3^2+Y3^2}, \end{cases} \tag{6}$$

where $X1$, $Y1$, $X3$ and $Y3$ are the coodinates of the 16-QAM signal as depicted in Fig. 4.

The modified equation is

$$c_{n+1} = c_n - \mu \cdot [|z_{r,n}| + |z_{i,n}| - \overline{R}_c] \cdot [\text{sgn}(z_{r,n}) + j\text{sgn}(z_{i,n})] x_n^*, \tag{7}$$

where $\overline{R}_c$ is estimated level value and the subscripts r and i denote the real and imaginary components.

The zero error contour of this algorithm is illustrated in Fig. 4. The modified equation (7) tends to drive the output data symbols to lie on an inner or outer square rotated by 45°. The residual error of this algorithm has a lower residual error than Sign_GA and GA, because this modified algorithm uses 45° rotated level DD error[4]. But the equalizer using this algorithm does not converge in coarse channel environments, because the modified algorithm uses the unreliable level DD error which is similar to DDA.

Thus we apply the SGA's updating rule which produces the relatively reliable DD error in this algorithm.

We can modify the formula (7) as

$$c_{n+1} = c_n - \mu \cdot f \cdot [|z_{r,n}| + |z_{i,n}| - \overline{R}_c] \cdot [\text{sgn}(z_{r,n}) + j\text{sgn}(z_{i,n})] x_n^*, \tag{8}$$

[4]level DD error $= [|z_{r,n}| + |z_{i,n}| - \overline{R}_c] \cdot [\text{sgn}(z_{r,n} + j\text{sgn}(z_{i,n})] x_n^*$

where f is the random variable which has "0" or "1". The random variable f which is similar to "stop and go" flag in SGA is defined by

$$f = \begin{cases} 0, \; if \sqrt{X1^2+Y1^2} < |z_{r,n}| + |z_{i,n}| < \sqrt{X2^2+Y2^2} \\ 1, \; \text{otherwise} \end{cases}, \tag{9}$$

where $X1$, $Y1$, $X2$ and $Y2$ are defined beneath (6).

Also estimated level $\overline{R}_c$ is controlled as follows

$$\overline{R}_c = \begin{cases} \sqrt{X1^2+Y1^2}, \; if \; |z_{r,n}| + |z_{i,n}| \le \sqrt{X2^2+Y2^2} \\ \sqrt{X3^2+Y3^2}, \; if \; |z_{r,n}| + |z_{i,n}| \ge \sqrt{X2^2+Y2^2}, \end{cases} \tag{10}$$

where $X1$, $Y1$, $X3$, $Y3$, $X2$ and $Y2$ are the coordinates of the 16-QAM signal as depicted in Fig 4. This situation is illustrated in Fig. 5. From the figure, it is clear that the proposed algorithm can update the equalizer tap coefficient in the blank area but cannot update it in the shaded area.

IV. PERFORMANCE COMPARISON

The performance of the new algorithm was compared to that of GA, Sign_GA, and SGA using computer simulation. Convergence rate and residual error after convergnce were examined. We assumed that the received signal's clock is perfectly recovered and its power is perfectly controlled. Also we assumed that a

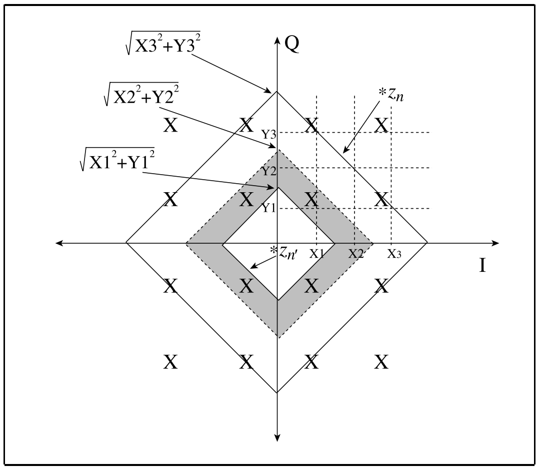

Fig. 5. Updating area of adaptive filter tap coefficient in the new algorithm (Blank area: updating coefficients, Shaded area: stop updating, $*z_{n'}$, $*z_n$: equalizer output, X: 16-QAM signal constellation).

