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Option Strategies: An Analysis of Naked Put Writing

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Abstract

Writing naked put options is a strategy employed either as a speculation to capture premium income, or as a method of placing a limit order to buy the underlying at the strike price in return for premium received. Using a Monte Carlo simulation, twenty thousand equity prices are generated under known volatility and return parameters. A binomial tree is constructed using the same volatility and return parameters. Put options on these "equities" are valued with the binomial methodology. The performance of various put writing strategies is evaluated on a risk-adjusted basis. Evidence presented suggests that the judicious use of put options may enhance returns during portfolio construction.

I. Introduction

The use of option trading strategies is quite widespread in the modern option markets. Whether the speculative use of options can enhance returns available to equity-oriented investors is an interesting empirical question. This paper addresses this question in the context of strategies involving naked put option writing. Three

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relatively conservative strategies involving the use of equity put options are explored. The evaluation is on a risk-adjusted performance basis, relative to a benchmark equity-oriented strategy.

The approach relies on simulating equity prices and valuing put options on these equities by using the well-known binomial methodology. The risk-adjusted performance of simulated portfolios involving the use of options is compared to that of simulated all-equity benchmark portfolios. This research also extends the literature on option valuation using a Monte Carlo simulation approach (see for example, Boyle, 1977 and Geske and Shastri, 1985). In contrast to earlier studies, this study employs a simulation methodology which does not rely on generating underlying equity prices using explicit distributional assumptions. These simulations rely on resampling from the observed equity prices - an approach which is likely to result in simulated prices which better reflect the distributional properties of actual stock prices.

Results presented in this paper suggest that the judicious use of equity put options can enhance the risk-adjusted returns available to an equity-oriented investor during the construction of a portfolio. In specific, these results indicate that the sale of naked put options which are in the money can enhance the portfolio performance by more than 90 basis points over expected returns under comparable levels of risk.

The rest of the paper is organized along the following lines. The next section discusses some of the various option strategies commonly used by market participants. Section 3 outlines the model of equity prices used in the paper while section 4 describes the data generation process in detail. The binomial option valuation methodology used in the paper is discussed in section 5. The results of the analysis are presented in section 6 while section 7 concludes.

II. Option Strategies

An option is a security which confers upon its owner the right, but not the obligation, to do something. The right, in the context of equity options, is usually to purchase or sell the underlying equity security at a fixed price for a limited time. Beyond the time stated in the contract, the right ceases to exist and the option expires.

Options come into existence when written by the party selling the option. This is substantially different from the creation of debt and equity securities: options can be created by almost anyone with an interest in doing so[1]. Writing options refers to the sale of an option which was previously unowned. In other words, it means creating a short position in a put or call option on an underlying security.

When such a position is created, the writer assumes the obligation to purchase or sell the underlying at the fixed price for a limited time. While the option purchasers liability is limited to the cost, or premium paid for the option, the writers liability is theoretically unlimited in the case of the sale of a call, and is limited to the difference between the strike price and zero in the case of the sale of a put.

The focus of this research is strategies involving the writing of put options when the writer has no existing exposure to the underlying asset. In the terminology of financial market participants, this is often referred to as "naked put writing", because of the unhedged nature of the position[2].

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1) Although almost anyone can create an option, such a strategy is suitable only for investors who understand the uses and risks of such contracts. Information on uses and risks is available from the major options exchanges, as well as through most brokerage firms.

2) For an excellent overview on the uses of exchange traded option contracts, see McMillan

Strategies involving the writing naked put options can be classified broadly in two general classes. One, they can be considered as a method of speculation in an attempt to capture premium income. In this case, the writer selects an underlying security which he or she believes will either maintain current value or increase in value, and writes a put option contract on the underlying. Options would likely be written at strike prices ranging from approximately at the money to well out of the money. Alternatively, one can view such writing as a method of placing a limit order to buy the underlying at the strike price, in return for premium received. In this latter case, the writer selects an underlying security which he or she desires to own, and writes the put option. Options written would be expected to have strike prices ranging from at-the-money to well in-the-money. These are general rules of thumb for put option writers.

In the following sections, the underlying motivation of the option writer is ignored, and the focus is directed toward the efficacy of the various strategies. The three basic strategies evaluated are writing puts which are *in*, *at*, and *out-of-the* money. The first step in this exercise is the specification of an equity price process, and this is done in the next section.

III. Underlying Price Process

Most evidence on equity market efficiency suggests that markets are semi-strong form efficient, but not strong form efficient[3]. Given this evidence, it is henceforth assumed that all publicly available information is impounded in the current equity

(1980).

3) Fama (1970) is widely credited for these definitions of market efficiency. Copeland and Weston (1988) present two and one-half decades of evidence both supporting and contradicting the notion of market efficiency on all three levels.

price. It follows that the future of such equity prices will depend only upon current price levels, and two identifiable parameters: expected (or required) return and the natural volatility of the equity. The foregoing can be characterized with a stochastic process incorporating expected return and volatility. Accordingly, the underlying equity prices used in this analysis are generated through the use of a stochastic process.

Stochastic processes may be either discrete time or continuous time. In the real world, equity prices change in discrete time: they change (or have the potential to change) with each trade. It is common for equity processes to be modeled using a specific type of stochastic process, known as a generalized Wiener process[4]:

$$P_t = P_{t-1} + \Delta P, \tag{1}$$

where,

$$\Delta P = P^*_{t-1}\mu^*\Delta t + P^*_{t-1}\sigma^*\varepsilon^*(\Delta t)^{0.5}, \tag{2}$$

P_{t-1} = initial equity price, end of period t-1,

μ = drift parameter,

Δt = time interval between observations,

σ = volatility of the underlying,

ε = standard normal random variable.

