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# COMPARATIVE TESTS OF TVD SCHEMES FOR MHD

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We develop numerical codes for astrophysical magnetohydrodynamics (MHD) using several total variation diminishing (TVD) schemes: Harten's upwind TVD scheme (UTVD), Yee's symmetric TVD scheme (STVD), and Van Leer's MUSCL approach. Our codes are based on the extension of the scalar TVD schemes using the local characteristic approximation and the construction of the numerical flux at the cell interface using the approximate Riemann solver. Our main interest is the performance of the TVD schemes in the case of using a simple averaging scheme for the Roe's approximate Riemann solver (there is no consistent analytic form of the averaged quantities in MHD with $\gamma \neq 2$ for the linearized Riemann problem (Brio & Wu 1988)).

The updating of the conservative variables $\mathbf{q}$ at the cell center using the numerical flux $\mathbf{f}^*$ at the cell boundaries is done as follows (see Yee (1989) and Ryu & Jones (1995) for details):

$$\mathbf{q}_i^{n+1} = \mathbf{q}_i^n - \lambda[\mathbf{f}^*_{i+1/2} - \mathbf{f}^*_{i-1/2}] , \tag{1}$$

for UTVD and STVD

$$\mathbf{f}^*_{i+1/2} = 0.5[\mathbf{f}(\mathbf{q}_i^n) + \mathbf{f}(\mathbf{q}_{i+1}^n) + \sum_{k=1}^{7} \beta_{k,i+1/2}\mathbf{r}_{k,i+1/2}] , \tag{2}$$

$$\beta_{k,i+1/2} = \begin{cases} \sigma(a_{k,i+1/2})(g_{k,i+1} + g_{k,i}) - Q(a_{k,i+1/2} + \gamma_{k,i+1/2})\alpha_{k,i+1/2} & \text{(UTVD)} \\ -\lambda g_{k,i+1/2} - Q(a_{k,i+1/2})[\alpha_{k,i+1/2} - g_{k,i+1/2}] & \text{(STVD)} , \end{cases} \tag{3}$$

$$\alpha_{k,i+1/2} = \mathbf{l}_{k,i+1/2} \cdot (\mathbf{q}_{i+1}^n - \mathbf{q}_i^n) , \tag{4}$$

$$\sigma(z) = 0.5[Q(z) - \lambda z^2] , \tag{5}$$

$$Q(z) = \begin{cases} \frac{z^2}{4\epsilon} + \epsilon , & \text{for } |z| < 2\epsilon \\ |z| , & \text{for } |z| \geq 2\epsilon , \end{cases} \tag{6}$$

$$\gamma_{k,i+1/2} = \sigma(a_{k,i+1/2}) \begin{cases} (g_{k,i+1} - g_{k,i})/\alpha_{k,i+1/2} , & \text{for } \alpha_{k,i+1/2} \neq 0 \\ 0 , & \text{for } \alpha_{k,i+1/2} = 0 , \end{cases} \tag{7}$$

$$g_{k,i+1/2} = \begin{cases} \text{minmod}(\alpha_{k,i-1/2}, \alpha_{k,i+1/2}) & \text{(UTVD)} \\ \text{minmod}(\alpha_{k,i-1/2}, \alpha_{k,i+1/2}) + \text{minmod}(\alpha_{k,i+1/2}, \alpha_{k,i+3/2}) - \alpha_{k,i+1/2} & \text{(STVD)} , \end{cases} \tag{8}$$

$$\text{minmod}(x, y) = \text{sign}(x)\max[0, \min(|x|, y\,\text{sign}(x))] , \quad \text{and,} \tag{9}$$

for the MUSCL

$$\mathbf{f}^*_{i+1/2} = 0.5[\mathbf{f}(\mathbf{q}^R_{i+1/2}) + \mathbf{f}(\mathbf{q}^L_{i+1/2}) + \sum_{k=1}^{7} \beta_{k,i+1/2}\mathbf{r}_{k,i+1/2}] , \tag{10}$$