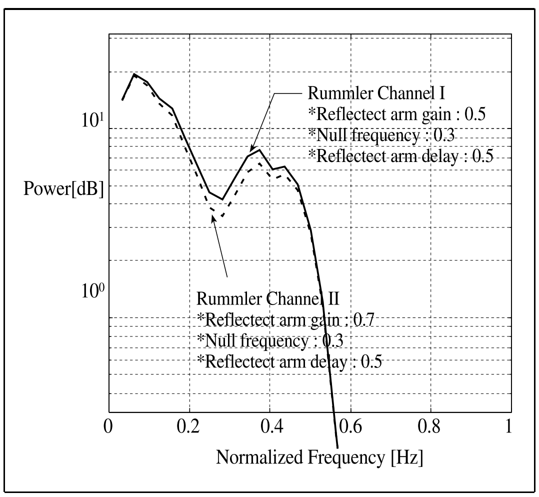

Fig. 6. The frequency response of the Rummler channel model (Reflected arm delay: 0.5[sec], Reflected arm gain: 0.5, 0.7, Null frequency: 0.3 [Hz]).

carrier phase offset does not effect the equalizer. In all the simulation examples, the complex equalizer has 11 taps. The middle tap of the equalizer was initialized to the nonzero value $1 + j0$, and the rest of the equlizer weights were initialized to $0 + j0$.

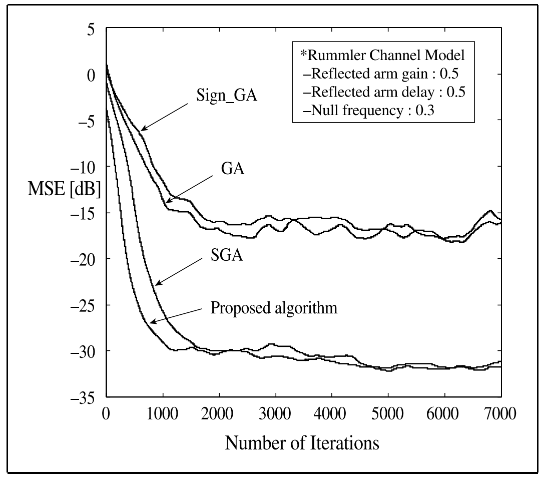

Fig. 7. The learning curve for Rummler channel model I (step size: GA (0.0001), Sign_GA (0.001), SGA (0.005), and Proposed (0.005); The step size is chosen to provide acceptable performance at the highest expected fading rate).

A simple 16-QAM constellation has been chosen and an additive white gaussian noise (AWGN) with a signal to noise ratio (SNR) of 30 dB is added. A 10% excess bandwidth square root raised cosine (SRRC) transmit filter is used. T/4-spaced transmit and receive filters and a T/2-spaced fractionally spaced equalizer (FSE) are employed. The step size of the algorithm is chosen to provide acceptable performance at the highest expected fading rate. The Rummler channel model [7] is simulated. The frequency response of the Rummler channel model is illustrated in Fig. 6.

Performance comparisons are presented with the GA, Sign_GA, and SGA in terms of the steady state mean squared error (MSE) and convergence rate. For comparison purpose, the corresponding GA, Sign_GA, and SGA are simulated in the presence of 30 dB noise.

First, the learning curve for Rummler channel model I is given in Fig 7. From the figure, it is clear that the proposed algorithm has a faster convergence rate than Sign_GA, GA and SGA. Also, the new algorithm has a much lower residual error than Sign_GA and GA. The computational requirement of the proposed algorithm is the same as that of SGA.

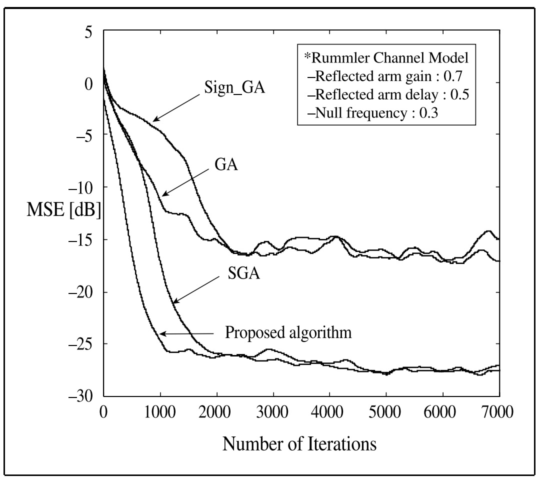

Fig. 8. The learning curve for Rummler channel model II (step size : GA (0.0001), Sign_GA (0.001), SGA (0.005), and Proposed (0.005)).

Second, the learning curve for Rummler channel model II is illustrated in Fig. 8. From the results, the new algorithm has a superior performance to Sign_GA, GA and SGA.

V. CONCLUSION

In summary, a new blind equalization algorithm has been presented. Simulation results demonstrate that the new algorithm has a faster convergence rate in comparison with such conventional algorithms as GA, Sign_GA and SGA. In addition, it has a lower residual error (approximately 15 dB) than that of GA and Sign_GA.

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