A process of this type incorporates a drift parameter, μ, and a random noise parameter, ε. The former can be thought of as the expected return on the underlying equity price, much as one may think of expected return under CAPM[5].

4) See Hull (1989) pp. 62-74.

5) A good working description of the Capital Asset Pricing Model (CAPM) can be found in

may be thought of as the root cause of unexpected price changes which occur randomly. Unexpected price changes, those which are not correlated with market returns, are known as the idiosyncratic component in asset returns. Both the return and the volatility are proportional to the equity price.

IV. Data Generation

Data simulation in this model economy is accomplished such that equity prices confirm to the above stochastic process with *one important difference*. Instead of drawing the random noise parameter, ε, from a normal distribution, the time-series of error terms are generated by drawing, *with replacement*, from the empirical distribution of the equity prices in the market. It is a well-known fact that stock returns (price changes) do not conform to the normal distribution[6]. This sampling approach, which is similar to the bootstrap method (Efron, 1979), is likely to result in simulated prices which better reflect the distributional properties of the actual equity prices.

The return and volatility parameters of the price process (μ and σ, respectively) are chosen so as to be consistent with the historical values observed for most developed equity and debt markets. The expected return on the equity, R_i, is specified in a CAPM format as:

$$E[R_i] = R_f + \beta_i[E[R_m] - R_f], \quad (3)$$

most corporate finance texts, such as Copeland and Weston (1988). For the original development of the model see Sharpe (1964).

6) See Fama (1965) for a discussion on the distribution of equity price changes.

where,

β_i = 1.0,

$E[R_m]$ = 10.0%,

R_f = 5.0%.

The volatility of the underlying is specified at 25%, and dividends are assumed to be zero. The data generation process is described in detail below.

Simulation in this model economy begins by re-sampling from the daily equity price series for *British Petroleum*, from the London Stock Exchange, over a period of 126 trading days - January 4, 1993 to July 2, 1993 (representing six months of trading history). The equity prices are generated using the following procedure.

(1) Using the 126 daily price observations a time-series of 125 error terms, ε_t, is calculated in accordance with equation (2) above:

$$\varepsilon_t = \frac{(P_t - P_{t-1}) - (P^*_{t-1}\mu^*\Delta t)}{P^*_{t-1}\sigma^*(\Delta t)} \qquad (4)$$

(2) The sample mean of the vector of 125 error terms obtained in step (1) is computed and is subtracted from each residual term to obtain a vector of mean-centered error terms, $e_t(t = 1, 2, \cdots 125)$. The standard deviation of these terms is also normalized to unity. The vector of mean-centered errors is used to estimate the population distribution of error terms in the simulations.

(3) Next, a time series of simulated prices is generated for each day, t (t = 1, 2. $\cdots$, 252) in the following manner. For each t, an integer is randomly generated from a uniform distribution over [1, 125] and set it equal to t^*. The

simulated price for day t is then given by:

$$P_t = P_{t-1} + \Delta P$$

where,

$$\Delta P = P^*_{t-1}\mu^*\Delta t + P_{t-1}\sigma^* e^*_t \cdot (\Delta t)^{0.5} \quad (5)$$

and, P_0 = 500 pence

(4) The above step gives a simulated price series representing 252 days (i.e., about one year) of trading.

Step (3) is repeated to obtain a set of 1,000 simulated equity price series, each representing a time series of 252 observations. This process is replicated 10 times yielding 10 sets of 1,000 price series each (so that there are a total of 10,000 simulated price series).

In the above construction of the model economy, the empirical distribution of the residuals from the model of interest is used to approximate the population distribution of these error terms. No explicit distributional assumptions for the error terms are made (the only assumption inherent in our sampling scheme is that the error terms are *iid*). In spite of the absence of explicit distributional assumptions, one must be sensitive to the possibility that results obtained may be functions of properties of the underlying randomly generated data[7]. To overcome such concerns the data set is expanded by relying on a variant of the antithetic variable technique. For example, one can obtain a sample by drawing n observations from a particular distribution of interest, obtaining $x_1, x_2, \cdots, x_n$. Next, one can create a second set of n observations $y_i = -x_i$, thereby insuring

7) See Boyle (1977) for a discussion on issues relating to data generation for Monte Carlo simulations.

that *in aggregate* the sample mean is zero. (The two pairs of error term data sets are mirror images of one another.) The use of such techniques can substantially mitigate any concerns which arise over the generation of random data. Accordingly, the size of the data set is doubled by generating a second set of 10,000 equity price time-series observations using the antithetic variable technique. Therefore, the resulting data set contains 20,000 time-series of equity prices. These equity price paths serve as the basis for the valuation of options using the binomial method (described later).

As indicated above, equity prices are generated so as to evolve over the course of a year, for a one year interval of performance evaluation. The compounding period for these prices is daily. All prices begin time at 500 pence per share (i.e., P_0 = 500p).

V. Option Pricing Models

The pricing of options is an inexact science[8]. Several analytical models have been proposed for the valuation of options. The Black-Scholes (1973) option pricing model is the most well known of these models. Many tests have been undertaken to assess the efficacy of this and other models. While such tests often find discrepancies between option pricing and model predictions, the models retain usefulness as such errors are usually less than the bid/ask spread[9].

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8) An evaluation of pricing efficiency of several option pricing models can be found in Geske and Shastri (1985).

