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A SPECIFICATION TEST OF AT-THE-MONEY OPTION IMPLIED VOLATILITY: AN EMPIRICAL INVESTIGATION

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Abstract

In this study we conduct a specification test of at-the-money option volatility. Results show that the implied volatility estimate recovered from the Black-Scholes European option pricing model is nearly indistinguishable from the implied volatility estimate obtained from the Barone-Adesi and Whaley's American option pricing model. This study also investigates whether the use of Black-Scholes implied volatility estimates in American put pricing model significantly affect the prediction the prediction of American put option prices. Results show that, as long as the possibility of early exercise is carefully controlled in calculation of implied volatilities prediction of American put prices is not significantly distorted. This suggests that at-the-money option implied volatility estimates are robust across option pricing model.

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1. Introduction

Efficiency of at-the-money options in estimating implied volatilities has been well documented. Since the price of at-the-money option is most sensitive to volatility of underlying stocks, it should provide more information about the true stock return volatility than the price of options away-from-the-money. Beckers (1981) examined various weighting schemes in calculating implied volatilities and conducted that best estimates are obtained by using only at-the-money options. Macbeth and Merville (1979) derived implied volatilities from the Black-Scholes European call option pricing model. They found that implied volatilities for out-of-the-money call options are less than implied volatilities obtained from at-the-money call options while implied volatilities for in-the-money call options are greater than those for at-the-money call options. Assuming that at-the-money options are correctly priced by the Black-Scholes model, they concluded that in-the-money call options are underpriced and out-of-the-money call options are overpriced. Their results are contingent upon the validity of at-the-money option implied volatilities recovered from the Black-Scholes European option pricing model. Due to the Black-Scholes model's restrictive assumptions, these estimates of implied volatilities inverted from the Black-Scholes model are subject to biases resulting from various sources: 1) stochastic nature of stock return volatilities, 2) misspecification of the terminal stock price distribution, and 3) presence of early exercise possibilities.

Based on observed linear relationship between at-the-money option prices and stock return volatilities[1], however, it has been shown that most of the problems mentioned above can be avoided or minimized if at-the-money options are used in

1) For a graphical illustration, see pp278-280 of Cox and Rubinstein (1985)

estimating implied volatilities. Corrado and Miller (1994) extended Feinstein's (1989, 1992) argument that the Black-Scholes option pricing model can recover virtually unbiased stock return volatility estimates when volatility is stochastic. They show that other biases, in estimating implied volatilities, resulting from misspecification of stock price dynamics can alse be minimized if implied volatilities are obtained from at-the-money options. They also show that the linear relationship between at-the-money option prices and stock return volatilities is well preserved for American options and claim that Feinstein's argument applies to American options, suggesting that implied volatility estimates are nearly indistinguishable across option option pricing models for at-the-money options.

The purpose of this paper is to empirically test the robustness of implied volatility estimates obtained from at-the-money options to option pricing models. Two different volatility estimates are derived here : one estimate is recovered from the Black-Scholes option pricing model and the other estimate is inverted from an American put valuation model developed by MacMillan (1986) and Barone-Adesi and Whaley (1987). To minimize biases induced from not accounting for early exercise possibility when recovering implied volatilities from the Black-Scholes European option pricing model, European option prices implied from observed American put prices using put-call parity are used in calculating implied volatilities. We then test whether implied volatilities recovered from the Black-Scholes European model are significantly different from those derived from the MacMillan/Barone-Adesi and Whaley's American put pricing model. Results show that these two estimates of implied volatilities are not significantly different from each other, suggesting that Corrado and Miller's argument that implied volatility estimates are nearly indistinguishable across option pricing model whether it is European or American model for at-the-money option is correct. To further investigate whether the use of different implied volatility estimates affects pricing of American put options, theoretical prices based on the two different estimates of volatilities are compared against observed market prices. It is shown that theoretical prices based on the Black-Scholes implied volatilities fall outside observed dealers bid-ask spread boundaries slightly more than do theoretical prices based on the American model implied volatilities. However, statistical tests show that theoretical option prices based on the two different volatility estimates are not statistically different from each other.

Section 2 describes estimation methodology and the data. Section 3 discusses empirical results and the final section concludes.

