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MAGNETIC FIELDS IN STARS AND DISKS

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ABSTRACT

Magnetic fields are thought to play a role in a wide variety of important astrophysical processes, from angular momentum transport and jet formation in accretion disks to corona formation in stars. Unfortunately, the dynamics of magnetic fields in astrophysical plasmas are extremely complicated, and the success of current theoretical models and computer simulations seems to be inversely correlated with the amount of observational detail available to us. Here I will discuss some of the more striking conflicts between numerical simulations and observations, and present an explanation for them based on an important dynamical process which is not adequately modeled in current numerical simulations. These processes will lead to the formation of flux tubes in stars and accretion disks, in accordance with observations. I will discuss some of the implications of flux tube formation for stellar and accretion disk dynamos.

Key Words : magnetic fields, turbulence, accretion

I. INTRODUCTION

Magnetic fields are seen in a wide variety of astrophysical objects and are commonly assumed to be present, at some level, almost everywhere. In distant objects their presence can be inferred from their effect on spectral features, or from synchrotron emission. In the Sun we can see a wealth of detailed structure which defies any simple explanation. These fields are not necessarily dynamically significant. For example, it would be hard to argue that the magnetic field of the Sun plays a major role in its structure or evolution. On the other hand, it does play a major role in the Sun's coronal structure. Moreover, magnetically fields are suspected of playing a major role in the dynamics of the interstellar medium and accretion disks. Given all this, it is somewhat disappointing that the dynamics of magnetized fluids are so complicated, even if we restrict ourselves to the sort of qualitative models that are so dear to astrophysicists.

In dense environments, like stars and accretion disks, the particle collision rate is high and the magnetized plasma can be described by the MHD approximation. Consequently, one possible solution is to simulate MHD dynamics numerically. In fact, in recent years there has been an explosion of interest in this approach as the available computing power has reached the point where purely hydrodynamical simulations can match laboratory simulations in considerable detail. Nevertheless, although such simulations have proven extremely in suggesting new approaches and stimulating further work, they have failed to achieve a reasonable degree of consistency with observations. For example, simulations of convection in a rotating magnetized fluid, meant as a model for the solar dynamo, have shown convincingly that this situation can lead to a dynamo (see e.g. Nordlund et al. 1992). However, the final results remain extremely sensitive to the numerical resolution of the simulation. Worse, the dynamo process seems to saturate when the magnetic energy density is less than %10 of the turbulent energy density associated with convection. In the Sun this ratio is observed to be close to one. Finally, there are grounds for believing that increased resolution will actually decrease this ratio (e.g. Cattaneo and Vainshtein 1991, see also Vishniac 1995). A somewhat different problem is encountered in simulations of magnetic field growth in shearing fluids (e.g. Hawley, Gammie, and Balbus 1995), meant as a model for magnetic field generation and angular momentum transport in accretion disks. There the magnetic field saturates with an energy almost ten times the turbulent energy density, which may or may not be realistic, but the actual level of angular momentum transport remains sensitive to numerical resolution, even at grid sizes sufficient to resolve transport coefficients in hydrodynamic turbulence. Also, such simulations typically give a dimensionless viscosity which is insensitive to the geometry of the computational box, whereas all available observational results indicate that real accretion disks have a dimensionless viscosity given by

$$\alpha \approx 50 \left(\frac{c_s}{r\Omega(r)}\right)^{1.5}, \qquad (1)$$

where c_s is the sound speed near the disk midplane, and $\Omega(r)$ is the orbital frequency (Cannizzo, Chen and Livio 1995, Vishniac and Wheeler 1996).

Here I will briefly discuss two effects tend to complicate attempts to build realistic numerical MHD simulations and give a qualitative sense of where they fit into the study of astrophysical MHD. The first is the repulsion and attraction of magnetic filaments previously discussed in Vishniac (1995). The second is a dynamo effect that may be particularly important in accretion disk simulations, but will have a considerably smaller role in real accretion disks (Vishniac and Brandenburg 1996).

II. FORCES BETWEEN MAGNETIC FILAMENTS

Observations of the solar photosphere suggest that magnetic fields tend to form filaments, a notion that led Parker (1979) to examine the hydrodynamic forces between two flux tubes. Since flux tubes are extended objects they will tend to move separately from the fluid around them, and two neighboring flux tubes will feel each other's wakes. Any obstruction in a flow will create a broadening wake of reduced velocity and drag. Consequently, when one flux tube is upstream from another they experience a kind of mock gravity in which shielding creates an attractive force. It is less obvious what forces exist between flux tubes that are arranged so that their separation is perpendicular to the ambient flow. When the flow is laminar then the increase in fluid velocity between the two leads to an attraction due to the Bernoulli effect. However, it turns out that when the flow is turbulent the two filaments will repel each other (Zdravkovich 1977). In other words, when the Reynolds number *on the scale of the filaments* is not very large, magnetic filaments always attract each other, aggregating into structures which will resist external hydrodynamic forces. When the Reynolds number on this scale is large (roughly speaking, larger than about π^3) magnetic filaments will be dispersed throughout the turbulent cell, arranged so that on the average their mutual forces are in equilibrium. The implication from this is that MHD simulations will be qualitatively misleading unless the Reynolds number on the scale of the filament radius is large. In contrast, purely hydrodynamic simulations will be qualitatively correct when the Reynolds number on the scale of the largest eddies is large, a criterion which is much easier to satisfy.