9) For example, Geske and Shastri (1985) find differences in option pricing model estimates on the order of one to five cents in their tests. If the spread is 1/16-the typical minimum value-this is 6.25 cents. Moreover, even on relatively large orders, the transaction fees are likely to represent an additional cost of a few cents per contract.

The Black-Scholes model is the most appropriate for the valuation of European-style[10] put and call options. It is also possible to use this model for the valuation of American-style call options on securities which do not pay dividends. For dividend paying equities, modifications may be undertaken so that the model may be utilized. The model is not appropriate for the valuation of American-style put options.

Lattice or binomial style option pricing models[11] can accommodate American-style put options. This is because the use of a binomial pricing methodology allows for the bounding of prices which must occur with American-style put options which may be exercised prior to maturity. The practical implication is that an option value can never be less than its intrinsic[12] value. A logical function at each node of the binomial lattice used in this research insures that this rule is enforced.

A. Binomial Tree Construction

The binomial option pricing tree used for option pricing is constructed as follows:

1. Nodes are specified, and branches reach out from each node representing up and down movements. Taken to the extreme, such nodes occur every time a trade occurs in the underlying equity. This could mean hundreds or even thousands of nodes per day. However, a sufficiently accurate approximation of this process may be derived with substantially fewer branches.
2. The degree of the up and down movement is governed by the volatility of the underlying, and the time elapsed between nodes.

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10) European-style options may only be exercised at expiration, while American-style options may be exercised at any time during the life of the contract.

11) This approach was first suggested by Cox, Ross, and Rubinstein (1979).

12) Intrinsic value is defined as the strike price minus the underlying price. Ignoringtransaction costs, if the option can be exercised at any time during its life, arbitrage conditions preclude the option value from being less than the intrinsic value.

3. The prices evolve through time until they reach the expiration date of the option n periods into the future.

4. The option terminal values, which are deterministic, are inserted at the expiration date nodes.

5. These values are devolved backward through time on the basis of the probability of an up or down movement in the equity, and are adjusted for the time value of money.

6. This process is replicated at each node back to the present or starting date. The value generated is the expected price of the option with n periods to expiration. Because this procedure works backward through time, it is sometimes called dynamic programming.

B. Numerical Example: Steps 1 - 3

Suppose that the year is divided into 24 periods, each 1/2 month long, and that volatility of the underlying were 25% per annum.

Define: Upward Movement $= u = \exp(\sigma\sqrt{\Delta t})$ (6)

So: $= u = \exp(0.25*(\sqrt{1/24})$

$= 1.05236$

Define: Downward Movement $= d = \exp(-\sigma\sqrt{\Delta t})$ (7)

So: $= d = \exp(-0.25*(\sqrt{1/24})$

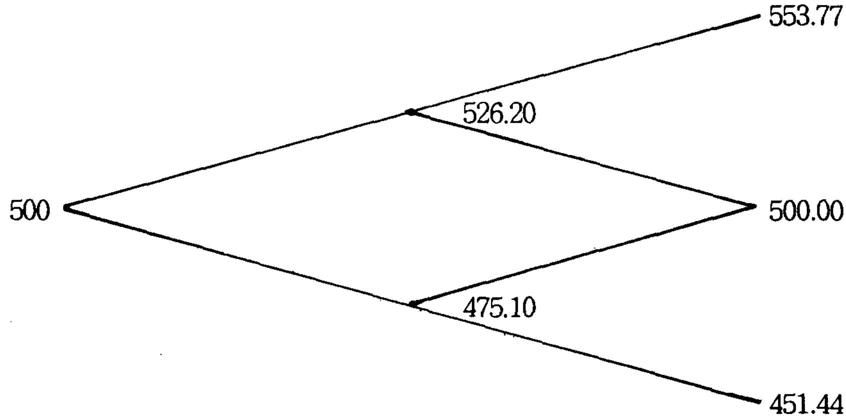
$= 0.95025$

$S_u = S_0^* u = 500*1.0524 = 526.20$

$S_0 = 500$

$S_d = S_0^* d = 500*0.9502 = 475.10$

For two periods:

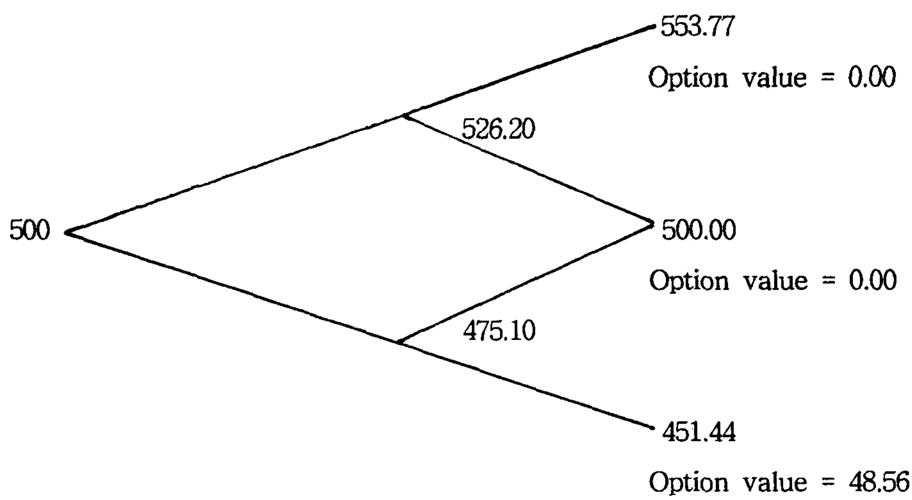


C. Numerical Example: Steps 4 - 6

Define: Option (put) strike price = 500

Risk-free rate = 5%

Next, insert the terminal option values (strike - ending price if > 0, else 0) at each of the ending nodes, assuming that this option had only two periods until expiration.