2. Estimation methodology and the data

2.1 Implied volatilities estimation

We esimate implied volatilities using Whaley's (1982) non-linear regression procedure which allows option prices to provide an implicit weighting scheme that yields an estimates of standard deviation with minimum prediction error. Let $P_{j(\sigma)}$ denotes the theoretical price of an put option given an estimate σ of the stock return volatility. The observed market price of put option is denoted by P_j. The prediction error, ε_j, is defined as follows;

$$\varepsilon_j = P_j - P_j(\sigma) \tag{1}$$

The estimate of σ is then determined by minimizing the sum of squared errors,

$$\sigma_{Whaley} = \sigma^*: \underset{(\sigma^*)}{Min.} \sum_{j=1}^{N} (P_j - P_j(\sigma^*))^2 \tag{2}$$

where N is the number of at-the-money option prices in each day and σ^* is the estimated parameter.

A numerical search routine is designed to find the optimal σ^*. Initialization value is set at $\sigma_0 = 0.30$, and then the equation (2) is solved iteratively using a Taylor expansion of P_j around the initialization value, σ_0, ignoring the higher-order terms, that is,

$$P_j - P_j(\sigma_0) = (\sigma_1 - \sigma_0) \left|\frac{\partial P_j}{\partial \sigma}\right|_{\sigma_0} + v_j \tag{3}$$

Applying ordinary least squares regression technique to equation (3) until $|(\sigma_1 - \sigma_0)/\sigma_0| < 0.0001$ yields an estimate of the optimal σ^*.

Using the above mentioned procedure, two types of implied volatility estimates are derived in this paper. One implied volatility estimate is recovered from a specific American option pricing model using observed market price of American put option. That is, the estimate of

$$\sigma_{MBAW} = \sigma^*: \underset{(\sigma^*)}{Min.} \sum_{j=1}^{N} (P_j^{AM} - P_j^{MBAW}(\sigma^*))^2 \tag{4}$$

where P_j^{AM} is the observed market price of American put option and P_j^{MBAW} is the theoretical price generated by the MacMillan/Barone-Adesi and Whaley American put pricing model.

And, the other implied volatility estimate is derived from the Black-Scholes option pricing model using the market price of European put option implied from the put-call parity. That is, the estimate of σ_{BS} is determined by minimizing the sum of squared errors,

$$\sigma_{BS} = \sigma^{*}: \underset{(\sigma^{*})}{Min.} \sum_{j=1}^{N} (P_j^{EU} - P_j^{BS}(\sigma^{*}))^2 \tag{5}$$

where P_j^{EU} is the market price of European put option and P_j^{BS} is the theoretical Black-Scholes put price.

2.2 Estimation of theoretical prices

Once implied volatility estimates are obtained using the estimation technique described above, a theoretical price of an American put option is generated using the quadratic approximation approach developed by MacMillan (1986) and Barone-Adesi and Whaley (1987). They claim that, given that both American and European option prices satisfy the well-known Black-Scholes partial differential equation, it must be true that the difference in prices between an American option and an otherwise identical European option (i.e. the early exercise premium) must also satisfy the same partial differential equation. Using a quadratic approximation technique, a solution for an American put valuation formula is derived as follows:

$$P^{MBAW}(\sigma^{*}) = P^{BS}(\sigma^{*}) + A_1(S/S^{*})^{q1} \quad \text{where} \quad S > S^{*}, \text{ and} \tag{6}$$

$$P^{MBAW}(\sigma^{*}) = X - S \quad \text{where} \quad S < S^{*}.$$

where $P^{MBAW}(\sigma^{*})$ denotes the MacMillan/Barone-Adesi and Whaley American put price, given the volatility estimate of σ^*, $P^{BS}(\sigma^*)$ denotes the Black-Scholes European put price, given the volatility estimate of σ^*,

$$A_1 = -(S^*/q_1)[\ 1 - N\{d_1(S^*)\}]\ ,$$

$$q_1 = \frac{1}{2}[\ -(M-1) - \{(M-1)^2 + 4M/K\}^{1/2}]\ ,$$

$$M = 2r/\sigma,$$

$$K = 1 - e^{-r(T-t)}$$

and S^* is a critical stock price which is obtained by solving the following equation,

$$X - S^* = P^{BS}(S^*) - [\ 1 - N\{-d_1(S^*)\}]\ /q_1. \qquad (7)$$

To investigate whether the use of different implied volatility estimates affects pricing of American put options, Equation (6) is used to yield two different theoretical prices, given σ_{MBAW} and σ_{BS} , respectively.