What determines flux tube properties? We begin by assuming viscosity is unimportant, an assumption which will be reasonable in stars and accretion disks, but which fails for computer simulations and possibly for the interstellar medium. Any individual flux tube of radius r_t and internal magnetic field B_t will interact with the surrounding fluid flow through turbulent drag. The radius of curvature of the field lines is obtained by balancing tension and drag. This implies

$$\left[\frac{B_t^2}{4\pi}\right](\pi r_t^2)\frac{4}{l} = C_d \rho V_l^2 r_t, \tag{2}$$

where l is the wavelength corresponding to a particular scale in the turbulent cascade, C_d is the coefficient of turbulent drag, and ρ is the density of the ambient medium. Flux tubes will are advected by turbulence on scales larger than l and will be rigid with respect to turbulence on smaller scales. Very crudely speaking, this implies that

$$V_{At} \sim V_l \left(\frac{l}{r_t}\right)^{1/2}, \tag{3}$$

where V_{At} is the Alfvén velocity associated with the tube (that is, using the magnetic field strength inside the tube and the density outside). In general, we expect that V_{At} will be much greater than the turbulent velocities in the surrounding fluid. We do expect equipartition between the field and the fluid turbulence, but only in the sense that the average magnetic energy density, which includes the small filling factor of the flux tubes, will be comparable to the average turbulent energy density in a saturated dynamo.

Balancing the average attraction and repulsion between flux tubes can be accomplished by requiring that the 'covering factor' f for a bundle of N flux tubes in a volume of radius r is roughly one, or

$$f \equiv N r_t / r \approx 1. \tag{4}$$

This implies that the flux tubes will be arranged in a fractal distribution of dimension one, like a plane. This equilibrium will be dynamic, not static, and individual flux tubes will be constantly shifting their position within the clustering hierarchy. The lower limit of the hierarchy is set by the flux tube radius, r_t. The upper limit is the curvature scale l. In this limit, when viscosity is unimportant and the magnetic field is in statistical equilibrium with its surroundings, flux tubes are not isolated objects. Ironically, the most striking observational confirmation of the concept of flux tubes comes when one of these conditions is violated and we can view a flux tube in isolation.

The physical meaning of the scale l can be recovered from these results. A turbulent cell of size l contains approximately l/r_t flux tubes. The total magnetic energy contained in these flux tubes is approximately

$$N B_t^2 r_t^2 l \sim B_t^2 l^2 r_t \sim \rho V_l^2 l^3. \tag{5}$$

The scale l is the scale of equipartition between the magnetic and turbulent energies.

The flux tubes will be in pressure equilibrium with the ambient thermal pressure or $P_{int} + B^2/8\pi = P_{ext}$. If the temperature of the gas is roughly constant, which will be true for stars and disks, but not for the interstellar medium, then this implies a density deficit in the flux tube. This deficit will be maintained by turbulent pumping. A flux tube in a turbulent medium will be stretched at a rate $\sim V_l/l$. In a stationary state the total length of flux tubes will be constant and the stretching of flux tubes will be balanced by the production of closed loops through twisting and self-intersection. These closed loops will tend to shrink through internal tension until they reach the limit where ohmic diffusion will dissolve them. In equilibrium turbulent pumping will be balanced by ohmic diffusion which puts matter into flux tubes at a rate $\sim \eta/r_t^2$. This implies a flux tube radius of

$$r_t \approx \left(\frac{\eta l}{V_l}\right)^{1/2}. \tag{6}$$

However, this will underestimate flux tube radii when flux tubes become largely empty. In this limit pressure equilibrium gives $B_t = (8\pi P)^{1/2}$ and

$$r_t \approx \left(\frac{V_l}{c_s}\right)^2 l. \tag{7}$$

We have derived this model with intense magnetic filaments arranged in a fractal distribution on the basis of hydrodynamic interactions between flux tubes. However, the same results can be obtained by considering a limited amount of magnetic flux passing through a turbulent cell and arranging it so as to maximize the dissipation. A uniform magnetic field inhibits turbulent dissipation by replacing eddies with weakly dissipative Alfvén waves (Kraichnan 1965). Individual flux tubes enhance dissipation through the creation of turbulent wakes. This process acts with maximum efficiency when the flux tubes are just marginally stiff with respect to the surrounding flow, when the available flux yields the maximum possible magnetic energy through its concentration into flux tubes, and when the flux tubes are arranged so that the covering factor of flux tubes across a turbulent cell is approximately one. We conclude that a fractal distribution of flux tubes constitutes a dissipative structure, an ordered state which maximizes the production of entropy. This conclusion also suggests that eddies with no long flux tubes will contain a set of closed loops which will facilitate dissipation within those eddies. However, since most of space is empty, the main difference from a hydrodynamic turbulent cascade will be to steepen the spectrum slightly. Magnetic fields do not inhibit the turbulent cascade of energy except possibly when they they dominate the kinetic energy by a significant factor.