The next step is to value the option at the preceding nodes. This requires that probabilities be assigned to the up and down movements for the underlying equity.

Define: Probability of up move = p = (a-d)/(u-d) (8)

Probability of down move = 1-p (9)

Where: $a = \exp(r\Delta t)$ (10)

u is as previously defined

d is as previously defined

Using the parameters previously specified:

a = exp(0.05*(1/24)) = 1.0021

p = (1.0021-0.9502)/(1.0524-0.9502) = 0.5078

1-p = 0.4922

Option value at preceding node is then defined as the greater of:

$$OV = ((OV^{*}_{u}p)+(OV^{*}_{d}(1-p)))/(1+(R_f/\#\text{periods per year})), \quad (11)$$

or strike price - equity price.

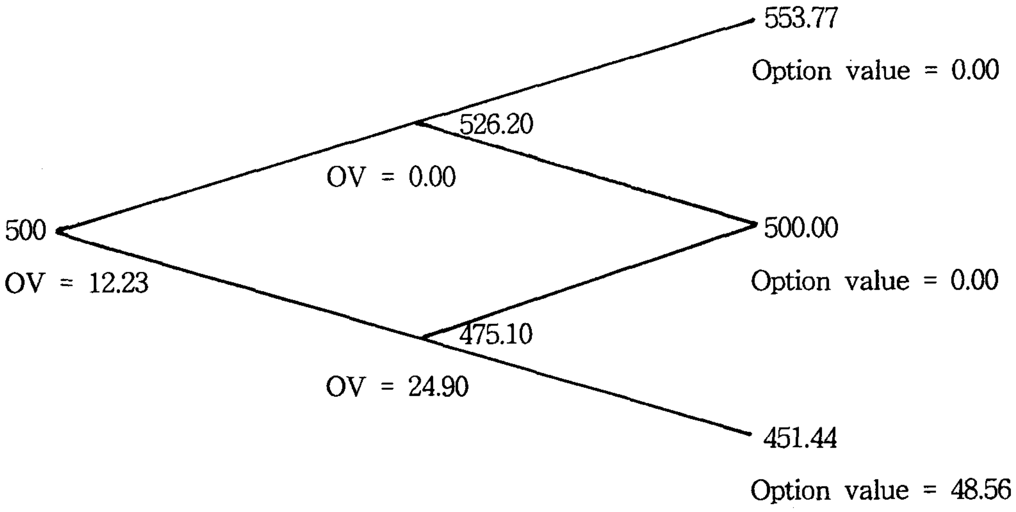
Therefore, using the two lower branches for the terminal values, one can determine the value at the preceding node by first using (12) as shown above:

OV = ((0*0.5078)+(48.56*0.4922))/(1+(0.05/24))

= 23.85

Since this is less than 500.00 - 475.10 = 24.90, the value of 24.90 is inserted at this node.

The tree would then be completed backwards similarly to its starting point to obtain the following:



Thus, the binomial model valuation is 12.23 pence for this two period put option.

The binomial option trees used to value the options in this analysis cover 6 months and 3 months. The branching of the binomial trees is 10 times per month. The number of branches was selected by evaluating the change in option price as each the number of branches was increased. With 10 branches per month, the change in option price was less than 0.01 pence on an at the money 6 month option.

It is worth emphasizing that all parameters are known with certainty for both the stock price evolution, and for the option pricing approximation. This means that any results obtained will not be a product of a mismatch in the volatility of the underlying equity compared with the volatility assumption of the option pricing model.

VI. Portfolio Performance Comparison

In this section, risk-adjusted performance statistics for the option strategy portfolios are compared with that for the all equity portfolio. All portfolios are defined to begin the period with 500 on the starting date. The primary performance measure is the Sharpe Ratio[13]. The Sharpe Ratio is a measure of excess return per unit of risk. In specific, the Sharpe Ratio is defined as:

$$SR = (R_p - R_f)/\sigma_p. \tag{12}$$

In words, this is the excess return on the portfolio (over the risk-free rate) divided by the standard deviation of the returns.

An additional measure of performance is provided for some of the portfolios, and this is the realized mean nominal portfolio returns versus beta-risk adjusted expected returns. The mean nominal return is determined from the gross return, which is found by calculating the final value of each portfolio at year end. The expected return is a function of percentage of the portfolio assets invested in equities midway through the year[14]. In specific, expected return is defined as:

$$E[\ R_p] = R_f + \beta[\ R_m - R_f]\ , \tag{13}$$

where:

β_p = (# Options Exercised*Underlying Price of Equities Exercised at Closing on Day 126) / Portfolio Value at Close on Day 126. (14)

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13) See Sharpe (1966).

14) In specific, this is done at the end of day 126. At this time, all options have been exercised or have expired. There are no options in any portfolio at this point: each portfolio is comprised of some weighting of equities and the risk-free asset.