2.3 The data

This study uses quotation data on the most heavily traded equity options on the Chicago Board of Trade Options Exchange (CBOE) from November 5 to November 30, 1990.[2][3] Real time price quotation data for IBM stock options are selected from the Berkeley Options Data Base[4]. The resorted format database is

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2) Among 30 most actively traded equity options during this period, 11 to 15 options were IBM options. Daily average trading volume for IBM options exceeds 10,000 contracts.

3) November 23 data are excluded from the sample due to extremely thin trading on that day which was the Friday immediately following the Thanksgiving holiday.

4) The Berkeley Options Data Base is derived from the Market Data Report of the CBOE. The database consists of bid-ask quotes and transaction data, time-stamped to the

used in this study.[5]

The market price of European put option, which is not directly observable from the market, is obtained from using put-call parity. Assuming that options markets are efficient in the sense that European put-call parity holds, and that investors are rational in the sense that holders of American call options do not prematurely exercise their call options when no dividends are to be paid until option maturity, the European put-call parity equation is reconstructed by replacing a European call with an American call:

$$C^A - P^E = S - Xe^{-rT} \tag{3.1}$$

where C^A is the value of an American call with a striking price of X and a maturity of T, and P^E is the value of an European put with a striking price of X and maturity T. By rearranging, we obtain the market value of an European put option:

$$P^E = C^A - S + Xe^{-rT} \tag{3.2}$$

To impute the market value of European put options, all put-call pairs that meet the following requirements are selected.

(1) Both put and call options in a put-call pair are options on the same underlying stock, with the same strike price and the same maturity.

(2) The length of time between put and call quotes for a put-call pair must be less than 2 minutes.

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nearest second.

5) The resorted data are sorted into files, one for each trading day. Within each file, the records are sorted by ticker symbol and by chronological order.

(3) Put and call prices are at least $1.00.[6]

(4) Put and call option prices within a put-call pair must satisfy the American put-call parity boundary condition.[7]

To isolate options on non-dividend paying stock from options on dividend paying stock, only options in a period where no dividends are paid before option maturity are selected.[8] The price of options used in this study are averages of bid and ask quotes. In any time interval during which the underlying stock price remains unchanged, the highest and lowest option prices are averaged[9], and all put-call pairs where stock prices are different, and put prices are unique and included. After the screening, 7,795 usable put-call pairs (daily average of 433 pairs) are identified. The average of bid and ask yield quotations on Treasury-bills that mature closest to option expiration are used to estimate a risk free rate of interest. Daily data on annualized T-bill rates are obtained from the Wall Street Journal.

3. Test hypothesis and results

Each day, implied volatilities for January contracts and December contracts are estimated separately using equation (4) and (5). Table1 presents the results of the

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6) Thinness in these options may result in unreasonable estimate due to the discreetness of the price change.

7) By filtering a sample based on this criterion, we avoide a joint test of market efficiency and model accuracy.

8) Since IBM stock went ex-dividend on November 5,1990 and February 5,1991, December and January option contracts traded during November 5 to November 30,1990 are selected.

9) In an efficient market, no option prices are to be changed during a short time interval where underlying stock prices remain unchanged.

Black-Scholes implied volatility estimates and the MBAW implied volatility estimates for IBM stocks during November,1990. It is shown that the MBAW implied volatilities are generally greater than the Black-Scholes implied volatilities during the first half of the month while the Black-Scholes implied volatilities are slightly greater than the MBAW implied volatilities during the rest of the month.