$$\beta_{k,i+1/2} = -Q(a_{k,i+1/2})\alpha_{k,i+1/2} \tag{11}$$

$$\alpha_{k,i+1/2} = \mathbf{l}_{k,i+1/2} \cdot (\mathbf{q}^R_{i+1/2} - \mathbf{q}^L_{i+1/2}) , \tag{12}$$

$$\mathbf{q}^R_{i+1/2} = \mathbf{q}^n_{i+1} - 0.5\text{minmod}[(\mathbf{q}_{i+1} - \mathbf{q}_i), (\mathbf{q}_{i+2} - \mathbf{q}_{i+1})] \tag{13}$$

$$\mathbf{q}^L_{i+1/2} = \mathbf{q}^n_i + 0.5\text{minmod}[(\mathbf{q}_i - \mathbf{q}_{i-1}), (\mathbf{q}_{i+1} - \mathbf{q}_i)] . \tag{14}$$

We do several tests in 1D and 2D for resolution at the discontinuities, convergence, and robustness.

Fig. 1 shows the results of the Brio and Wu's (1988) MHD shock tube test for each TVD schemes. As can be seen in the figure, the UTVD and the MUSCL give better resolution at the discontinuities than the STVD. The MUSCL, however, produce small oscillations near the fast rarefaction wave. From our numerical test we find that the UTVD is more stable (runs on higher CFL number) than other two TVD methods.

Table 1 shows the convergence test for the UTVD following the method introduced in Stone et al. (1992). We get a similar results to the results of the ZEUS code used in Stone et al. (1992). Smaller in integrated errors with

Table 1. MHD Shock Tube Density Errors

| Number of Zones | Integrated Error* | Convergence Rate |
|---|---|---|
| 64 | 3.9369E-03 | ... |
| 128 | 1.9608E-03 | 1.01 |
| 256 | 9.7847E-04 | 1.01 |
| 512 | 4.8876E-04 | 1.01 |
| 1024 | 2.4426E-04 | 1.01 |

*The errors are computed from the difference between two numerical solutions which computed with twice difference in resolution.

the coarse grid than the result of Stone et al. (1992), however, indicates a better resolution at the discontinuities of the TVD methods than the method used in the ZEUS code.

To test the multidimension performance, we run 2D shock tube test and MHD explosion problem. The results reveals a robustness of TVD schemes such as keeping a good symmetry even with the directional splitting and the good handling of the strong shock waves. We find, however, that the TVD-MHD schemes using the simple averaging can produce negative pressure in some test problems.

## REFERENCES

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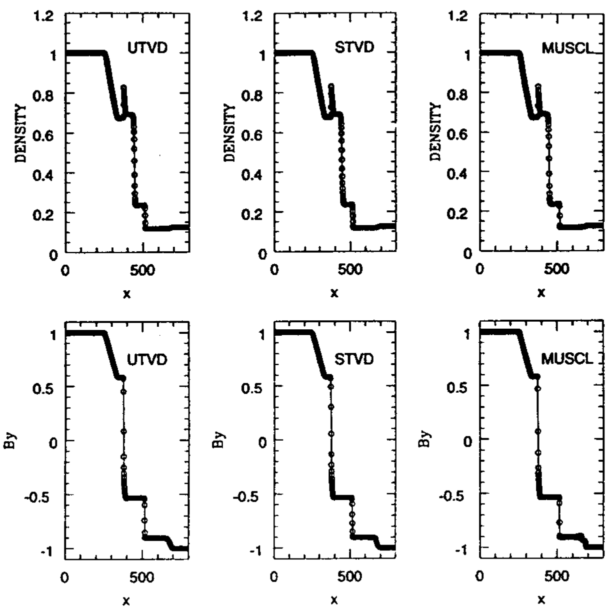


**Fig. 1.**— Brio and Wu's (1988) MHD shock tube test for each TVD schemes.