Table I All Equity Portfolio Returns

Portfolio Number	Compound Return	Standard Deviation	Sharpe Ratio
1	10.420	27.340	0.198
2	11.083	27.320	0.223
3	9.388	26.402	0.166
4	9.942	28.367	0.174
5	10.120	27.540	0.186
6	10.324	29.018	0.183
7	10.815	28.453	0.204
8	9.621	27.812	0.166
9	10.621	28.597	0.197
10	11.055	28.021	0.216
11	10.435	27.687	0.196
12	9.882	28.290	0.173
13	11.151	27.085	0.227
14	11.052	27.766	0.218
15	10.746	27.733	0.207
16	11.081	28.711	0.212
17	10.669	29.114	0.195
18	11.541	28.969	0.226
19	10.627	28.154	0.200
20	9.840	27.487	0.176
	Mean		0.1972
	Standard Deviation		0.0197

Table II Option Portfolio Returns: 6 Month at the Money

Portfolio Number	Compound Return	Standard Deviation	Sharpe Ratio
1	7.544	13.375	0.190
2	7.858	13.585	0.210
3	7.821	13.357	0.211
4	7.107	12.852	0.164
5	7.329	12.456	0.187
6	7.610	14.178	0.184
7	7.375	13.652	0.174
8	7.410	13.347	0.181
9	7.176	13.279	0.164
10	7.866	12.867	0.223
11	7.306	12.504	0.184
12	7.236	12.073	0.185
13	8.405	12.853	0.265
14	7.184	12.879	0.170
15	7.789	13.303	0.210
16	7.559	12.991	0.197
17	7.215	13.305	0.166
18	7.729	13.073	0.209
19	7.147	12.932	0.166
20	6.873	12.932	0.145
	Mean		0.1892
	Standard Deviation		0.0262

Table III Option Portfolio Returns: 3 Month at the Money

Portfolio Number	Compound Return	Standard Deviation	Sharpe Ratio
1	7.397	15.040	0.159
2	7.580	15.177	0.170
3	7.998	15.309	0.196
4	7.161	15.185	0.142
5	7.198	14.125	0.156
6	8.055	16.591	0.184
7	7.547	17.022	0.150
8	7.294	15.142	0.151
9	6.878	14.712	0.128
10	7.954	15.942	0.185
11	7.258	15.375	0.147
12	7.045	15.296	0.134
13	8.237	14.949	0.217
14	7.528	15.378	0.164
15	7.604	15.473	0.168
16	8.309	16.145	0.205
17	7.255	14.969	0.151
18	8.059	16.891	0.181
19	6.902	14.923	0.127
20	7.334	14.910	0.157
	Mean		0.1636
	Standard Deviation		0.0242

This performance measure is utilized to give economic content to the size of the performance improvement when the Sharpe Ratios show a statistically significant improvement over the benchmark.

A. Performance Benchmark

The benchmark case for comparison is the all equity portfolio. This portfolio is one for which the cash was used to purchase one share of each of the 1,000 securities at a starting value of 500p each.

The results for these benchmark portfolios are shown in Table I. The column of particular note is that of the Sharpe Ratio. The mean Sharpe Ratio on the benchmark portfolio group is 0.1972, which is not statistically different (t=0.657) from the known true Sharpe Ratio[15] of 0.1943. One can interpret the meaning of the mean Sharpe Ratio as being that these benchmark portfolios provided 0.1972% excess return for each 1% standard deviation of returns. This provides a stand-alone measure of comparative performance for the option strategy portfolios.

It is also noteworthy, but not displayed in Table I, that the mean nominal return for the benchmark portfolio is 10.004%, versus the expected return of 10%. Nine of the twenty individual portfolios have nominal returns which are less than 10%, and eleven have returns which are greater than 10%.

B. At the Money Put Performance

The at the money portfolios are those for which one option with a strike price of 500p was sold short on each of the 1,000 representative securities on the first day of the year. The gross premiums for these options are 30.037p and 22.179p for the six month and the three month expirations, respectively. The premiums received

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15) The true ratio is known, since the return and volatility parameters are specified in the price generation process.

on the sale of the options are invested in the risk-free asset, along with the initial 500 cash balance. In the event that the options are in the money on expiration, the underlying security is purchased at the strike price and a sufficient amount of the risk-free asset is sold to pay for the cost of the purchase. Gross returns are determined from the terminal value of each portfolio.

Calculation of gross returns:

If underlying price > strike price,

$$GR = (\text{Premium} + \text{Principal}) * (1 + R_f/252)^{252}.$$

If underlying price < strike price,

$$GR = (((\text{Principal}) * (1 + R_f/252)^{t}) - \text{Principal}) * (1 + R_f/252)^{(252-t)}$$

$$+ (\text{Premium}) * (1 + R_f/252)^{252}$$

$$+ \text{Terminal Value of Underlying},$$

where: t = time to expiration of the option in half months.

Table II contains the results for the sale of options with six months to expiration, and Table III contains the results for the sale of options with three months to expiration.

The mean Sharpe Ratios are 0.1892 and 0.1636 for the six month and three month portfolios, respectively. These ratios indicate that there is an apparent decrease in performance relative to the benchmark. The difference between the ratio on the six month option portfolio and the benchmark is not significant in a statistical sense, while that between the three month portfolio and the benchmark is significant at the 0.01 level (t-statistic = -5.535). On the basis of these results, it would not appear that there are significant performance improvements to be realized from a strategy employing short at the money options.

C. In the Money Put Performance

The in the money portfolios are those for which options with strike prices of 525, 550, and 575 are sold short on each of the 1,000 representative securities on the first day of the year. These options are 5%, 10%, and 15% in the money, and have positive intrinsic value. The gross premiums for these options are 43.898p, 60.589p, and 79.879p for the six month expirations, respectively. For the three month expirations the corresponding premiums are 36.679p, 54.899p, and 76.240p.