The formation of flux tubes implies that the magnetic field can move relative to the fluid, although with significant drag. This validates the usual assumption of turbulent diffusivity in mean field dynamo theory. It also implies that the advection of magnetic fields is not necessarily very efficient. Accretion disks cannot acquire very large poloidal fields merely through radial infall. This is especially true for radiation pressure dominated disks where the outward buoyancy of the field lines is enhanced (Park and Vishniac 1996). Unfortunately, this also implies that numerical simulations are unable to achieve realistic magnetic field dynamics unless they are three dimensional and have large hydrodynamic Reynolds numbers on the filament scale. That is

$$\frac{\nu}{\eta} \gtrsim \frac{\pi^6}{(V_T/k_T\nu)}\left(\frac{E_T}{E_B}\right)^2, \tag{8}$$

where k_T is the wavenumber of the largest turbulent eddies. Even assuming equipartition, this criterion cannot be met by the current generation of numerical simulations.

III. THE INCOHERENT DYNAMO

The preceding section explained why turbulent MHD simulations fail to converge at resolutions sufficient to capture the broad features of hydrodynamic turbulence. However, this is not a complete explanation for what is seen in simulations of magnetically induced turbulence in accretion disks. There the magnetic field energy grows at a rate Ω, which is the eddy turnover rate associated with the Balbus-Hawley instability. Eventually this growth ends without erasing all memory of the initial conditions. On longer time scales, comparable to a few tens of orbital periods, the simulations show a regeneration of the large scale field, which then persists indefinitely, albeit with occasional field reversals. The magnetic field energy saturates at some modest fraction of the thermal pressure and the associated dimensionless viscosity is a bit less than 1%. The initial growth can be explained as the result of the evolution of a uniform, volume filling field, into an intermittent and disordered one. The saturation values of field energy and α are dependent on the degree of resolution and require no fundamental explanation. However, the existence of a local dynamo is troubling. These simulations do not show the kind of symmetry breaking normally required for dynamo activity. Moreover, the existence of a purely local dynamo would seem to be inconsistent with observational evidence for a viscosity which depends on global parameters.

The resolution to this lies in the fact that the volume average helicity is nonzero at any one time, even if the long term average is strictly zero. Normally the radial component of the magnetic field is driven from the azimuthal component by the $\theta\theta$ component of the fluid helicity, which expresses the tendency of the field lines to twist in a spatially coherent pattern. The equation for the evolution of B_r is

$$\partial_t B_r = -\partial_z(\alpha_{\theta\theta}B_\theta) + \nabla D\nabla B_r, \tag{9}$$

where D is the turbulent diffusion coefficient. Even if $\alpha_{\theta\theta}$ has a long term average of zero, at any one time we expect it to have an rms value of roughly $V_T/N^{1/2}$, where V_T is the eddy velocity and N is the number of eddies in a magnetic domain. Consequently, given a large scale azimuthal field B_θ, the radial field will grow in a random walk. The Balbus-Hawley instability has an eddy turnover rate of about Ω and an eddy size $\sim V_A/\Omega$. For an azimuthally symmetric magnetic domain of radial and vertical width $\sim h$ this implies

$$\partial_t B_r^2 \approx B_\theta^2 \frac{V_A^3}{rh^2\Omega^2}. \tag{10}$$

Since B_r drives B_θ through shearing at a rate of $(3/2)\Omega$ this gives a dynamo growth rate of approximately

$$\tau_{dynamo}^{-1} \sim \left(\frac{V_A}{c_s}\right)^{5/3}\left(\frac{h}{r}\right)^{1/3}\Omega. \tag{11}$$

Since the turbulent damping rate and buoyancy loss rate both scale as $(V_A/c_s)^2\Omega$, this implies a saturation magnetic pressure, and dimensionless viscosity, which scales as $(h/r)^2$. In this way the global dimensions of the disk enter into a purely local process. Current simulations involve just a few eddies, at most, in the azimuthal direction, which leads to an overestimate of the importance of this process.

IV. CONCLUSION

The two mechanisms described here may both be important in stars and disks. They both present difficulties for attempts to model turbulent dynamos with insufficient resolution or without a full consideration of the geometry of the modeled object.

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