3.1 Test of difference between the MBAW implied volatility and the Black-Scholes implied volatility

To investigate whether the two implied volatility estimates are statistically different from each other, we test the following null hypotheses that the mean value of the MBAW implied volatilities are equal to the mean value of the Black-Scholes implied volatilities:

$$H_0 : \quad \sigma_{MBAW} = \sigma_{BS}$$

Rejection of the null would imply that the MBAW implied volatilities are statistically significantly different from the Black-Scholes implied volatilities. The result reported in table 2 shows that, although the mean value of the Black-Scholes implied volatilities is slightly higher than the mean value of the MBAW implied volatilities, the two implied volatility estimates are not significantly different from each other, suggesting that Corrado and Miller's (1994) argument that implied volatility estimates are nearly indistinguishable across option pricing model whether it is European or American model for at-the-money options is correct. This might also suggest that option prices generated from the Black-Scholes European option pricing formula and the MBAW American option pricing model provide the same information about the future stock return volatility.

3.2 Test of difference between theoretical option prices given σ_{MBAW} and σ_{BS}

Equation (6) is used to yield two different theoretical prices, given σ_{MBAW} and σ_{BS}, respectively. Table 3 presents the observed market price and the theoretical prices, given the two volatility estimates, for all options, for in-the-money options, for at-the-money options, and out-of-the-money options. Both of theoretical prices overvalue in-the-money options and undervalue out-of-the-money options. The degree of mispricing is slightly greater for the theoretical prices based on the Black-Scholes implied volatility than for the theoretical prices based on the MBAW implied volatility. To examine whether the two theoretical price predictions are indistinguishable from each other, we test the null hypothesis that the mean value of theoretical prices generated from using the Black-Scholes implied volatility is equal to the mean value of theoretical prices based on the MBAW implied volatility. The results in Table 4 shows that, although the mean value ($2.85) of theoretical prices based on the Black-Scholes implied volatility is slightly greater than the mean value ($2.84) of theoretical prices based on the MBAW implied volatility, both price predictions, on average, are not statistically different from each other.

To further investigate whether the use of different implied volatility estimates affects pricing of American put options, theoretical prices based on the two different estimates of volatilities are compared against observed bid and ask quotes. Table 5 shows that proportions of theoretical prices based on each of the two implied volatility estimates outside dealers bid-ask spread boundaries are nearly indistinguishable (79.6% vs 79.8%) for in-the-money options, although, for in-the-money options, theoretical prices based on the American model implied volatility fall outside the observed bid-ask spread boundaries slightly more (97.2%) than do theoretical prices based on the Black-Scholes implied volatility (94.6%). These results suggest that predictions of American put pricing are not significantly affected by the estimation of implied volatility whether the volatility estimate is recovered from the Black-Scholes European option pricing model or from a specific American put pricing model.

4. Conclusion

We statistically test the robustness of implied volatility estimates across option pricing models for at-the-money put options. The results of the specification tests show that the implied volatility estimate recovered from the Black-Scholes European option pricing model is nearly indistinguishable from the implied volatility estimate obtained from the MacMillan/Barone-Adesi and Whaley's American put pricing model. We also investigate whether the use of Black-Scholes implied volatility estimates in American put pricing model significantly affect the prediction of American put option prices. It is shown that, as long as the possibility of early exercise are carefully controlled in calculation of implied volatilities, predictions of American put prices are not significantly distorted when the black-Scholes implied volatility estimates are used in American put option pricing model.

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Table 1

Daily Implied Volatilities (σ) of IBM stocks during 11/05/90 – 11/30/90*

σ_{MBAW} is implied volatility inverted from MBAW American model using market prices of American put options.
σ_{BS} is implied volatility inverted from Black-Scholes European model using market prices of European put options.

Trade date	Option maturity	Number of obs.	σ_{MBAW}	σ_{BS}
November 5,1990	Dec. '90	224	0.2612	0.2566
	Jan. "91	98	0.2670	0.2718
November 5,1990	Dec. '90	224	0.2612	0.2566
	Jan. "91	98	0.2670	0.2718
November 5,1990	Dec. '90	224	0.2612	0.2566
	Jan. "91	98	0.2670	0.2718
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	Jan. "91	98	0.2670	0.2718
November 5,1990	Dec. '90	224	0.2612	0.2566
	Jan. "91	98	0.2670	0.2718
November 5,1990	Dec. '90	224	0.2612	0.2566
	Jan. "91	98	0.2670	0.2718
November 5,1990	Dec. '90	224	0.2612	0.2566
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	Jan. "91	98	0.2670	0.2718
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	Jan. "91	98	0.2670	0.2718
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	Jan. "91	98	0.2670	0.2718
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	Jan. "91	98	0.2670	0.2718
November 5,1990	Dec. '90	224	0.2612	0.2566
	Jan. "91	98	0.2670	0.2718

November 23 data were excluded from the sample due to extremely thin trading during that day which was Friday after the Thanksgiving holiday.