As was the case for the at the money options, the premiums received on the sale are invested in the risk-free asset, along with the initial 500 cash balance. In the event that the options are in the money on expiration, the underlying security is purchased at the strike price and a sufficient amount of the risk-free asset is sold to pay for the cost of the purchase. Gross returns are determined from the terminal values of each portfolio, exactly as was done for the at the money portfolios.

Tables IV through VI contain the relevant performance statistics for the six month expirations. Tables VII through IX contain the corresponding results for the three month expirations.

The portfolios utilizing the six month options have mean Sharpe Ratios of 0.2089, 0.2263, and 0.2417 for the 5%, 10%, and 15% in the money classes, respectively. The corresponding t-statistics on these differences are 2.500, 5.972, and 7.732, indicating that the improvement over the benchmark is significant at the 0.05 level for the 5% in the money portfolio, and at the 0.01 level for the 10% and 15% in the money portfolios.

The portfolios utilizing the three month options have mean Sharpe Ratios of 0.1860, 0.2033, and 0.2147 for the 5%, 10%, and 15% in the money classes respectively. The corresponding t-statistics on these differences are -1.377, 1.541, and 4.157, indicating that the 5% and 10% in the money portfolios are not significantly different from the benchmark Sharpe Ratio, while the 15% in the money portfolio shows an improvement which is significant at the 0.01 level.

Table IV Option Portfolio Returns: 6 Month 5% in the Money

Portfolio Number	Compound Return	Standard Deviation	Sharpe Ratio
1	8.155	15.551	0.203
2	8.902	16.181	0.241
3	8.399	15.512	0.219
4	7.537	15.149	0.167
5	8.123	15.129	0.206
6	7.991	16.181	0.185
7	8.262	16.355	0.199
8	8.082	15.732	0.196
9	7.902	16.034	0.181
10	8.384	14.978	0.226
11	7.980	15.025	0.198
12	7.910	14.753	0.197
13	9.316	15.165	0.285
14	8.176	15.351	0.207
15	8.420	15.526	0.220
16	8.296	15.605	0.211
17	8.062	15.782	0.194
18	8.876	15.771	0.246
19	7.855	15.363	0.186
20	8.293	15.698	0.210
	Mean		0.2089
	Standard Deviation		0.0255

Table V Option Portfolio Returns: 6 Month 10% in the Money

Portfolio Number	Compound Return	Standard Deviation	Sharpe Ratio
1	8.911	17.645	0.222
2	9.917	18.752	0.262
3	9.131	17.624	0.234
4	8.164	17.375	0.182
5	8.692	17.247	0.214
6	9.121	18.991	0.217
7	9.135	18.495	0.224
8	8.615	17.892	0.202
9	9.022	18.737	0.215
10	8.961	17.268	0.229
11	8.970	17.656	0.225
12	8.640	17.033	0.214
13	9.954	17.157	0.289
14	9.207	17.677	0.238
15	9.301	17.915	0.240
16	9.313	18.054	0.239
17	8.457	18.022	0.192
18	9.504	17.791	0.253
19	8.935	17.979	0.219
20	8.867	17.853	0.217
	Mean		0.2263
	Standard Deviation		0.0234

Table VI Option Portfolio Returns: 6 Month 15% in the Money

Portfolio Number	Compound Return	Standard Deviation	Sharpe Ratio
1	9.891	20.000	0.245
2	10.820	20.629	0.282
3	9.991	20.087	0.248
4	8.929	19.491	0.202
5	9.537	19.538	0.232
6	9.566	20.750	0.220
7	9.713	20.408	0.231
8	8.887	19.675	0.198
9	10.006	20.939	0.239
10	10.110	19.386	0.264
11	9.916	19.984	0.246
12	9.040	18.832	0.215
13	10.893	19.508	0.302
14	10.475	20.212	0.271
15	10.012	19.883	0.252
16	10.096	20.053	0.254
17	9.266	20.011	0.213
18	10.381	19.891	0.271
19	9.637	19.883	0.233
20	9.231	19.543	0.216
	Mean		0.2417
	Standard Deviation		0.0267

Table VII Option Portfolio Returns: 3 Month 5% in the Money

Portfolio Number	Compound Return	Standard Deviation	Sharpe Ratio
1	8.096	17.781	0.174
2	8.943	18.633	0.212
3	8.407	17.970	0.190
4	7.884	18.448	0.156
5	8.204	18.260	0.175
6	8.714	19.187	0.194
7	8.844	20.270	0.190
8	7.840	18.330	0.155
9	8.137	19.081	0.164
10	9.193	18.962	0.221
11	8.549	18.490	0.192
12	7.515	18.540	0.136
13	9.494	18.105	0.248
14	8.575	18.227	0.196
15	8.839	19.013	0.202
16	9.064	19.173	0.212
17	7.967	18.605	0.159
18	9.047	19.527	0.207
19	8.007	18.324	0.164
20	8.059	17.793	0.172
	Mean		0.1860
	Standard Deviation		0.0263

Table VIII Option Portfolio Returns: 3 Month 10% in the Money

Portfolio Number	Compound Return	Standard Deviation	Sharpe Ratio
1	8.975	20.801	0.191
2	10.357	22.401	0.239
3	9.093	21.007	0.195
4	8.359	20.480	0.164
5	9.583	21.716	0.211
6	9.193	21.840	0.192
7	9.927	22.832	0.216
8	8.654	21.422	0.171
9	9.127	22.097	0.187
10	9.885	20.869	0.234
11	9.499	21.181	0.212
12	8.446	21.403	0.161
13	10.456	21.221	0.257
14	9.740	20.796	0.228
15	9.473	21.424	0.209
16	10.004	22.307	0.224
17	8.904	21.582	0.181
18	9.802	22.088	0.217
19	9.364	21.507	0.203
20	8.566	20.468	0.174
	Mean		0.2033
	Standard Deviation		0.0256

Table IX Option Portfolio Returns: 3 Month 15% in the Money

Portfolio Number	Compound Return	Standard Deviation	Sharpe Ratio
1	9.982	23.138	0.215
2	11.168	24.368	0.253
3	9.450	22.908	0.194
4	9.096	22.428	0.183
5	10.262	23.835	0.221
6	9.474	23.530	0.190
7	10.616	24.782	0.227
8	9.401	23.433	0.188
9	10.028	24.077	0.209
10	10.227	22.791	0.229
11	10.347	23.316	0.229
12	9.440	23.633	0.188
13	10.733	23.142	0.248
14	10.624	23.329	0.241
15	10.357	23.388	0.229
16	10.522	25.146	0.220
17	9.914	24.444	0.201
18	10.633	24.099	0.234
19	10.112	24.095	0.212
20	9.208	22.941	0.183
	Mean		0.2147
	Standard Deviation		0.0214

In order to give these results some economic content, note that the improvement in the Sharpe Ratios on the six month portfolios represent 29 basis points, 64 basis points, and 95 basis points of return improvement for a portfolio with a standard deviation of 20% per annum.

These results are corroborated by comparing the mean nominal returns realized on the six month portfolios with the risk-adjusted expected returns. In the case of the 5% portfolio, the mean improvement of the nominal realized return over the risk-adjusted expected return is 59 basis points. For the 10% portfolio, the mean improvement is 81 basis points, and for the 15% portfolio, the mean improvement is a noteworthy 104 basis points.

The results provided here suggest that it may be possible enhance portfolio performance to a meaningful extent with in the money put option strategies. Since, on average, 53.3%, 63.5%, and 72.7% of the respective 5%, 10%, and 15% six month options expired in the money, it is worth noting that such strategies will only be appropriate for investors wishing to increase the level of equity exposure in their portfolios.

D. Out of the Money Put Performance

The out of the money portfolios are those for which options with strike prices of 475 and 450 are sold short on each of the 1,000 representative securities on the first day of the year. These options are 5% and 10% out of the money, and have no positive intrinsic value. The gross premiums on these options are 19.289p and 11.462p for the six month expirations, respectively. For the three month expirations the corresponding premiums are 11.966p and 5.601p.

As was the case for the at the money options, the premiums received on the sale are invested in the risk-free asset, along with the initial 500 cash balance. In the event that the options are in the money on expiration, the underlying security is purchased at the strike price and a sufficient amount of the risk-free asset is sold to pay for the cost of the purchase. Gross returns are determined from the terminal values of each portfolio, exactly as was done for the at the money portfolios.

Tables X and XI contain the relevant performance statistics for the six month expirations. Tables XII and XIII contain the corresponding results for the three month expirations.

The mean Sharpe Ratios on the six month portfolios are 0.1715 for the 5% portfolio, and 0.1473 for the 10% portfolio. The t-statistics on the difference between these and the benchmark portfolio are -3.605 and -7.407, both of which are significant at the 0.01 level. The mean Sharpe Ratios on the three month portfolios are 0.1380 for the 5% portfolio, and 0.1059 for the 10% portfolio. The t-statistics on these differences are -12.303 and -16.958, both of which are significant at the 0.01 level.

These results suggest that strategies which involve the writing of naked puts which are out of the money-a strategy usually employed by those seeking premium income rather than with a desire to purchase the security underlying-provide an inferior risk/reward trade-off.

VII. Summary and Conclusions

This research is based upon a Monte Carlo simulation of equity prices incorporating known return and risk parameters to generate 20,000 equity price paths over the course of one year.

Using the known underlying parameters, put options on these hypothetical equities are valued. The options are then employed in three basic naked put writing strategies: at the money, in the money, and out of the money.

Table X Option Portfolio Returns: 6 Month 5% Out of the Money

Portfolio Number	Compound Return	Standard Deviation	Sharpe Ratio
1	6.771	10.390	0.170
2	7.141	11.417	0.188
3	7.312	10.931	0.212
4	6.878	10.753	0.175
5	6.579	9.936	0.159
6	6.919	11.734	0.164
7	6.417	11.283	0.126
8	6.546	10.828	0.143
9	6.550	10.474	0.148
10	6.995	10.335	0.193
11	6.721	10.222	0.168
12	6.863	9.676	0.193
13	7.640	10.584	0.249
14	6.473	10.450	0.141
15	6.924	10.323	0.186
16	6.899	10.963	0.173
17	6.538	10.974	0.140
18	6.878	10.959	0.171
19	6.894	10.520	0.180
20	6.624	10.747	0.151
	Mean		0.1715
	Standard Deviation		0.0276

Table XI Option Portfolio Returns: 6 Month 10% Out of the Money

Portfolio Number	Compound Return	Standard Deviation	Sharpe Ratio
1	6.178	8.086	0.146
2	6.363	9.284	0.147
3	6.434	8.085	0.177
4	6.431	8.434	0.170
5	6.341	8.063	0.166
6	6.181	8.970	0.132
7	6.011	9.108	0.111
8	6.197	8.202	0.146
9	5.996	8.455	0.118
10	6.204	8.145	0.148
11	6.121	7.966	0.141
12	6.331	7.404	0.180
13	6.791	7.854	0.228
14	6.101	8.360	0.132
15	6.315	7.999	0.164
16	6.023	8.346	0.123
17	5.855	8.358	0.102
18	6.053	8.024	0.131
19	6.143	8.053	0.142
20	6.211	8.436	0.144
	Mean		0.1473
	Standard Deviation		0.0276