Table 2

Results of T-Tests That Mean Values of σ_{MBAW} and σ_{BS} are equal

σ_{MBAW} is implied volatility inverted from MBAW American model using market prices of American put options.

σ_{BS} is implied volatility inverted from Black-Scholes European model using market prices of European put options.

T-Test	σtype	N	Mean	Std. Dev	t-value	Prob.>|t|
$H_0: \sigma_{MBAW} = \sigma_{BS}$	σ_{MBAW}	36	0.2262	0.0188	- 0.1678	0.8672
$H_0: \sigma_{MBAW} = \sigma_{BS}$	σ_{BS}	36	0.2269	0.0171		

Table 3
Mean Value of MBAW theoretical prices of IBM put options 11/05/90 – 11/30/90*

$P(\sigma_{MBAW})$ is a theoretical price generated from MBAW American model given σ_{MBAW}. $P(\sigma_{BS})$ is a theoretical price generated from MBAW American model given σ_{BS}. P_{obs} is an observed market price of American put options.

Moneyness	N	P_{obs}($)	$P(\sigma_{MBAW})$($)	$P(\sigma_{BS})$($)	$P(IV_{BS}^{EU})$($)
All Option	7795	2.9610	2.8374	2.9007	2.8518
In-the-money Option	1351	5.1238	5.3427	5.4021	5.3716
At-the-money Option	2864	3.2307	3.2306	3.2944	3.2617
Out-of-the-money Options	3580	1.9289	1.5774	1.6418	1.5729

σ_{MBAW} is implied volatility inverted from MBAW American model using market prices of American put options.
σ_{BS} is implied volatility inverted from Black-Scholes European model using market prices of European put options.

Table 4

Results of T-Tests That Mean Values of theoretical prices are equal

$P(\sigma_{MBAW})$ is a theoretical price generated from MBAW American model given σ_{MBAW}.

$P(\sigma_{BS})$ is a theoretical price generated from MBAW American model given σ_{BS}

T-Test	Model Price	N	Mean($)	Std. Dev	t-value	Prob.>|t|
$H_0: \sigma_{MBAW} = \sigma_{BS}$	$P(\sigma_{MBAW})$	7795	2.8374	1.6545	- 0.5452	0.5856
$H_0: \sigma_{MBAW} = \sigma_{BS}$	$P(\sigma_{BS})$	7795	2.8518	1.6511		

Table 5

Mean Value of MBAW theoretical prices outside bid-ask dealer spread boundaries.

$P(\sigma_{MBAW})$ is a theoretical price generated from MBAW American model given σ_{MBAW}.

$P(\sigma_{BS})$ is a theoretical price generated from MBAW American model given σ_{BS}

P^{bid}_{obs} is an observed bid price of American put options.

P^{ask}_{obs} is an observed ask price of American put options.

Moneyness	P^{bid}_{obs}	P^{ask}_{obs}	Proportion of $P(\sigma_{MBAW})$			Proportion of $P(\sigma_{BS})$		
			N	%	Ave.Dev.($)	N	%	Ave.Dev.($)
All Option	2.8972	3.0247	6056	77.7	0.1041	6365	81.7	0.0781
In-the-money Option	5.0189	5.2288	1076	79.6	0.1625	1078	79.8	0.2021
At-the-money Option	3.1625	3.2990	1502	52.4	0.0	1899	66.3	0.0
Out-of-the-money Options	1.8843	1.9735	3478	97.2	0.3177	3388	94.6	0.3327

σ_{MBAW} is implied volatility inverted from MBAW American model using market prices of American put options.

σ_{BS} is implied volatility inverted from Black-Scholes European model using market prices of European put options.