Table XII Option Portfolio Returns: 3 Month 5% Out of the Money

Portfolio Number	Compound Return	Standard Deviation	Sharpe Ratio
1	6.827	12.127	0.151
2	6.409	11.563	0.122
3	6.803	11.467	0.157
4	6.704	12.418	0.137
5	6.377	11.267	0.122
6	6.923	12.566	0.153
7	6.611	13.368	0.121
8	6.280	11.824	0.108
9	6.478	11.683	0.126
10	6.614	11.597	0.139
11	6.199	11.632	0.103
12	6.598	10.247	0.156
13	6.885	11.253	0.168
14	6.372	11.933	0.115
15	7.016	12.153	0.166
16	7.130	12.884	0.165
17	6.620	11.288	0.143
18	6.729	11.497	0.150
19	6.256	11.171	0.112
20	6.716	11.835	0.145
	Mean		0.1380
	Standard Deviation		0.0199

Table XIII Option Portfolio Returns: 3 Month 10% Out of the Money

Portfolio Number	Compound Return	Standard Deviation	Sharpe Ratio
1	6.285	9.491	0.135
2	5.907	8.520	0.107
3	5.739	7.330	0.101
4	5.869	8.270	0.105
5	5.866	7.647	0.113
6	5.568	7.667	0.074
7	5.757	8.765	0.086
8	5.591	8.070	0.073
9	5.879	8.123	0.108
10	5.789	8.516	0.093
11	5.891	8.244	0.108
12	5.970	7.056	0.137
13	6.058	7.460	0.142
14	5.753	9.414	0.080
15	6.223	8.401	0.146
16	5.661	8.592	0.077
17	6.092	8.356	0.131
18	6.057	8.700	0.121
19	5.782	8.295	0.094
20	5.750	8.608	0.087
	Mean		0.1059
	Standard Deviation		0.0227

When the put strategy portfolios are compared to an all equity benchmark, it can be seen that the at the money returns offer little hope for improvement of portfolio performance. For both sets, the Sharpe Ratios suggest that there is modest deterioration in the return per unit of risk assumed. This deterioration is significant for the three month portfolios.

The out of the money results suggest that one may experience substantially inferior performance from such a strategy on a risk-adjusted basis. The evidence is the dramatic deterioration of the Sharpe Ratios under this strategy. It is concluded on this basis that there is little merit in the writing of naked out of the money put options.

Most interesting are the results for in the money strategies. The in the money returns are significantly superior, both in terms of excess over expected returns and the Sharpe Ratio. Moreover, the risk-adjusted return improvement on the six month expirations appears large enough to remain economically significant, even after allowing for additional transaction costs due to the use of options contracts.

The key conclusion of this paper is that it appears possible to significantly enhance portfolio returns utilizing in the money put options. This is true, however, only if the writer is willing to increase his or her overall equity exposure. The options were exercised approximately 53.3%, 63.5%, and 74.8% of the time for the six options, respectively. For a patient investor in the process of constructing an equity portfolio, the use of six month options provides a risk-adjusted pickup of approximately 59 to 104 basis points during the first year. This should be substantially in excess of transaction costs for investors able to trade even modest numbers of option contracts. An examination of the feasibility of such a strategy using actual option prices from the equity markets is part of our ongoing research.

Bibliography

Black, Fischer, and Myron Scholes, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy* 81, (1973), 637-59.

Boyle, Phelim, "Options: A Monte Carlo Approach," *Journal of Financial Economics* 4, (1977), 323-38.

Copeland, Thomas, and J. Fred Weston, *Financial Theory and Corporate Policy,* (Addison-Wesley, Reading, Massachusetts, USA), (1988).

Cox, John, Stephen Ross, and Mark Rubinstein, "Option Pricing: A Simplified Approach," *Journal of Financial Economics* 7, (1979), 229-64.

DeGroot, Morris, *Probability and Statistics,* (Addison-Wesley, Reading, Massachusetts, USA), (1986).

Efron, B., "Bootstrap Methods: Another look at the Jackknife," *The Annals of Statistics*, 7, (1979), 1-26.

Fama, Eugene, "The Behaviour of Stock Prices," *Journal of Business* 38, (1965), 34-105.

Fama, Eugene, "Efficient Capital Markets: A Review of Theory and Empirical Work," *Journal of Finance* 25, (1970), 383-417.

Geske, Robert and Kuldeep Shastri, "Valuation by Approximation: A Comparison of Alternative Option Valuation Techniques," *Journal of Financial and Quantitative Analysis* 20, (1985), 45-71.

Hull, John, *Options, Futures, and Other Derivative Securities*, (Prentice-Hall, Englewood Cliffs, New Jersey, USA), (1989).

McMillan, Lawrence, *Options as a Strategic Investment: A Comprehensive Analysis of Listed Stock Option Strategies*, (New York Institute of Finance, New York, New York, USA), (1980).

Sharpe, William, "Mutual Fund Performance," *Journal of Business* 39, (1966), 119-38.

Sharpe, William, "Capital Asset Prices: A theory of Market Equilibrium Under Conditions of Risk," *Journal of Finance* 19, (1964), 425-42.